15 *Real Options: The Value of Flexibility*

After this chapter you will be able to:

1. explain what a single NPV assumes: a committed plan that cannot later be deferred, abandoned, or expanded
2. distinguish the four standard real options: defer, abandon, expand, and stage
3. compute the risk-neutral probability $q = ((1+r) - d)/(u - d)$, and check that $d < 1 + r < u$
4. compute a one-period option value from $C_0 = (q\,C_u + (1-q)\,C_d)/(1+r)$
5. build a two-period lattice and roll it back, taking at each node the larger of exercising and continuing
6. choose between the risk-neutral lattice and the decision-tree rollback, according to whether the uncertainty has a traded twin

Every present worth in this book so far has priced a committed plan: invest at year 0, operate to year N, collect the salvage, no changes of mind allowed. Real managers do not run projects that way. They wait for a price to recover before developing a deposit, stop a pit when the metal no longer pays for the diesel, add a second mill line when the ore justifies it, and build a plant in stages so the later money can watch what the earlier money learned. Each of these is a *decision made after the uncertainty resolves*, and a single NPV computed today, with no branch points in it, prices none of them. chapter 12 measured how uncertain a committed plan is; chapter 13 let plans branch on future information and rolled the branches back. This chapter gives that flexibility a market discipline and a name: a *real option*, the right, without the obligation, to take an action on a real asset at a known cost. The chapter builds the standard one-period and two-period binomial valuations fully by hand, shows where the curious "risk-neutral" probability comes from and when it is defensible for real assets, sets the same numbers side by side with the decision-tree rollback of chapter 13, and closes with the field's founding application, a mine whose value lives as much in the right to close it as in the plan to run it.

When a positive NPV is not enough.

Q1. A deposit can be developed today for a net present worth of +4,000 SAR. The firm may instead wait a year and decide then, when the metal price is known. Does the PW rule of chapter 5 say "invest now, since $+4{,}000 > 0$"?

Q2. A project value moves up by the factor $u = 1.25$ or down by $d = 0.85$ over one year, and the risk-free rate is 5%. Compute the risk-neutral probability $q = \big((1+r) - d\big)/(u-d)$.

Q3. In the down branch, exercising an option would lose money. Can the option itself ever have a negative value?

Answers.

Q1. *No.* Invest-now and wait-then-decide are mutually exclusive alternatives, and chapter 5's own rule for mutually exclusive alternatives is to take the one with the higher worth, not the first positive one. If waiting is worth +9,000 SAR because the bad price branch can be declined, then investing now costs the firm 5,000 SAR of value even though its NPV is positive. The NPV rule was never wrong; the menu was incomplete.

Q2.

$$q = \frac{1.05 - 0.85}{1.25 - 0.85} = \frac{0.20}{0.40} = 0.50.$$

Check that it does its job: $0.5\,(1.25) + 0.5\,(0.85) = 1.05$, so under q the asset grows at exactly the risk-free rate. Note that q was computed from u, d, and r alone; nobody's opinion about the chance of the up move entered.

Q3. *No.* An option is a right without an obligation: in the branch where exercising loses, the holder declines and the payoff is zero, not negative. The worst possible option value is therefore zero, and flexibility, priced before whatever it costs to create, can never subtract value from a project. What is bought at a price is the *creation* of the option; the audit exercises show how that price can exceed what the option is worth.

15.1 *What a Single NPV Assumes*

Real option. The right, without the obligation, to take a defined action on a real asset (invest, expand, abandon, wait) at a known cost, within a known time.

The committed-plan NPV of chapter 5 makes a quiet assumption: *now or never, and no exits*. Every cash flow in the diagram is promised, in both directions, before the first uncertain year unfolds. Under that assumption the analysis of chapter 12 is the right response to uncertainty, because measurement is all that is left once the plan cannot bend. But when the plan *can* bend, when the firm may delay, stop, or grow the project after watching prices, an analysis with no branch points undervalues the project, and always in the same direction: it prices the downside as if the firm will stand still and take it.

Example 15.1 — The deposit that was worth more unspent

Najd Copper holds an undeveloped satellite deposit. Developing it costs 95,000 SAR, and a developed deposit is worth $V_0 = 100{,}000$ SAR today, the present worth at the 10% MARR of its production plan (equivalently, what developed properties of this size trade for). Over the next year the copper price will resolve: the planning team sees the developed value moving to $V_u = 130{,}000$ SAR (price up) or $V_d = 80{,}000$ SAR (price down), each with probability 0.5, and the development cost staying 95,000 SAR. Compare investing now with waiting one year and deciding then, using the rollback of chapter 13 at the MARR.

Solution.

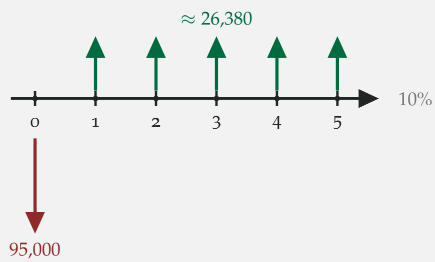


Figure 15.1. Invest now: the development cost buys the production plan whose five annual returns are worth $V_0 = 100{,}000$ SAR at the 10% MARR.

Step 1: invest now.

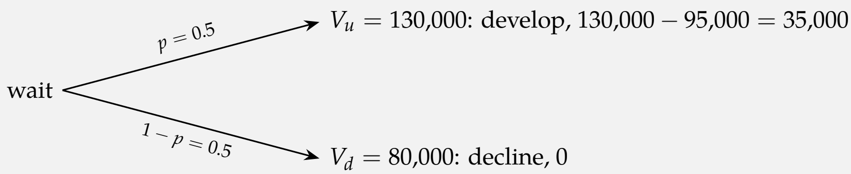
The committed-plan answer is one line:

$$\mathrm{NPV}_{\mathrm{now}} = 100{,}000 - 95{,}000 = \boxed{+5{,}000 \text{ SAR}},$$

and by chapter 5's single-project rule, accept.

Step 2: wait, watch, and decide.

Waiting converts the project into a branch: develop next year only if the price went up.



Rolling back at the MARR, as in chapter 13:

$$\mathrm{NPV}_{\mathrm{wait}} = \frac{0.5\,(35{,}000) + 0.5\,(0)}{1.10} = \frac{17{,}500}{1.10} = \boxed{+15{,}909 \text{ SAR}}.$$

Step 3: the verdict.

Waiting is worth $15{,}909 - 5{,}000 = 10{,}909$ SAR more than investing today. Please note what created the gap: in the down branch the committed plan swallows a value of $80{,}000 - 95{,}000 = -15{,}000$ SAR, while the waiting firm simply declines. Flexibility earns its value entirely by truncating the downside; the upside branch is identical in both plans.

Insight. Why did the NPV rule seem to fail? It did not. "Invest now" and "wait and decide" are mutually exclusive alternatives, and chapter 5 always demanded the full set of alternatives before choosing. What real-options thinking adds is the reminder that *timing and exit policies are alternatives too*, plus, in Section 15.3, a market argument about which probabilities and which discount rate a branch payoff deserves.

15.2 *The Option Zoo, in Minerals*

Four option types cover most engineering practice, and the minerals industry supplies a clean example of each.

option	minerals example	what is truncated
defer	hold the lease; develop when price recovers	bad development timing
abandon	close the pit when price $<$ cash cost	operating losses
expand	add a mill line if grades hold	missed upside
stage	pilot plant $\rightarrow$ demonstration $\rightarrow$ refinery	later stages of a failure

Table 15.1. The four standard real options with their minerals-industry forms. Each one either truncates a downside or opens an upside, and each is exercised only after information arrives.

The defer option is the lease that is not yet a mine: development is postponed until the price, the permit, or the drilling result justifies it, exactly Example 15.1. The abandonment option is priced every morning at a marginal pit: when the metal price falls below the cash cost of production, the right to stop, and to redeploy or sell the fleet, replaces an obligation to lose money. The expansion option is engineered in advance, a concentrator built with oversized foundations and permitted throughput headroom so that a second line is a decision rather than a redesign. And staging is how refineries and first-of-a-kind plants are actually built: the pilot's few million buys the right to a demonstration plant, whose results buy the right to full scale, and a failure at any stage strands only the money already spent.

The field's origin is a mine. Brennan and Schwartz (1985) valued a copper mine as a claim on the uncertain copper price, with the rights to open it, close it temporarily, and abandon it treated as options and priced by the same no-arbitrage logic as traded contracts, and they solved for the optimal open/close/abandon price thresholds as part of the valuation. Their machinery was continuous-time stochastic control; this chapter's lattice is the same idea with one or two time steps, small enough to work by hand.

15.3 *One Period, By Hand: The Binomial Lattice*

The tree of Example 15.1 used the planning team's probability 0.5 and the firm's MARR, both judgment calls. Finance contributes a sharper tool for the special case where the underlying uncertainty is a *traded price*, because then the market itself

Option to defer. The right to delay an investment and decide after prices or information resolve.

Option to abandon. The right to stop operating, avoid the remaining losses, and collect an exit value.

Option to expand. The right to enlarge capacity at a known cost if conditions turn out well.

Staged investment. A chain of options: each stage buys the right, not the duty, to fund the next stage. Also called a compound option.

In practice. An abandonment option is worth what the assets fetch *elsewhere*, and redeployability is an engineering property. Mobile crushers, haul trucks, and modular plant resell into a liquid market; a fixed conveyor poured into a specific pit wall resells as scrap steel. Two projects with identical cash flows can carry very different abandonment options because one was designed on wheels.

Reference. Brennan & Schwartz (1985), *The Journal of Business*, 58(2), 135–157. Full reference at the end of the chapter.

Underlying. The asset whose uncertain value the option is written on; here, the developed project's value V.

Exercise price. The known cost of taking the action, here the development cost I; in option notation, the strike.

Binomial lattice. A model in which the underlying value moves to exactly two states per period: up by the factor u or down by the factor d.

prices the branches. Model the developed value over one period as a binomial move,

$$V_1 = \begin{cases} uV_0 & \text{up state,} \\ dV_0 & \text{down state,} \end{cases} \qquad d < 1 + r < u,$$

Risk-neutral probability. The weight $q = \big((1+r) - d\big)/(u-d)$ under which the underlying's expected growth equals the risk-free rate. A pricing device, not a forecast.

where r is the risk-free rate. The option to develop pays $\max(V_1 - I, 0)$ in each state, the "decline the bad branch" payoff of Example 15.1 written as a formula.

Here is the pricing argument in words. A portfolio of the traded twin asset and risk-free lending can be built to repay exactly the option's two payoffs, one branch each, so the option must cost what that portfolio costs; anything else offers a riskless profit to whoever spots it. Working out that portfolio's cost turns out to be equivalent to a two-line recipe: weight the payoffs not by anyone's forecast but by the *risk-neutral probability*

$$q = \frac{(1+r) - d}{u - d}, \tag{15.1}$$

and discount the weighted average at the risk-free rate:

$$C_0 = \frac{q\,C_u + (1-q)\,C_d}{1+r}. \tag{15.2}$$

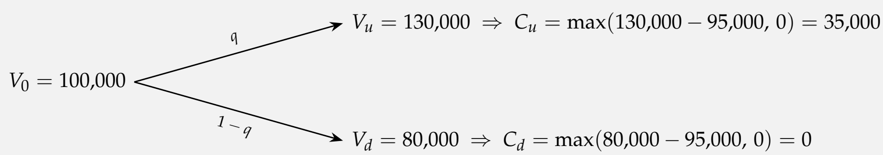
The weight q is manufactured from u, d, and r so that the underlying itself, priced by the same recipe, grows at exactly the risk-free rate; that is the sense in which the market's price of risk is already inside it.

Example 15.2 — Pricing the deferral, one period

Price Example 15.1's option to wait with the binomial machinery: $V_0 =$ 100,000 SAR, $u = 1.3$, $d = 0.8$ (so $V_u = 130{,}000$ and $V_d = 80{,}000$ SAR), development cost $I = 95{,}000$ SAR, risk-free rate $r = 5\%$.

Solution.

Step 1: the lattice and the payoffs.



Step 2: the risk-neutral probability.

By Equation (15.1),

$$q = \frac{1.05 - 0.80}{1.30 - 0.80} = \frac{0.25}{0.50} = 0.50.$$

Check it does its job on the underlying: $0.5\,(130{,}000) + 0.5\,(80{,}000) = 105{,}000 = 100{,}000\,(1.05)$. Under q, holding the deposit's twin earns exactly the risk-free rate, as required.

Step 3: the option value.

By Equation (15.2),

$$C_0 = \frac{0.5\,(35{,}000) + 0.5\,(0)}{1.05} = \frac{17{,}500}{1.05} = \boxed{+16{,}667\text{ SAR}}.$$

Step 4: the verdict sentence.

Hold the lease: the right to develop next year is worth 16,667 SAR against 5,000 SAR for developing today, so waiting adds $16{,}667 - 5{,}000 = 11{,}667$ SAR, and the firm should invest today only if a year's delay carries costs (lost output, expiring permits) worth more than that premium.

Insight. Why q and not the planning team's p? Because the option's payoffs can be manufactured from the traded twin and a bank account, identical in every state, and two assets with identical payoffs in every state must trade at one price no matter what anyone believes about the states' chances. The forecast p is not wrong; it is simply not needed once a market already prices the underlying risk. The moment there is no traded twin, this argument loses its footing, and Section 15.6 takes over.

Rollback. Valuing a lattice from its final period backward, one node at a time, choosing at each node the better of exercising and continuing.

15.4 *Two Periods: The Value of Waiting Longer*

Stretching the lattice to two periods adds one new ingredient: at the middle date the firm holds a choice between *exercising now* and *keeping the option alive*, and the rollback must take the larger of the two at every node.

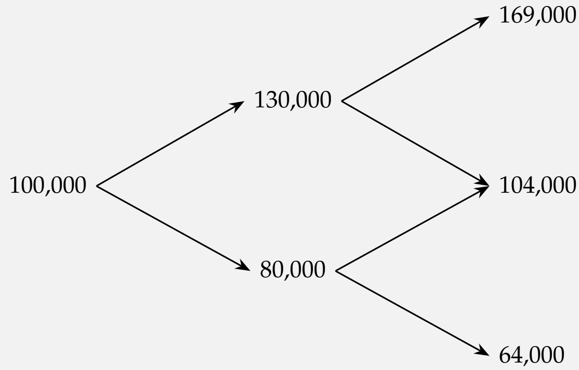
Example 15.3 — The two-year lease

Extend Example 15.2: the lease now allows development at $I = 95{,}000$ SAR at the end of year 1 or year 2, with the same $u = 1.3$, $d = 0.8$, $r = 5\%$ each year (so $q = 0.5$ throughout). Value the option and the exercise policy.

Solution.

Step 1: the value lattice.

The lattice recombines ($ud = du$):



Step 2: rollback, worked in a table.

Year-2 payoffs are $\max(V_2 - 95{,}000, 0)$; year-1 nodes compare exercising with continuing (Equation (15.2) applied at each node):

node	V (SAR)	exercise now	continue	option value
year 2, up-up	169,000	74,000	—	74,000
year 2, up-down	104,000	9,000	—	9,000
year 2, down-down	64,000	0	—	0
year 1, up	130,000	35,000	$\frac{0.5(74{,}000)+0.5(9{,}000)}{1.05} = 39{,}523.81$	**39,523.81**
year 1, down	80,000	<0: decline	$\frac{0.5(9{,}000)+0.5(0)}{1.05} = 4{,}285.71$	**4,285.71**
year 0	100,000	5,000	$\frac{0.5(39{,}523.81)+0.5(4{,}285.71)}{1.05} = 20{,}861.68$	**20,861.68**

Table 15.2. Rollback of the two-year deferral option. At every interior node the option value is the larger of exercising and continuing; bold marks the choice taken.

Step 3: read the policy.

The two-year option is worth $\boxed{+20{,}862 \text{ SAR}}$, against 16,667 for one year and 5,000 for developing today. And the table contains a policy, not just a price: even after a good first year ($V = 130{,}000$, exercise worth 35,000), continuing is worth more (39,524), so the patient firm develops nothing before year 2 in this model. Please note what made early exercise unattractive: the deposit pays nothing while undeveloped, so waiting sacrifices no cash flow; a producing asset, or a lease that expires, tilts the comparison toward exercising.

In practice. Deferral is not free in the field even when the model says waiting wins. Permits lapse, pit walls weather, dewatering restarts cost real money, and the owner's team disperses to other projects. Each of these is a leak in the option's value, and the engineer is the one who can size the leak: put a yearly holding cost into the lattice and the ''never exercise early'' conclusion of Example 15.3 starts to bend.

15.5 *Abandon and Expand: Flexibility in Operations*

The defer option guards the front gate. The other two workhorses live inside an operating project, and both price cleanly with the chapter 13 rollback at the MARR, the practical default defended in the next section.

Example 15.4 — The pit that can stop

A satellite pit at *Najd Copper* has one year of reserves left. Next year's operating outcome is worth +30,000 SAR if the copper price holds (probability 0.5) and −8,000 SAR if it falls below the cash cost (probability 0.5), both stated as year-1 values. The mobile fleet can be redeployed to the main pit at any time for a year-1 value of 6,000 SAR. MARR = 10%. Value the final year with and without the right to abandon.

Solution.

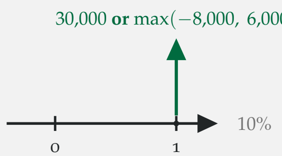


Figure 15.2. The pit's final year: one uncertain year-1 value. Abandonment replaces the bad branch with the redeployment value.

Step 1: committed to operate.

$$\mathrm{PW}_{\text{no option}} = \frac{0.5\,(30{,}000) + 0.5\,(-8{,}000)}{1.10} = \frac{11{,}000}{1.10} = \boxed{+10{,}000\ \text{SAR}}.$$

Step 2: free to stop.

In the bad branch the firm compares operating ($-8{,}000$) with redeploying ($+6{,}000$) and takes the larger:

$$\begin{aligned}\mathrm{PW}_{\text{option}} &= \frac{0.5\,(30{,}000) + 0.5\ \max(-8{,}000,\ 6{,}000)}{1.10}\\ &= \frac{0.5\,(30{,}000) + 0.5\,(6{,}000)}{1.10} = \boxed{+16{,}364\ \mathrm{SAR}}.\end{aligned}$$

Step 3: the option's worth.

The right to stop is worth $16{,}364 - 10{,}000 = 6{,}364$ SAR, earned entirely in the branch where the plan would have lost: $6{,}000 - (-8{,}000) = 14{,}000$ SAR of avoided damage, weighted by its 0.5 chance and discounted one year. A fixed plant with no redeployment value (exit value 0) would still hold an abandonment option worth $0.5\,(8{,}000)/1.10 = 3{,}636$ SAR, the avoided loss alone; the wheels added the rest.

Example 15.5— The concentrator that can grow

Wadi Phosphate's new concentrator will reveal within a year whether the ore grades hold across the deposit. If they do (probability 0.6), a second flotation line can be added at a year-1 cost of 20,000 SAR and would add a year-1 value of 32,000 SAR; if grades disappoint, expansion has no value. MARR = 10%. Value the expansion right, and state what it required at design time.

Solution.

Step 1: the branch payoffs.

Expansion is exercised only in the good branch, where it is worth $\max(32{,}000 - 20{,}000,\ 0) = 12{,}000$ SAR at year 1; in the bad branch the right lapses at zero.

Step 2: roll back.

$$\text{expansion option} = \frac{0.6\,(12{,}000) + 0.4\,(0)}{1.10} = \frac{7{,}200}{1.10} = \boxed{+6{,}545\ \mathrm{SAR}}.$$

Step 3: what it cost to own.

This value exists only because the plant was *designed expandable*: oversized foundations, a permitted footprint, tie-in points on the slurry lines. If those design allowances cost 4,000 SAR today, the option is cheap at the price $(6{,}545 > 4{,}000)$; if the plant was built tight and expansion means a rebuild, the "option" was never created. Flexibility is bought at design time and exercised later, or it is not owned at all.

15.6 *Trees with p or Lattices with q: Which to Use*

Traded twin. A traded asset or portfolio whose price moves with the project's underlying uncertainty, e.g. the metal price behind a mine's value. Its existence is what licenses risk-neutral pricing.

The same deferral was now priced twice: 15,909 SAR by the chapter 13 rollback with the team's $p = 0.5$ at the 10% MARR (Example 15.1), and 16,667 SAR by the risk-neutral recipe with $q = 0.5$ at the 5% risk-free rate (Example 15.2). Same lattice, same payoffs, different frameworks, different numbers, and here the two happen to sit close only because p happened to equal q; a team that believed $p = 0.6$ would have rolled back to $0.6\,(35{,}000)/1.10 = 19{,}091$ SAR, and one that also demanded a 15% risk-adjusted rate, $21{,}000/1.15 = 18{,}261$ SAR. The comparison is worth a table:

framework	probability	discount rate	option value (SAR)
risk-neutral lattice	$q = 0.50$ (from u, d, r)	5% (risk-free)	16,667
tree rollback	$p = 0.50$ (team's)	10% (MARR)	15,909
tree rollback	$p = 0.60$ (team's)	10% (MARR)	19,091
tree rollback	$p = 0.60$ (team's)	15% (risk-adj.)	18,261

Table 15.3. One deferral, four defensible-looking valuations. The framework must be chosen and declared, and its two ingredients, probability and rate, must come from the same framework.

Insight. The handbook's honest position. Risk-neutral pricing is the right tool when the underlying uncertainty has a *traded twin*, a metal price, an oil price, a power price, because then u, d, and q are read from markets and no committee forecast enters. For real assets whose uncertainty is not traded, ore grade, a permit decision, a technology's yield, there is no twin, no arbitrage argument, and risk-neutral machinery becomes borrowed clothing. There the decision-tree rollback of chapter 13, with the team's honest p and the firm's MARR, is the practical default: less elegant, but every input is one the firm can defend. What is never defensible is mixing the frameworks, a subjective p discounted at the risk-free rate flatters every option it touches, and the first audit exercise prices exactly that error.

15.7 Rules of Thumb: When Flexibility Is Worth Money

Three regularities fall out of every example in this chapter and are worth carrying as instincts.

1. **Flexibility feeds on uncertainty.** Shrink the spread of Example 15.2 from (130,000, 80,000) to (110,000, 100,000), keeping $I = 95{,}000$, and the option to decline never bites: both branches would be exercised, and whatever waiting still adds is only a year's interest on the unspent 95,000 SAR, not flexibility. In chapter 12's language: when the sensitivity lines are flat, options are not worth engineering.

2. **Flexibility needs a decision that can actually be deferred or reversed.** An option exists only if the action can wait (the lease does not expire tomorrow) or be undone (the fleet rolls to another pit). Irreversible, now-or-never commitments carry no option value no matter how uncertain the world is; they are priced, and stress-tested, as chapter 12 committed plans.

3. **Option value is downside truncation.** Every worked example earned its premium in the bad branch: declining development, redeploying the fleet, letting the expansion lapse. If a proposal's "option value" grows out of the good branch, look again; the good branch was already in the NPV.

Figure 15.3 puts both spreads of rule 1 on one axis, holding the mean outcome at 105,000 SAR so that only the width changes. Look at where the curve stops being flat: that kink, at a spread of 20,000 SAR, is where the bad branch first falls below the 95,000 SAR development cost, and everything the option earns above the flat stretch comes from declining it.

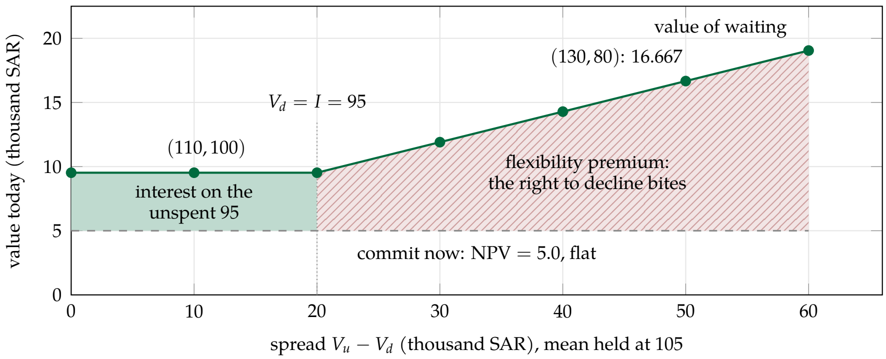


Figure 15.3. Why flexibility feeds on uncertainty. The value of waiting climbs with the spread between the good and the bad outcome — the chapter's stand-in for volatility, which the lattice never parameterizes — while the NPV of committing today stays at 5,000 SAR; below a spread of 20,000 SAR the premium is only interest on the unspent development cost, and only past that does the right to decline the bad branch start paying.

In practice. A kill criterion agreed before the experiment is an abandonment option with the exit value written down: which metric, by which date, releases the remaining budget back to the firm. A team that refuses to predefine it converts a staged project back into the committed plan of chapter 12, and pays for flexibility it no longer owns.

15.8 *Case Study from the Literature*

Real-options studies of mines are published continuously in the open literature; this section reads one with the chapter's eyes. Zapata Quimbayo and Mejía Vega (2019), in the open-access *Business and Economic Research*, value a hypothetical open-pit gold mine in Western Australia: total investment \$97.5 million, about 450,000 recoverable ounces mined over six years, valued first as a committed plan and then with flexibility under three correlated uncertainties, the gold price, the USD/AUD exchange rate, and operating costs. Amounts are in Australian dollars, the paper's reporting currency.

Reference. Zapata Quimbayo & Mejía Vega (2019), *Business and Economic Research*, 9(3), 204–218. Open access; full reference at the end of the chapter.

Example 15.6 — Case study: a gold mine's flexibility premium

The paper reports: a static DCF value of the mine's cash flows of \$161.4 million at a 15% risk-adjusted rate, hence a committed-plan NPV of \$63.87 million on the \$97.5 million investment; annual volatilities of 16.21% (gold price), 12.08% (exchange rate), and 4.51% (operating cost); a risk-free rate of 2.57%; and, from a multinomial lattice in which the investment is made only when it pays (payoff $\max(V_t - I, 0)$, this chapter's deferral right), a project value with flexibility of \$146.1 million, so that the option adds \$82.23 million. (a) Verify the arithmetic links among the reported numbers. (b) Verify the lattice's up factor for the gold price from its volatility. (c) Read the result with this chapter's rules of thumb.

Solution.

Step 1: the links.

The committed plan checks: $161.4 - 97.5 = 63.9 \approx 63.87$, the paper's NPV. The flexibility premium checks the same way: the option value is reported as the value with flexibility minus the static NPV, $146.1 - 63.87 = \boxed{\$82.23 \text{ million}}$, exactly "the worth of the flexibility over the committed plan" in this chapter's language.

Step 2: the up factor from the volatility.

Binomial lattices in practice set $u = e^{\sigma\sqrt{\Delta t}}$ and $d = 1/u$ from the measured volatility σ, the industrial answer to where u and d come from. For the paper's gold price, with $\sigma = 16.21\%$ and one-year steps,

$$u = e^{0.1621} = \boxed{1.176}, \qquad d = 1/u = 0.850,$$

so each lattice year moves the gold price up 17.6% or down 15.0%. The paper then discounts risk-neutral expectations at its 2.57% risk-free rate, Equation (15.2) at industrial scale, with the risk-neutral weights computed from all three volatilities and their correlations at once (its Table 6).

Step 3: the reading.

The flexibility premium, \$82.23 million, is *larger than the entire committed-plan NPV* of \$63.87 million. That is not an exotic result; it is rule of thumb 1 at full strength. Three correlated uncertainties give the mine's value a wide spread, and the right to decline development in the bad states truncates the whole lower half of it. Two cautions complete the reading. First, the mine is hypothetical and holding the undeveloped property is modeled as costless; real leases expire, and care-and-maintenance and permitting costs leak option value, the chapter 13-style detail an owner would add. Second, the framework is properly matched: market-measured volatilities, risk-neutral weights, risk-free discounting, the traded twins here (gold, currency) are as liquid as twins get, which is why this is the setting where risk-neutral machinery is at home.

15.9 Chapter Summary

- A single NPV prices a **committed plan**: now or never, no exits. When the firm can defer, abandon, expand, or stage, the committed-plan NPV **undervalues the project**, always by ignoring the right to decline a bad branch.

- A **real option** is a right without an obligation, so its value is never negative; its worth is earned by **truncating the downside** after information arrives.

- Four workhorses: **defer** (develop when the price recovers), **abandon** (stop when price falls below cash cost and redeploy), **expand** (add capacity if grades hold), **stage** (each phase buys the right to the next). Abandonment value depends on the assets' redeployability; expansion value must be built into the design before it can be owned.

- One-period binomial pricing: $q = \big((1+r) - d\big)/(u-d)$, then $C_0 = \big(q\,C_u + (1-q)\,C_d\big)/(1+r)$. The weight q makes the traded twin grow at the risk-free rate; forecasts do not enter. Check $d < 1+r < u$ before using it.

- Multi-period lattices are solved by **rollback**: at each node take the larger of exercising and continuing. An underlying that pays nothing while waiting favors late exercise; holding costs and expiring rights pull exercise earlier.

- **Framework discipline**: risk-neutral q with the risk-free rate when a traded twin exists (metal, oil, power); chapter 13 rollback with the team's p and the MARR when it does not. For real assets the tree-with-MARR is the practical default; mixing the frameworks (p with the risk-free rate) flatters every option.

- Rules of thumb: flexibility is worth most when **uncertainty is high** and the decision is **deferrable or reversible**; zero spread or irreversible commitments carry no option value; and any claimed option value that comes from the good branch was already in the NPV.

- Numbers to remember from the running example: develop now +5,000; wait one year 16,667 (risk-neutral) or 15,909 (tree at MARR); wait two years 20,862 SAR.

15.10 Exercises

These problems price the chapter's three workhorse options: defer, abandon, and expand. They open with one-step lattices and risk-neutral probabilities, extend to multi-period lattices, and close with operating options embedded in a project already running.

Lattice warm-ups and lecture practice

Exercise 15.1. An underlying moves with $u = 1.2$ and $d = 0.9$ over one year. (a) Compute the risk-neutral probability at a risk-free rate of 5%, and verify it reprices the underlying. (b) Recompute q at a risk-free rate of 25% and explain what the result means.

Exercise 15.2. *Wadi Lithium* can develop a brine lease for $I = 58{,}000$ SAR. The developed project is worth $V_0 = 60{,}000$ SAR today; over one year the lithium price moves the value by $u = 1.25$ or $d = 0.8$, and the risk-free rate is 5%. (a) Value developing today. (b) Value the option to wait one year, by hand. (c) State the verdict in one sentence.

Quick checks (multiple choice)

Exercise 15.3. A concentrator is built with oversized foundations and a permitted throughput ceiling well above the initial line, so a second mill line can be added later if ore grades hold. In this chapter's terms the design allowance creates:

(A) an option to defer.
(B) an option to abandon.
(C) an option to expand.
(D) a staged investment in which each phase is obligatory.

Exercise 15.4. Which single change **reduces** the value of an option to defer a development?

(A) The spread between the up and down values widens.
(B) The spread between the up and down values narrows.
(C) The lease is extended, allowing the decision to wait another year.

(D) The deposit pays nothing while undeveloped.

Operating options practice

Exercise 15.5. A dragline at *Najd Copper* has one more year of planned service. The year's operating outcome, stated as a year-1 value, is +25,000 SAR with probability 0.6 and −12,000 SAR with probability 0.4. At any time the machine can instead be released to a contractor project for a year-1 value of +7,000 SAR. MARR = 10%. (a) Value the final year as a committed plan. (b) Value it with the release right. (c) State the option's worth and where it is earned.

Exam-style problems

Exercise 15.6. *Wadi Lithium* holds a two-year development lease on a spodumene property. The developed project is worth $V_0 = 200{,}000$ SAR today; each year the value moves by $u = 1.25$ or $d = 0.80$; the development cost is $I = 190{,}000$ SAR at any exercise date; the risk-free rate is 5% per year, and lithium prices trade on liquid markets. (a) (*4 pts*) Value developing today. (b) (*6 pts*) Compute the risk-neutral probability and verify it reprices the underlying. (c) (*10 pts*) Value the lease as a two-year option by rollback, checking early exercise at each year-1 node. (d) (*5 pts*) State the exercise policy and the verdict sentence.

Exercise 15.7. A consultant and the *Najd Copper* planning team value the same one-year deferral: developed value $V_0 = 200{,}000$ SAR moving by $u = 1.25$ / $d = 0.80$, development cost 190,000 SAR. The team believes the up move has probability 0.7; the firm's MARR is 12%; the risk-free rate is 5%. (a) (*6 pts*) Value the deferral by the chapter 13 rollback with the team's probability and the MARR. (b) (*8 pts*) Value it with the risk-neutral lattice. (c) (*7 pts*) Explain why the two answers differ, and state which framework this decision should report and why. (d) (*4 pts*) Write the verdict sentence.

Assessing outside recommendations

Real recommendations arrive finished: a memo, a sheet, a verdict. The two problems below are audits. Decide whether to accept the recommendation, name every flaw, and correct the numbers. The flawed work is quoted exactly as received.

Exercise 15.8. A consultant's memo values the deferral of Example 15.2 ($V_0 =$ 100,000 SAR, $u = 1.3$, $d = 0.8$, $I = 95{,}000$ SAR, risk-free rate 5%, MARR 10%) as follows:

> *"Our commodity desk assesses the probability of the up move at 0.6. Option value =* $[0.6 \times 35{,}000 + 0.4 \times 0]/1.05 = 21{,}000/1.05 = 20{,}000$ *SAR. Because we discount at the risk-free rate, this valuation is conservative."*

Accept or reject the memo? Name each flaw and correct the numbers both defensible ways.

Exercise 15.9. A vendor's proposal for the crushing unit of chapter 12 (investment 55,000 SAR; PW at the base case +8,757 SAR including the 5,000 SAR salvage collected at year 5; MARR 10%) adds a flexibility section:

> *"Real-options uplift: the unit can be abandoned at end of life for its 5,000 SAR salvage value. Valuing this abandonment option as the present worth of the salvage,* $5{,}000 \times (P/F, 10\%, 5) = 5{,}000 \times 0.6209 = 3{,}105$ *SAR, the project's true value is* $8{,}757 + 3{,}105 = 11{,}862$ *SAR."*

Company engineers add two facts: the unit could also be sold at the end of year 3 for about 12,000 SAR, and in a weak-demand state (probability 0.3) the value at year 3 of continuing through years 4–5 would be only 9,500 SAR. Accept or reject the flexibility section? Name each flaw and compute a defensible abandonment option value.

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