16 Replacement Decisions

After this chapter you will be able to:

1. distinguish the defender from the challenger, and keep sunk costs out of the comparison
2. compute the defender's investment as its current market value MV, using the outsider viewpoint
3. choose between defender and challenger by comparing their annual equivalent costs AEC
4. compute the economic service life N^* that minimizes $AEC(N) = CR(i) +$ operating AE
5. compute the marginal cost of one more year of the defender, and decide in which year to replace it
6. convert a replacement comparison to an after-tax basis, using the after-tax market value and the depreciation shield $t\,D$

Assets age. Trucks burn more fuel, pumps need more repairs, crushers spend more days in the workshop. The old machine still works, so the question every operations engineer eventually faces is not *whether* to replace it but *when*. This chapter gives that question a vocabulary and an arithmetic. The asset we own and operate now is the *defender*; the best replacement available on the market is the *challenger*. The chapter first clears away the numbers that do not belong in the comparison, the sunk costs, and prices the defender correctly through the opportunity-cost viewpoint. It then compares defender and challenger by annual equivalent cost, finds the *economic service life* at which an asset is cheapest to own, and the marginal rule that says exactly when to replace. Two closing pieces extend the framing: taxes restate every cash flow after tax but change none of the logic, and a published mining study shows the economic-service-life table at work on a real drill rig. Everything here runs on the machinery of chapter 6: capital recovery plus operating cost, stated per year. What is new is the framing. The running example is an ore-transfer truck at *Najd Copper*, the concentrate producer of Chapter 5.

Which cost still counts today.

Q1. Najd Copper bought a truck three years ago for 85,000 SAR. It would sell today for 28,000 SAR. The firm is deciding whether to keep it. Which of the two numbers belongs in the analysis, and why?

Q2. A machine costs 50,000 SAR, will be resold for 20,000 SAR after 3 years, and $i = 10\%$. What is its capital recovery cost per year?

Q3. A machine is fully paid off. Is keeping it for another year free?

Answers.

Q1. Only the 28,000 SAR. The 85,000 SAR is spent whatever the firm decides; it is the same in both futures, keep or replace, so it cannot separate them. The 28,000 SAR is a real amount the firm gives up by keeping the truck, so it is the cost of the keep option today.

Q2. From chapter 6, $CR(10\%) = (50{,}000 - 20{,}000)(A/P, 10\%, 3) + 0.10\,(20{,}000)$, which is $30{,}000\,(0.40211) + 2{,}000 = 14{,}063$ SAR per year. This capital-plus-interest chain is the engine of every computation in this chapter.

Q3. *No.* Keeping it means not collecting its market value, so the capital parked inside it earns no interest; the machine also loses resale value over the year and consumes operating costs. ''Paid off'' describes the past; the cost of one more year is made of future flows only.

Lecture 24. This section is covered by Lecture 24.

Reference. Park, 3rd ed., Sections 12.1–12.3 (pp. 530–550).

Defender. The asset the firm owns and operates now.

Challenger. The best replacement asset available on the market.

16.1 Defender, Challenger, and Sunk Costs

The vocabulary comes from boxing. The *defender* is the asset we own and operate now; the *challenger* is the best replacement available on the market. The contest between them is a comparison of mutually exclusive alternatives, and both deliver the same required service, the ore gets hauled either way, so these are *service* *projects* in the sense of Chapter 5: compare annual equivalent costs, and the least cost wins.

Sunk cost. Money already spent. It is the same in every future course of action, so it can never separate the alternatives.

Before any arithmetic, the books must be cleared of numbers that do not belong. A *sunk cost* is money already spent: it is the same in both futures, keep or replace, so it cannot separate them. Only *future* cash flows count.

Example 16.1 — Four numbers on one truck

Najd Copper's mine-site ore-transfer truck carries four numbers. Decide which of them are relevant to the keep-or-replace decision.

	amount (SAR)	relevant?
purchase price, 3 years ago	85,000	
engine repair, last year	6,000	
book value on the ledger	34,000	
market value today	28,000	

Solution.

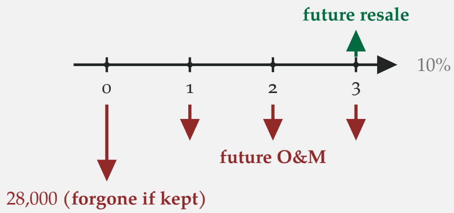

Figure 16.1. What the keep option looks like once the books are cleared: only future flows remain, and the only year-0 amount is the market value the firm gives up by keeping the truck.

Take the four numbers in order.

	amount (SAR)	verdict
purchase price, 3 years ago	85,000	**no** — sunk
engine repair, last year	6,000	**no** — sunk
book value on the ledger	34,000	**no** — accounting, not cash
market value today	28,000	**yes** — a real, forgone amount

Table 16.1. Clearing the books for the ore-transfer truck. Three of the four numbers never enter the analysis.

The purchase price is spent whatever the firm decides: sunk. The engine repair is sunk for the same reason. The book value is an accounting figure from the depreciation schedule of chapter 10, not a cash flow; nobody will pay the firm 34,000 SAR because the ledger says so. The only relevant number is the last one: the truck *would sell for* $\boxed{\text{28,000 SAR}}$ today, a real amount the firm gives up by keeping it. What the defender *cost* is history; what it *would sell for today* is the price of keeping it.

Insight. The pull of sunk costs is psychological. Having spent 85,000 SAR feels like a reason to keep going, "otherwise the money is wasted." It is not a reason. The money is gone in both futures, keep or replace, so it cannot separate them. Exam questions plant a purchase price or a past repair in the problem statement precisely to test whether you will leave it out.

Outsider viewpoint. Treat the defender's current market value as its investment. Keeping the asset means forgoing that cash, and giving up cash is the same sacrifice as spending it.

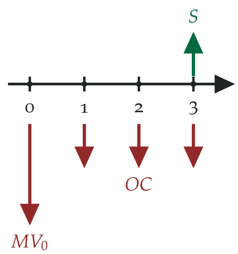

Figure 16.2. The defender through an outsider's eyes: its market value is the investment at year 0, followed by operating costs and a final resale.

In practice. A codebase a company owns is a defender with a market value near zero: keeping it forgoes almost nothing, so it only has to justify its maintenance cost. That is why a full rewrite, which must earn back its whole build cost, loses to the patched legacy system more often than engineers expect.

16.2 *The Opportunity-Cost (Outsider) Viewpoint*

The viewpoint that makes the comparison fair is called the *opportunity-cost* or *outsider* viewpoint: treat the defender's current market value as its investment. Keeping the truck means choosing *not* to collect 28,000 SAR today, and giving up cash is the same sacrifice as spending it. The test question is: *what would an outsider pay to step into our position?* The outsider owns nothing; to get the defender's service, they would have to buy the truck at its market value. So that is the defender's price tag, 28,000 SAR, not 85,000 SAR and not zero. This puts defender and challenger on equal footing: each option starts with its own investment at $n = 0$.

Example 16.2 — The defender's AEC from the outsider viewpoint

Keeping Najd Copper's ore-transfer truck for 3 more years involves the following estimates, at $i = 10\%$: market value now 28,000 SAR; operating costs 14,000, 16,000, and 18,000 SAR in years 1–3; resale value 10,000 SAR at the end of year 3. Find the defender's annual equivalent cost.

Solution.

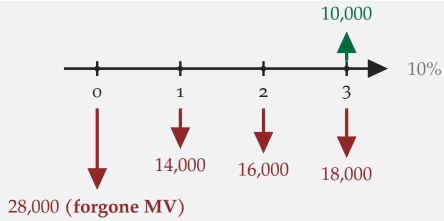

Figure 16.3. The defender as an outsider sees it: the market value is the year-0 investment, the operating costs rise with age, and the truck is still worth 10,000 SAR at the end.

This is the capital-plus-operating chain of Section 6.4, with one change only: the investment is the market value, not a purchase price.

Step 1: the capital.

With $(A/P, 10\%, 3) = 0.40211$:

$$\begin{aligned} CR(10\%) &= (28{,}000 - 10{,}000)\,(A/P,\ 10\%,\ 3) + 0.10\,(10{,}000) \\ &= 18{,}000\,(0.40211) + 1{,}000 = 8{,}238 \text{ SAR per year.} \end{aligned}$$

Step 2: the rising operating costs, through present worth.

$$\begin{aligned} P_{\text{opr}} &= 14{,}000\,(0.9091) + 16{,}000\,(0.8264) + 18{,}000\,(0.7513) \\ &= 12{,}727.40 + 13{,}222.40 + 13{,}523.40 = 39{,}473.20 \text{ SAR,} \end{aligned}$$

$$AE_{\text{opr}} = 39{,}473.20\,(0.40211) = 15{,}873 \text{ SAR per year.}$$

Step 3: add.

$$AEC_D = 8{,}238 + 15{,}873 = \boxed{24{,}111 \text{ SAR per year}}.$$

Keeping the truck costs about 24,100 SAR per year over the next three years. Please note where the 28,000 SAR entered and where the 85,000 SAR did not.

16.3 *The Challenger, and the Verdict*

The challenger goes through exactly the same chain, with its own price and its own estimates.

Example 16.3 — The challenger's AEC and the comparison

The challenger is a new ore-transfer truck: price 72,000 SAR; operating costs 8,000, 11,000, and 15,000 SAR in years 1–3; resale 33,000 SAR after 3 years; $i = 10\%$. Find the challenger's annual equivalent cost and state the keep-or-replace verdict.

Solution.

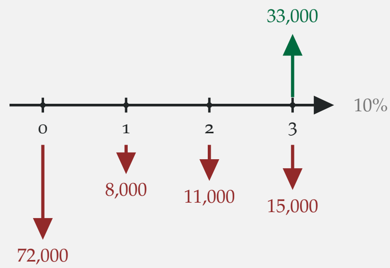

Figure 16.4. The challenger: a large purchase, lower operating costs, and a strong resale value after three years.

Step 1: the capital.

$$\begin{aligned} CR(10\%) &= (72{,}000 - 33{,}000)\,(0.40211) + 0.10\,(33{,}000) \\ &= 15{,}682.29 + 3{,}300 = 18{,}982.29 \text{ SAR per year.} \end{aligned}$$

Step 2: the operating costs.

$$\begin{aligned} P_{\text{opr}} &= 8{,}000\,(0.9091) + 11{,}000\,(0.8264) + 15{,}000\,(0.7513) \\ &= 7{,}272.80 + 9{,}090.40 + 11{,}269.50 = 27{,}632.70 \text{ SAR,} \end{aligned}$$

$$AE_{\text{opr}} = 27{,}632.70\,(0.40211) = 11{,}111.38 \text{ SAR per year.}$$

Step 3: add, and compare.

$$AEC_C = 18{,}982.29 + 11{,}111.38 = 30{,}093.67 \approx \boxed{30{,}094 \text{ SAR per year}}.$$

Compare: $AEC_D = 24{,}111 < AEC_C = 30{,}094$, so **keep the defender**, for now. Notice what decided it. The new truck is cheaper to *operate* by a wide margin (11,111 against 15,873 SAR per year), but its capital recovery cost is more than double the defender's (18,982 against 8,238 SAR per year). A new asset must earn back a large price; an old one only has to justify its market value.

Insight. The textbook also shows a *cash-flow* approach, in which the defender's sale proceeds are credited against the challenger's price instead of charged to the defender. With equal comparison periods the two approaches give the same verdict, because both options shift by the same amount at year 0. This book uses the opportunity-cost view because it also works when the lives differ, which is the usual case.

16.4 *The Economic Service Life*

Before deciding *when* to replace, a sharper question must be answered: how long should an asset be kept at all? A fifty-year-old truck can be kept running, but almost nobody does, because ownership has two opposing cost currents as the keep-period N grows.

- The **capital recovery cost** $CR(i) = (I - S_N)(A/P, i, N) + i\,S_N$ **falls** with N: the purchase price is spread over more years.
- The **annual equivalent of operating costs rises** with N: maintenance, fuel, and downtime climb as the asset ages.

Economic service life. The keep-period N^* that minimizes the total annual equivalent cost $AEC(N)$ of owning and operating an asset.

Their sum, the total annual equivalent cost as a function of N,

$$AEC(N) = \underbrace{(I - S_N)(A/P,\, i,\, N) + i\,S_N}_{\text{falls with } N} + \underbrace{\left(\textstyle\sum_{n=1}^{N} OC_n\,(P/F,\, i,\, n)\right)(A/P,\, i,\, N)}_{\text{rises with } N}, \tag{16.1}$$

falls at first and rises later, with a minimum in between. The N that achieves the minimum is the *economic service life* N^*. Keep the asset that long, and each year of service costs as little as possible. Please note that both S_N (the market value if sold at age N) and OC_n (the operating cost during year n) are year-by-year estimates, so the minimization is done by building a table.

In practice. The OC_n column is engineering data before it is money: deterioration curves, failure rates, and the maintenance log supply the estimates. The minimum moves when a failure mode changes: a gearbox that begins failing at year 4 steepens the operating column and pulls N^* earlier, so the table is rebuilt when the maintenance history changes, not on a fixed schedule.

Example 16.4 — The challenger's economic service life

For the challenger truck of Example 16.3 ($I = 72{,}000$ SAR, $i = 10\%$), the market value and operating cost evolve by year as follows:

keep for N (years)	1	2	3	4
market value S_N (SAR)	54,000	42,000	33,000	26,000
operating cost OC_N (SAR)	8,000	11,000	15,000	20,000

Build the $AEC(N)$ table for $N = 1, \ldots, 4$ and find the economic service life.

Solution.

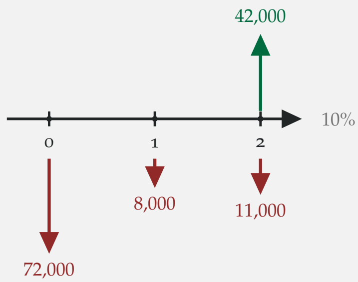

Figure 16.5. One row of the table: the "keep for $N = 2$ years" policy. Each row of the $AEC(N)$ table prices a diagram like this one, with its own final resale value.

Work row by row; each row uses Equation (16.1) with its own $(A/P, 10\%, N)$.

$N = 1$:

$(A/P, 10\%, 1) = 1.10$. The first-year operating cost is already an annual amount.

$$CR = (72{,}000 - 54{,}000)(1.10) + 0.10\,(54{,}000) = 19{,}800 + 5{,}400 = 25{,}200,$$

$$AEC(1) = 25{,}200 + 8{,}000 = 33{,}200 \text{ SAR per year.}$$

$N = 2$:

$(A/P, 10\%, 2) = 0.57619$.

$$CR = 30{,}000\,(0.57619) + 4{,}200 = 21{,}485.70,$$

$$AE_{\text{opr}} = \left[8{,}000\,(0.9091) + 11{,}000\,(0.8264)\right](0.57619)$$
$$= 16{,}363.20\,(0.57619) = 9{,}428.30,$$

so $AEC(2) = 30{,}914.00$ SAR per year.

$N = 3$ and $N = 4$:

the 3-year row is Example 16.3; the 4-year row repeats the chain with $(A/P, 10\%, 4) = 0.31547$. Collecting all four rows:

keep N	S_N	OC_N	$CR(10\%)$	op. AE	$AEC(N)$	
1	54,000	8,000	25,200.0	8,000.0	33,200.0	
2	42,000	11,000	21,485.7	9,428.3	30,914.0	
3	33,000	15,000	18,982.3	11,111.4	**30,093.7**	$\leftarrow N^*$
4	26,000	20,000	17,111.6	13,026.6	30,138.2	

Table 16.2. The challenger's economic-service-life table at 10%. The capital column falls, the operating column rises, and their sum bottoms out at $N = 3$.

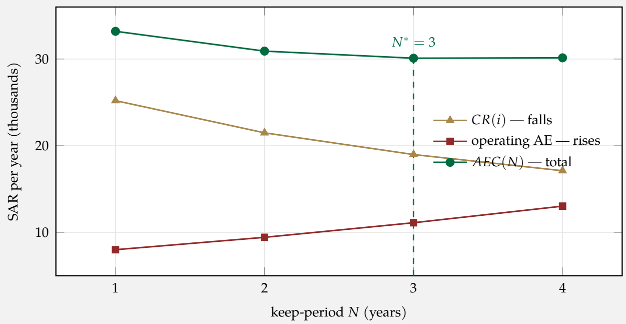

Figure 16.6. $AEC(N)$ for the challenger truck: falling capital recovery meets rising operating cost, and the total bottoms out at $N^* = 3$ years.

Scan the last column of Table 16.2: it falls, bottoms out, and turns up. The economic service life is $\boxed{N^* = 3 \text{ years, with } AEC^* = 30{,}094 \text{ SAR per year}}$. This minimum is the challenger's *best possible annual cost*, the benchmark that the defender will have to beat in the next section.

Insight. Two readings of Figure 16.6. First, the bottom is flat: 30,094 at $N = 3$ against 30,138 at $N = 4$, a difference of only about 45 SAR per year. Being a year off the economic life usually costs little, so a near tie is not worth worrying about; what matters for the replacement decision is the *value* at the bottom. Second, two special cases mark the ends of the pattern. If the market value stayed equal to the price while operating costs rise, the minimum would sit at $N = 1$: replace as fast as possible. If operating costs were flat while the market value keeps falling, $AEC(N)$ would keep falling: hold the asset as long as possible.

Marginal cost of one more year. $MC = MV_{\text{now}}(1+i) - MV_{\text{in 1 yr}} + OC_{\text{next yr}}$: the cost of keeping the defender for exactly one more year, built from future flows only.

16.5 Replace Now or Later: The Marginal Rule

The trigger question is: replace now, or later? The tool is the *marginal cost* of keeping the defender one more year, and it has three pieces, all of them future flows:

$$MC_{1\text{ yr}} = \underbrace{MV_{\text{now}}\,(1+i)}_{\text{forgone sale, with interest}} - \underbrace{MV_{\text{in 1 yr}}}_{\text{recovered at year-end}} + \underbrace{OC_{\text{next yr}}}_{\text{operations}}. \tag{16.2}$$

Keeping the truck means not collecting its market value now, so that amount is charged, carried one year at interest. At the end of the year the truck can still be sold, so its year-end market value is credited back. And the year's operating cost is paid. The decision rule is short:

- **keep** the defender another year if $MC_{1\text{ yr}} < AEC_C^*$, the challenger's minimum annual cost;
- **replace** in the first year whose marginal cost exceeds AEC_C^*.

Example 16.5 — One more year of the defender

For Najd Copper's current truck: market value now 28,000 SAR, market value in one year 22,000 SAR, operating cost next year 14,000 SAR, at $i = 10\%$. Compute the marginal cost of one more year and compare it with the challenger's benchmark from Example 16.4.

Solution.

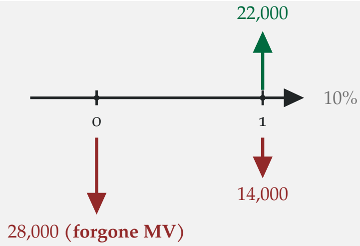

Figure 16.7. One more year of the defender: forgo the sale now, recover the year-end market value, pay the year's operating cost. The marginal cost states all three at the end of the year.

Stack the three pieces of Equation (16.2):

$$\begin{aligned} MC &= 28{,}000\,(1.10) - 22{,}000 + 14{,}000 \\ &= 30{,}800 - 22{,}000 + 14{,}000 = \boxed{22{,}800 \text{ SAR}}. \end{aligned}$$

Compare: $MC = 22{,}800 < AEC_C^* = 30{,}094$, so keeping the truck one more year is cheaper than switching to the challenger at its best. **Keep the defender this year**, and re-run the check next year with fresh market values.

Insight. The first piece of Equation (16.2) causes the most errors. Keeping the truck is not free even if it is fully paid off, because the 28,000 SAR locked inside it could be earning 10% somewhere else. The charge $MV_{\text{now}}(1 + i)$ is the outsider viewpoint compressed into a single year.

''Later'' can be made precise: project the marginal cost forward year by year, using the defender's expected market values, and find the first year it crosses the challenger's benchmark.

Example 16.6 — Year by year: when does the verdict flip?

The defender's market value is expected to fall along the path 28,000 $\to$ 22,000 $\to$ 15,500 $\to$ 10,000 SAR over the next three years, with operating costs 14,000, 16,000, and 18,000 SAR. Project the marginal cost of each year and decide when Najd Copper should replace the truck ($AEC_C^* = 30{,}094$ SAR per year, $i = 10\%$).

Solution.

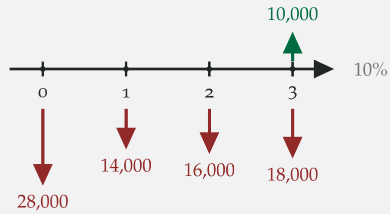

Figure 16.8. The defender's remaining three years, with the market value falling $28 \to 22 \to 15.5 \to 10$ thousand SAR. Each year of the table below charges the opening market value with interest, credits the closing one, and adds that year's operating cost.

Each row applies Equation (16.2) to one year:

keep year	$MV\,(1.10)$	$-\,MV_{\text{end}}$	$+\,OC$	marginal cost
1	30,800.00	22,000	14,000	22,800.00
2	24,200.00	15,500	16,000	24,700.00
3	17,050.00	10,000	18,000	25,050.00

Table 16.3. The one-more-year ledger for the defender. The marginal cost climbs each year, as it should for an aging truck, but never reaches the challenger's benchmark of 30,094 SAR per year.

Every year's marginal cost stays below $AEC_C^* = 30{,}094$ SAR per year, so the answer to "now or later?" is $\boxed{\text{later}}$: keep the truck to the end of its remaining life, then buy the challenger and run it on its 3-year rhythm. Two remarks. First, the defender's own economic service life is *not* automatically the replacement moment; the comparison against the challenger decides the timing. Second, the plan is provisional: market values and challenger prices move, so the check is re-run every year.

Reference. Park, 3rd ed., Section 12.4 (pp. 552–559).

After-tax market value. The net proceeds $MV - t\,(MV - BV)$ of selling the defender today: the cash the firm actually forgoes by keeping it.

16.6 After-Tax Replacement

Everything so far was computed before taxes. Taxes change the numbers in three places and the framing in none. All three changes come straight from chapter 10.

1. **The defender's price tag becomes its after-tax market value.** Selling the defender today triggers the disposal rules of Section 10.11: a sale above book value pays tax on the gain, a sale below book value earns a credit on the loss, and the cash the seller keeps is the net proceeds of Equation (10.7), $MV - t\,(MV - BV)$. Under the outsider viewpoint, *that* is what keeping the defender forgoes, so the

net proceeds, not the raw market value, become the defender's investment at year 0. The book value BV comes from the depreciation schedule of chapter 10. This is the one place where the book value, dismissed as "accounting, not cash" in Example 16.1, re-enters the analysis: it moves cash through the tax on disposal.

2. **Operating costs shrink by the tax rate.** O&M is tax-deductible, so a cost of OC per year costs only $OC\,(1-t)$ after tax.

3. **Each option carries its own depreciation shield.** Whichever machine is owned keeps depreciating, and each year's charge saves $t \times D$ in tax (Section 10.10.1). A freshly purchased challenger usually carries larger shields than a defender whose schedule is nearly written off, and this is the challenger's quiet tax advantage.

The comparison itself is unchanged: outsider viewpoint, annual equivalent costs, least cost wins, discounted at an *after-tax* MARR.

Example 16.7 — After-tax defender versus challenger at Wadi Phosphate

Wadi Phosphate's apron feeder at the primary crusher (the defender) was bought three years ago for 60,000 SAR and is being depreciated straight-line over 6 years toward a zero salvage estimate, so the annual charge is 10,000 SAR and the book value today is 30,000 SAR. Its market value today is 22,000 SAR. Kept for 3 more years, it costs 12,000 SAR per year to run and will sell for 6,000 SAR at the end (book value then: zero). The challenger costs 48,000 SAR, is depreciated straight-line over 3 years toward a 12,000 SAR salvage estimate (annual charge 12,000 SAR), costs 6,000 SAR per year to run, and will sell for 12,000 SAR after 3 years. The tax rate is $t = 20\%$ and the after-tax MARR is 10%. Compare the two after tax, and against the same comparison before tax.

Solution.

Step 1: the defender's after-tax price tag.

Selling today at 22,000 SAR, below the 30,000 SAR book value, realizes a loss of 8,000 SAR and a tax credit of $0.20\,(8{,}000) = 1{,}600$ SAR:

$$\text{net proceeds} = 22{,}000 - 0.20\,(22{,}000 - 30{,}000) = 22{,}000 + 1{,}600 = 23{,}600\,\text{SAR}.$$

Keeping the feeder forgoes 23,600 SAR, *more* than its market value: by not selling, the firm also gives up the loss credit.

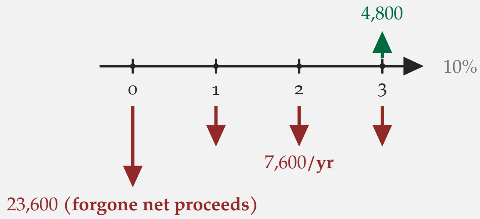

Figure 16.9. The defender after tax: the year-0 investment is the net proceeds forgone, the annual outflow is the after-tax O&M net of the depreciation shield, and the final recovery is the after-tax resale.

Step 2: the defender's after-tax annual flows.

The O&M costs $12{,}000\,(1-0.20) = 9{,}600$ SAR after tax; the remaining three years of depreciation each shield $0.20\,(10{,}000) = 2{,}000$ SAR. Net annual outflow: $9{,}600 - 2{,}000 = 7{,}600$ SAR. The year-3 sale at 6,000 SAR against a zero book value pays $0.20\,(6{,}000) = 1{,}200$ SAR of gain tax, leaving net proceeds of 4,800 SAR.

Step 3: the defender's after-tax AEC.

With $(A/P, 10\%, 3) = 0.40211$:

$$CR = (23{,}600 - 4{,}800)\,(0.40211) + 0.10\,(4{,}800) = 7{,}559.67 + 480 = 8{,}039.67,$$
$$AEC_D^{\mathrm{AT}} = 8{,}039.67 + 7{,}600 = \boxed{15{,}640 \text{ SAR per year}}.$$

Step 4: the challenger after tax.

Its O&M costs $6{,}000\,(0.80) = 4{,}800$ SAR; its shield is $0.20\,(12{,}000) = 2{,}400$ SAR per year; net annual outflow 2,400 SAR. The year-3 resale forecast of 12,000 SAR happens to equal the book value, so the sale is tax-free and the net proceeds are 12,000 SAR.

$$\begin{aligned} CR &= (48{,}000 - 12{,}000)\,(0.40211) + 0.10\,(12{,}000) \\ &= 14{,}475.96 + 1{,}200 = 15{,}675.96, \\ AEC_C^{\mathrm{AT}} &= 15{,}675.96 + 2{,}400 = \boxed{18{,}076 \text{ SAR per year}}. \end{aligned}$$

Step 5: compare, both ways.

Before tax, the same estimates give $AEC_D = (22{,}000 - 6{,}000)(0.40211) + 600 + 12{,}000 = 19{,}034$ SAR per year against $AEC_C = 15{,}675.96 + 6{,}000 = 21{,}676$ SAR per year.

AEC (SAR/yr)	before tax	after tax
defender	19,034	15,640
challenger	21,676	18,076
margin for keeping	2,642	2,436

Table 16.4. The same replacement comparison before and after tax. The verdict survives; the margin tightens.

Keep the defender on either basis. The after-tax margin, 2,436 SAR per year, is about 8% thinner than the before-tax margin of 2,642 SAR per year: the tax authority softened the defender's advantage but did not overturn it.

Insight. The pre-tax verdict usually survives the after-tax pass, but the margins tighten. Taxes scale both options' operating costs by $(1 - t)$ and hand both a depreciation shield, so most of the effect cancels in the comparison. What does not cancel is the disposal tax on the gap between the defender's market and book values, and the difference between a fresh depreciation schedule and a nearly exhausted one. When that gap is wide, or the challenger enjoys sharply accelerated depreciation (Section 10.7), the after-tax pass can flip a close pre-tax verdict, and that is exactly when it is mandatory.

16.7 Unequal Lives: AE under Repeatability

Repeatability. The assumptions that the service is needed indefinitely and that each asset is replaced by an identical one at the end of its life, so its AEC repeats as a level annual stream.

One loose end remains. We compared the defender's cost over its remaining 3 years against the challenger's cost at its own economic life. Here both happened to be 3 years, but usually they differ: an old machine may have 2 years left while the new one is best kept for 5. Why is comparing the two annual costs still fair, with no common horizon in sight?

Two assumptions do the work, together called *repeatability*:

1. the service is needed **indefinitely**: the hauling does not stop when a truck dies;

2. each asset is **replaced by an identical one** at the end of its life, so its annual equivalent cost repeats as a level stream, year after year.

Under these assumptions, each option is a flat annual cost stretching into the future, and flat annual costs are directly comparable whatever the cycle lengths are. This is exactly the one-cycle argument of Section 6.10: no least common multiple is needed. Compare the defender's AEC over its remaining years with the challenger's AEC at its economic life, and pick the smaller. When repeatability is unreasonable, for example when the mine itself closes in 4 years, a stated study period overrides everything, as in Chapter 5.

16.8 *Case Study from the Literature*

The $AEC(N)$ table of Section 16.4 is not only a classroom device. Mining companies build the same minimization from their maintenance databases and publish the results. This section reads one such study with the chapter's eyes: an open-access economic replacement time model for an underground production drill rig (Al-Chalabi, 2022), built on the same 10% discount rate this chapter uses.

Example 16.8 — Case study: the economic replacement time of a mine drill rig

Al-Chalabi (2022) develops a replacement model for an Epiroc Simba production drill rig used underground in Tara Mine, Ireland, from 42 months of cost records (April 2016 to September 2019) in the mine's computerized maintenance management system; the costs are disguised as ''currency units'' (cu) for confidentiality. The model finds the replacement age that minimizes the discounted total ownership cost, purchase price plus operating and maintenance costs, over a 120-month planning horizon at a 10% discount rate, assuming each rig is replaced by an identical one. Headline results: the economic replacement time is 81 months, with a 6-month replacement range (months 79–84) over which the total cost is almost constant; a regression on the sensitivity runs gives, in months,

$$ERT = 81.95 + 0.19\,IFPP + 0.06\,DFOC + 0.27\,DFMC,$$

where *IFPP*, *DFOC*, and *DFMC* are the percentage increase in purchase price and the percentage decreases in operating and maintenance cost for a new rig model. (a) Map the model onto this chapter's machinery. (b) Convert the results to operating hours using the paper's utilization data. (c) Read the regression coefficients as a sensitivity ranking.

Solution.

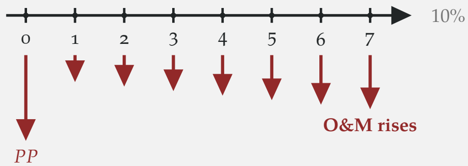

Figure 16.10. One ownership cycle of the drill rig as the paper models it: the purchase price, then operating and maintenance costs that climb with age, with no resale value, repeated identically cycle after cycle. Each candidate replacement age prices one such cycle.

Step (a): the model is this chapter's, with two local choices.

Minimizing the discounted ownership cost per identical cycle is the economic-service-life minimization of Equation (16.1), and the identical-replacement assumption is the repeatability of Section 16.7 stated as a modeling premise. Two local choices differ from the truck examples. The model carries no resale value, so the falling current is purely the purchase price spreading over more months, and the maintenance costs come month by month from the mine's work-order records rather than from a forecast, which is the engineering-judgment point of Section 16.4 in practice: the OC_n column is condition data before it is money. The result has the same shape as Figure 16.6: total cost falls, bottoms out at $N^* = 81$ months (about 7 years), and turns up. The 6-month replacement range is the flat bottom of Example 16.4's insight reported as a formal result: near the minimum, being a few months early or late costs almost nothing, so the mine plans a replacement *window*, not a date.

Step (b): from months to operating hours.

The paper reports 12,121 operating hours over the 42 recorded months:

$$\frac{12{,}121}{42} = 288.6 \approx 289\,\text{hours per month}, \qquad 81 \times 288.6 = \boxed{23{,}376\text{ operating hours}},$$

matching the paper's reported figure exactly, and the replacement window of months 79–84 converts to $79 \times 288.6 \approx 22{,}800$ through $84 \times 288.6 \approx 24{,}242$ hours (the paper reports 22,800–24,250). Please note the unit discipline: the calendar answer is what the budget needs; the hour answer is what the maintenance planner needs.

Step (c): the coefficients are a tornado.

Read the regression the way Example 12.3 reads a swing table: each coefficient is the per-percent effect of one input on the decision variable.

factor (1% change)	months of ERT	rank
maintenance cost (-1%)	$+0.27$	1
purchase price ($+1\%$)	$+0.19$	2
operating cost (-1%)	$+0.06$	3

Table 16.5. The regression coefficients of Al-Chalabi (2022) read as a sensitivity ranking: months of economic replacement time per one-percent change in each input.

Maintenance cost dominates, exactly the paper's stated ranking, and the directions match the two currents of Equation (16.1): a dearer rig is kept longer because a larger purchase price takes more months to spread thin, and a cheaper-to-maintain rig is kept longer because the rising current flattens. The paper's own worked case, a new rig model 10% dearer to buy, 7% cheaper to operate, and 10% cheaper to maintain, gives

$$ERT = 81.95 + 0.19\,(10) + 0.06\,(7) + 0.27\,(10) = \boxed{87 \text{ months}},$$

matching the paper's 86.9. The ranking directs the effort, as in chapter 12: the biggest lever on the rig's economic life is held by the maintenance department, not the purchasing department.

16.9 *Chapter Summary*

- The **defender** is the asset the firm owns; the **challenger** is the best replacement on the market. Both deliver the same service, so the comparison is by **annual equivalent cost**, and the least cost wins.
- **Sunk costs never enter.** The purchase price and past repairs are the same in both futures; the book value is accounting, not cash. Only future cash flows count.
- The **outsider (opportunity-cost) viewpoint** prices the defender at its **current**

market value: keeping the asset means forgoing that cash, and forgoing cash is the same sacrifice as spending it.

- The **economic service life** N^* minimizes $AEC(N) = CR(i) +$ operating AE, where the capital recovery cost falls with N and the operating AE rises with N. The minimum AEC^* of the challenger is the benchmark for the replacement decision; the bottom of the curve is usually flat.

- The **marginal cost of one more year** of the defender is $MV_{\text{now}}(1+i) - MV_{\text{in 1 yr}} + OC_{\text{next yr}}$, all future flows. Keep the defender while this stays below the challenger's AEC^*; **replace in the first year it crosses**. Re-run the check every year, because market values and challenger prices move.

- With **unequal lives**, the AE comparison is fair under **repeatability**: the service continues indefinitely and each asset repeats identically, so each AEC is a level annual stream (Section 6.10). A stated study period overrides repeatability.

- **After-tax replacement** keeps the framing and restates the cash flows with the tools of chapter 10: the defender's investment becomes its **after-tax market value** (the net proceeds $MV - t\,(MV - BV)$), operating costs cost $(1-t)$ on the SAR, and each option carries its own depreciation shield $t\,D$. The pre-tax verdict usually survives, with a tighter margin; a wide market-to-book gap or an accelerated challenger schedule is what can flip it.

16.10 Exercises

These problems run the chapter's three instruments in order: the outsider viewpoint that prices the defender at its market value, the $AEC(N)$ table that locates the economic service life, and the marginal cost of one more year. Later items reverse the question, asking for the operating cost at which a keep-or-replace verdict flips.

Warm-up and lecture practice

Exercise 16.1. A defender's market value is 9,000 SAR now and 6,500 SAR in one year; its operating cost next year is 4,000 SAR; $i = 10\%$. Find the marginal cost of keeping it one more year.

Exercise 16.2. Najd Copper's tailings pump is the defender, at $i = 10\%$: market value now 13,000 SAR; market value in one year 10,000 SAR; operating cost next year 7,500 SAR. The best challenger's minimum annual equivalent cost is $AEC^* = 11{,}000$ SAR per year at its economic life. (a) Compute the defender's marginal cost of one more year. (b) Keep or replace now? One decision sentence. (c) At what next-year operating cost would the verdict flip?

Quick checks (multiple choice)

Exercise 16.3. A machine bought four years ago for 120,000 SAR has a book value of 40,000 SAR and could be sold today for 22,000 SAR. Under the opportunity-cost (outsider) viewpoint, the defender's investment is taken as:

(A) 0 SAR
(B) 22,000 SAR
(C) 40,000 SAR
(D) 120,000 SAR

Exercise 16.4. Which statement about the marginal (one-more-year) rule is **true**?

(A) The defender should always be kept to the end of its physical life.
(B) Keep the defender another year if the cost of that year, the forgone market value with interest, plus the operating cost, minus the year-end market value, is below the challenger's AEC^*.
(C) The defender must be replaced as soon as a newer challenger appears on the market.
(D) The defender's own economic service life alone fixes the replacement date.

Exercise 16.5. Najd Copper is deciding whether to replace a conveyor drive. Which of the following numbers is *relevant* to the decision?

(A) The 48,000 SAR paid for the drive five years ago.
(B) The 7,000 SAR overhaul completed last year.
(C) The drive's 18,000 SAR book value on the ledger.
(D) The 11,000 SAR a dealer would pay for the drive today.

Replace the truck? (in-class activity)

Ore haulage at *Najd Copper* has doubled since the mine opened a satellite pit, and one mine-site ore-transfer truck is showing its age. The company bought it three years ago for 65,000 SAR. Today the defender's market value is 20,000 SAR, its operating cost is 9,000 SAR per year, and it is usable for 3 more years, after which it is worth 5,000 SAR. The challenger costs 50,000 SAR, with market values and operating costs by keep-period given in Exercise 16.7. The MARR is 10%. The 65,000 SAR purchase price is sunk and must not appear anywhere in the analysis. The factors used in this activity are $(A/P, 10\%, n) = 1.1000$, 0.5762, 0.4021, 0.3155 and $(P/F, 10\%, n) = 0.9091, 0.8264, 0.7513, 0.6830$ for $n = 1, \ldots, 4$. (The activity closes with a break-even question on the defender's resale value, worked as Exercise 12.6 in chapter 12.)

Exercise 16.6. The defender's AEC (outsider viewpoint). (i) Draw the defender's cash flow diagram as an outsider would see it. (ii) Compute the defender's annual equivalent cost.

Exercise 16.7. The challenger's economic service life. The challenger costs $I = 50{,}000$ SAR, with this forecast:

keep for n (years)	1	2	3	4
market value S_n (SAR)	36,000	27,000	20,000	14,000
operating cost in year n (SAR)	5,000	7,000	10,000	15,000

Most of the AEC table is done; fill the four missing cells and find the minimum:

keep for n	$CR(n)$	annualized op. cost AOC(n)	$AEC(n)$
1	19,000.0	5,000.0	24,000.0
2	15,952.6	5,952.3	21,904.9
3	14,063.0	?	?
4	?	8,861.9	?

Exercise 16.8. The decision. Compare AEC_D from Exercise 16.6 with AEC^* from Exercise 16.7 and write the decision sentence.

Exam-level practice

Exercise 16.9. Replace Wadi Phosphate's rock-breaker? The defender's market value is 18,000 SAR now and 13,000 SAR in one year, and its operating cost next year is 9,500 SAR. The challenger costs 56,000 SAR, with the estimates below; $i = 10\%$.

keep for n (years)	1	2	3
market value S_n (SAR)	42,000	33,000	26,000
op. cost in year n (SAR)	4,800	6,200	8,000

(a) Build the challenger's economic-service-life table, $CR(n)$, annualized operating cost, and $AEC(n)$ for $n = 1, 2, 3$, and state N^* and AEC^*. (b) Compute the defender's marginal cost of one more year. (c) Keep or replace now? One decision sentence comparing the two numbers. (d) At what next-year defender operating cost would the verdict flip?

Exercise 16.10. The front-end loader's own economic life. Wadi Phosphate's front-end loader (the defender) has a market value of 26,000 SAR today. If kept, its market value and operating cost evolve as follows, at $i = 10\%$:

end of year	1	2	3
market value if sold then (SAR)	19,000	13,500	9,500
operating cost during the year (SAR)	8,500	10,500	13,000

The best challenger's minimum annual equivalent cost is 18,700 SAR per year. (a) Compute the defender's $AEC(1)$, $AEC(2)$, and $AEC(3)$ from the outsider viewpoint, and identify its economic service life N^* and minimum AEC^*. (b) Verify that $AEC(1)$ equals the marginal cost of keeping the loader one more year, and explain why that is no coincidence. (c) Should the loader be replaced now? One decision sentence.

Exercise 16.11. After-tax keep-or-replace. Najd Copper's concentrate slurry pump (the defender) has a market value of 15,000 SAR today and a book value of 10,000 SAR on the tax ledger; its remaining straight-line schedule charges 5,000 SAR in each of the next two years. Kept for 3 more years, the pump costs 8,000 SAR per year to run and will sell for 3,000 SAR at the end of year 3 (book value then: zero). The tax rate is $t = 20\%$ and the after-tax MARR is 10%. The best challenger's minimum *after-tax* annual equivalent cost is 11,400 SAR per year. (a) Compute the after-tax opportunity cost of keeping the pump. (b) Compute the defender's after-tax AEC over the 3 years. (c) Keep or replace? One decision sentence.

Exam-style problems

Exercise 16.12. Replace Wadi Lithium's brine transfer pump? The defender's market value is 21,000 SAR now and 15,000 SAR in one year, and its operating cost next year is 11,000 SAR. The challenger costs 58,000 SAR, with the estimates below; $i = 10\%$.

keep for n (years)	1	2	3
market value S_n (SAR)	44,000	34,000	26,500
op. cost in year n (SAR)	5,000	6,500	8,500

(a) (*12 pts*) Build the challenger's economic-service-life table, $CR(n)$, annualized operating cost, and $AEC(n)$ for $n = 1, 2, 3$, and state N^* and AEC^*. (b) (*6 pts*) Compute the defender's marginal cost of one more year. (c) (*4 pts*) Keep or replace now? One decision sentence comparing the two numbers. (The pump's purchase history is sunk and must not appear.) (d) (*3 pts*) At what next-year defender operating cost would the verdict flip?

Exercise 16.13. The flotation blower's own economic life. Najd Copper's flotation blower was bought four years ago for 70,000 SAR and stands on the ledger at a book value of 21,000 SAR. Its market value today is 16,000 SAR. If kept, its market value and operating cost evolve as follows, at $i = 10\%$:

end of year	1	2	3
market value if sold then (SAR)	11,500	8,000	5,500
operating cost during the year (SAR)	7,000	8,800	11,200

The best challenger's minimum annual equivalent cost is 14,500 SAR per year. (a) (*12 pts*) Compute the defender's $AEC(1)$, $AEC(2)$, and $AEC(3)$ from the outsider viewpoint, and identify its economic service life N^* and minimum AEC^*. (b) (*5 pts*) Verify that $AEC(1)$ equals the marginal cost of keeping the blower one more year, and explain why. (c) (*4 pts*) Keep or replace now? One decision sentence. (d) (*4 pts*) Two amounts in the statement are irrelevant to the decision. Name them and say why.

Assessing outside recommendations

Replacement studies often arrive as someone else's memo. The two problems below are audits: decide whether to accept the recommendation, name every flaw, and correct the numbers. The flawed work is quoted exactly as received.

Exercise 16.14. Wadi Phosphate's slurry pump (the defender) can serve 3 more years: market value now 14,000 SAR; resale after 3 years 5,000 SAR; operating cost 12,400 SAR per year. The best challenger's minimum annual equivalent cost is 15,900 SAR per year at its economic life; $i = 10\%$. The pump was bought five years ago for 60,000 SAR and is fully depreciated. A consultant's memo reads:

> *"The pump was purchased for 60,000 SAR and is fully depreciated, so keeping it carries no capital cost; selling it now would waste the 60,000 SAR already invested. The defender's annual cost is therefore its operating cost, 12,400 SAR per year, well below the challenger's 15,900 SAR per year. Recommendation: keep the pump."*

Accept or reject the memo? Name each flaw and correct the numbers.

Exercise 16.15. Najd Copper asked an AI assistant when to replace a blasthole drill rig, at $i = 10\%$. Defender: market value now 30,000 SAR, market value in one year 24,000 SAR, operating cost next year 27,500 SAR. Challenger: cost 90,000 SAR, with the workshop's full five-year forecast:

keep for n (years)	1	2	3	4	5
market value S_n (SAR)	63,000	47,000	36,000	28,000	22,000
op. cost in year n (SAR)	9,000	12,000	15,500	19,500	32,000

The assistant returned this memo:

> *"Challenger economic-service-life table:* $AEC(1) = 45{,}000$; $AEC(2) = 39{,}905$; $AEC(3) = 37{,}274$ *SAR per year. The minimum is* $N^* = 3$ *with* $AEC^* = 37{,}274$ *SAR per year. Defender marginal cost of one more year:* $30{,}000\,(1.10) - 24{,}000 + 27{,}500 = 36{,}500$ *SAR, below the benchmark, so keep the defender and re-check next year."*

Accept or reject the memo? Name each flaw and correct the numbers.

References

Al-Chalabi, H. (2022). Development of an economic replacement time model for mining equipment: A case study. *Life Cycle Reliability and Safety Engineering*, 11(2), 203–217. `https://link.springer.com/article/10.1007/s41872-022-00188-1`

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