12 *Handling Project Uncertainty*

After this chapter you will be able to:

1. compute the base-case present worth of a project from the most-likely estimate of every input
2. build a one-way sensitivity table by moving one input at a time by $\pm 10\%$ and $\pm 20\%$
3. read a sensitivity graph by slope, and rank the inputs by their total PW swing
4. compute the break-even value of an uncertain input by setting $\mathrm{PW} = 0$, then state the cushion as a percentage of the base estimate
5. compute the expected present worth $E[\mathrm{PW}] = \sum_j p_j \, \mathrm{PW}_j$ from a table of pessimistic, most-likely, and optimistic scenario worths
6. explain how the cost of capital k and a risk premium together set the MARR

Every number used so far in this book was an estimate. The revenues, the operating costs, the salvage values, the lives: each one was somebody's best guess about the future, written down as if it were a fact. Chapters 5 through 10 took those guesses, computed one present worth, and trusted it. Managers rarely get to do that. They are asked how confident the answer is, and this chapter shows how that question is answered with numbers. Four tools are developed, in order of increasing ambition. *Sensitivity analysis* varies one input at a time and shows which estimate the verdict depends on most. *Break-even analysis* solves $\mathrm{PW} = 0$ for an uncertain input and reports how far the estimate can slip before the decision changes. *Scenario analysis* bundles the inputs into coherent stories, one present worth per story. And the *expected present worth* weights those stories by their probabilities. None of these tools removes the uncertainty. They measure it, and they tell the analyst which uncertainty matters. The chapter then opens the MARR itself, showing where the hurdle rate comes from, the firm's cost of capital, and how firms raise it for riskier projects, and closes by reading a published techno-economic study whose risk section runs exactly these tools.

What a single present worth hides.

Q1. A project's present worth at the MARR, computed from best-guess estimates, is +5,000 SAR. Does this guarantee that the project will create value?

Q2. A machine costs 20,000 SAR and is expected to save A SAR per year for 5 years, with no salvage. The MARR is 10%. What is the smallest annual saving for which the machine is worth buying?

Q3. A project has two possible outcomes: PW $= -10{,}000$ SAR with probability 0.4, and PW $= +20{,}000$ SAR with probability 0.6. Compute the expected present worth. Does a positive average remove the risk?

Answers.

Q1. *No.* The +5,000 SAR is only as reliable as the estimates behind it. If the actual revenues come in below the forecast, the actual present worth can be negative even though the estimated one was positive. A single PW carries no measure of how likely that is, which is the gap this chapter fills.

Q2. Set PW $= 0$: $-20{,}000 + A^* (P/A, 10\%, 5) = 0$, so $A^* = 20{,}000/3.7908 = 5{,}276$ SAR per year. Below this value the machine destroys value; above it, the machine earns more than the MARR. Please note that the naive answer $20{,}000/5 = 4{,}000$ SAR ignores interest and is too low.

Q3. $E[\text{PW}] = 0.4\,(-10{,}000) + 0.6\,(20{,}000) = +8{,}000$ SAR. The average is positive, but the risk remains: there is a 40% chance of losing 10,000 SAR. A complete answer reports both numbers.

Lecture 23. This section is covered by Lecture 23.

Reference. Park, 3rd ed., Sections 11.1–11.3 (pp. 482–495).

Point estimate. A single best-guess value written down for an uncertain input, used as if it were a fact.

12.1 *Why Point Estimates Mislead*

Our habit so far has been fixed: write one best estimate for each input, compute one present worth, and decide. That habit has two weaknesses.

1. **There is no guarantee the estimates will match the actual values.** A revenue forecast is a forecast, not a fact. The machine may cost more than quoted; the market for the output may shrink.

2. **A single present worth carries no measure of risk.** It cannot say how likely the project is to lose money, or how large the loss could be.

What-if analysis. Recomputing the decision measure after changing one or more input estimates: if demand is 10% lower than forecast, what happens to the PW?

The repair is *what-if analysis*. If demand comes in 10% below the forecast, what happens to the present worth? If the equipment costs more than quoted, what happens? You have already met one what-if question: the break-even usage problem of Section 6.9 asked how many operating hours justify the more expensive machine. This chapter asks that style of question about every input of a project.

Insight. A single PW hides how *fragile* it is. Two projects can have the same present worth of +8,000 SAR, yet one survives a 30% revenue error while the other flips sign at 6%. The tools of this chapter measure the fragility, so the verdict can be reported together with the amount of estimation error it can absorb.

Base case. The present worth computed from the most-likely estimate of every input. All what-if questions are measured against it.

12.2 *The Base Case*

Every tool in this chapter starts from the same reference point: the *base case*, the present worth computed from the most-likely estimate of every input. The running example of this chapter is a mobile ore-crushing unit at *Wadi Phosphate*, the phosphate producer of Chapter 5.

Example 12.1 — The base case: a mobile crushing unit

Wadi Phosphate plans a mobile ore-crushing unit that can be moved between pits. The most-likely estimates are:

investment	55,000 SAR	annual revenue	42,000 SAR
annual operating cost	26,000 SAR	salvage (year 5)	5,000 SAR
life	5 years	MARR	10%

Compute the base-case present worth and state the verdict.

Solution.

The annual net cash flow is $42{,}000 - 26{,}000 = 16{,}000$ SAR per year. Netting first keeps the arithmetic small, but revenue and operating cost stay separate *inputs*, because each can be misestimated on its own.

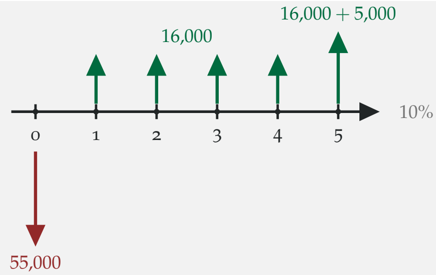

Figure 12.1. The crushing unit's base case: one investment, five net annual returns, and the salvage added to the last return.

Step 1: factor notation.

$$\mathrm{PW}(10\%) = -55{,}000 + 16{,}000\,(P/A,\ 10\%,\ 5) + 5{,}000\,(P/F,\ 10\%,\ 5).$$

N	(P/A)	(P/F)
4	3.1699	0.6830
5	**3.7908**	**0.6209**
6	4.3553	0.5645

Table 12.1. Excerpt from the $i = 10\%$ interest factor table. These two factors do all the work of this chapter's running example.

Step 2: substitute.

$$\begin{aligned}\mathrm{PW}(10\%) &= -55{,}000 + 16{,}000\,(3.7908) + 5{,}000\,(0.6209)\\ &= -55{,}000 + 60{,}652.80 + 3{,}104.50 = \boxed{+8{,}757\ \mathrm{SAR}}.\end{aligned}$$

The base-case verdict is **accept**. The question of this chapter is different: how much of a misestimate does that verdict survive?

One-way sensitivity analysis. Vary one input at a time by fixed percentages of its base estimate, recompute the PW, and hold every other input at base.

In practice. The table moves every input by the same percentages, but which inputs deserve worry is known before any spreadsheet: ore grade varies with the block being mined, equipment availability with the maintenance backlog. Field knowledge decides which rows of the table matter.

12.3 *One-Way Sensitivity Analysis*

The first tool follows a strict recipe. Vary *one input at a time* by ±10% and ±20% of its base estimate, recompute the present worth, and hold every other input at its base value. Each input is moved by the same *percentages*, not by the same amounts, so inputs with different units, a revenue, a cost, an investment, land on one comparable scale.

Insight. No cell of a sensitivity table needs a full recomputation. The present worth is linear in each input, so the change in PW is the change in the input times the factor that carries it to year 0. For the crushing unit, each 1% of revenue is 420 SAR per year, worth $420 \times 3.7908 \approx 1{,}592$ SAR of present worth; each 1% of the investment is 550 SAR at year 0, worth exactly 550 SAR of present worth. Compute the base case once, then add and subtract multiples of these per-percent amounts.

Example 12.2 — Sensitivity table and graph for the crushing unit

For the crushing unit of Example 12.1, vary the annual revenue, the annual operating cost, the investment, and the salvage value, one at a time, by $\pm 10\%$ and $\pm 20\%$. Tabulate the present worths, draw the sensitivity graph, and read it.

Solution.

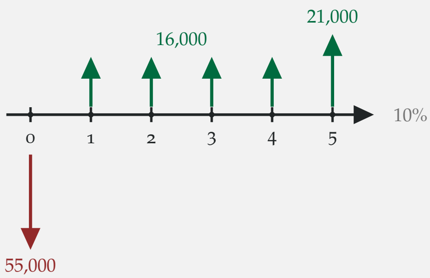

Figure 12.2. The base-case flows again. Each row of the sensitivity table perturbs exactly one of the four inputs behind these arrows.

Step 1: the per-percent effect of each input.

Through $(P/A, 10\%, 5) = 3.7908$, a 10% revenue change ($\pm 4{,}200$ SAR per year) moves the PW by $4{,}200\,(3.7908) = 15{,}921$ SAR; a 10% operating-cost change ($\pm 2{,}600$ SAR per year) moves it the opposite way by $2{,}600\,(3.7908) = 9{,}856$ SAR. A 10% investment change is 5,500 SAR at year 0, already in present-worth terms. A 10% salvage change is $500\,(0.6209) = 310$ SAR.

Step 2: build the table.

Starting from the base PW of $+8{,}757$ SAR:

PW (SAR)	−20%	−10%	base	+10%	+20%
annual revenue	−23,085	−7,164	8,757	24,679	40,600
operating cost	28,469	18,613	8,757	−1,099	−10,955
investment	19,757	14,257	8,757	3,257	−2,243
salvage	8,136	8,447	8,757	9,068	9,378

Table 12.2. One-way sensitivity of the crushing unit's PW. Each row varies one input; every other input stays at base.

Step 3: plot the graph.

Each input becomes one line through the base point.

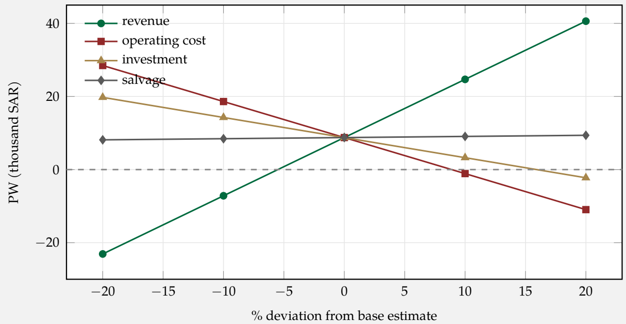

Figure 12.3. The sensitivity graph of the crushing unit. All four lines pass through the base case at 0% deviation; the dashed line is PW = 0.

Step 4: read the graph.

Read it by slope. The revenue line is the steepest, so revenue is the most dangerous input: a small estimation error produces a large swing in PW. The operating-cost line slopes downward, less steeply; the investment line is shallower still; and the salvage line is nearly flat. Where a line crosses the dashed PW = 0 line is that input's *break-even* point: the revenue line crosses near −6%, while the investment line does not cross until about +16% over budget.

Please pay attention to the pattern in Table 12.2: a 10% revenue slip already turns the PW *negative* (−7,164 SAR), while a 20% salvage error moves it by only about ±621 SAR.

Example 12.3 — Ranking the inputs by swing

Using Table 12.2, rank the four inputs of the crushing unit by their total PW swing across the ±20% band, and state what the ranking tells the planning team.

Solution.

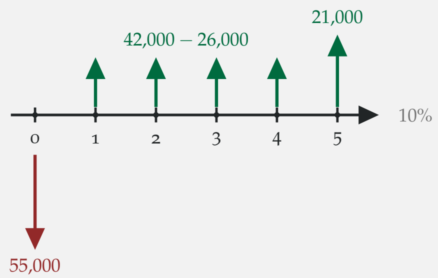

Figure 12.4. The same project, with the annual net kept as revenue minus operating cost. The ranking below sorts these four estimates by how hard each one can move the PW.

For each input, take the full range of present worths across the ±20% band and sort by the width of that range, widest on top:

rank	input	PW range (SAR)	swing (SAR)
1	annual revenue	−23,085 → 40,600	**63,685**
2	operating cost	−10,955 → 28,469	39,424
3	investment	−2,243 → 19,757	22,000
4	salvage	8,136 → 9,378	1,242

Table 12.3. The inputs of the crushing unit ranked by total PW swing across ±20%. Sorted this way and drawn as horizontal bars, the ranking is called a tornado chart.

The tornado of Figure 12.5 plots the same four inputs as the sensitivity graph of Figure 12.3: read the graph for each input's slope, and read the tornado for the rank order and the width of each swing at a glance.

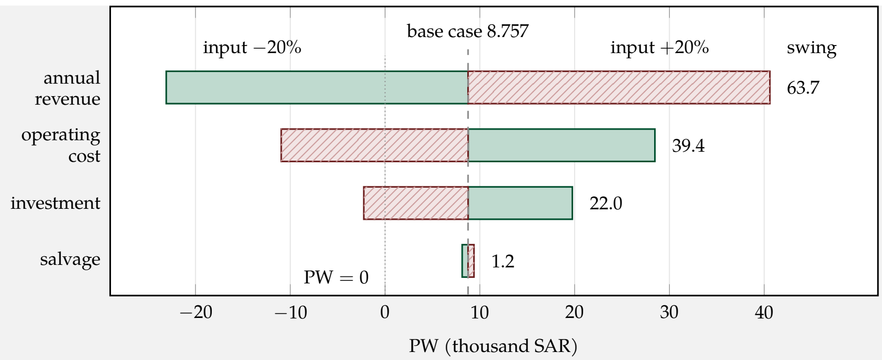

Figure 12.5. Tornado diagram of the crushing unit. Each bar runs from the present worth at the input's −20% value to the present worth at its +20% value, widest swing on top; the dashed line is the base case PW = +8,757 SAR, and revenue alone can move the verdict further than the other three inputs combined.

Revenue swings the PW across $\boxed{\text{63,685 SAR}}$, about half again as much as the operating cost and three times as much as the investment; the salvage barely registers. The ranking tells the team where to spend effort. If one more week of study is available before the decision, that week belongs to the revenue forecast: a contract survey, a trial crushing campaign, anything that firms up the top of the ranking. Refining the salvage estimate would be a wasted week. The analysis does not remove the uncertainty; it tells the team *which* uncertainty matters.

Break-even value. The value of one uncertain input at which PW = 0: the point where the verdict flips.

Cushion. The gap between the base estimate and the break-even value, reported as a percentage of the base estimate.

12.4 *Break-Even Analysis*

Sensitivity analysis says revenue is dangerous. Break-even analysis asks the follow-up question: how far can it fall before the project starts destroying value? The move is one you already know. In Chapter 7 we set $\text{PW}(i) = 0$ and solved for the interest rate; here we hold the MARR fixed and solve $\text{PW} = 0$ for the uncertain *input*. It is also the same algebra as the break-even usage problem of Section 6.9. One idea, many unknowns.

Example 12.4 — Break-even annual net for the crushing unit

For the crushing unit of Example 12.1, find the break-even annual net cash flow A^*, the percentage cushion below the 16,000 SAR base estimate, and the corresponding break-even revenue.

Solution.

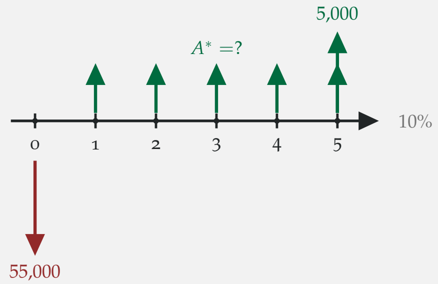

Figure 12.6. The break-even setup: the annual net is now the unknown, chosen so that the PW is exactly zero.

Step 1: set PW $= 0$ ***and solve for*** A^*.

$$-55{,}000 + A^*\,(3.7908) + 5{,}000\,(0.6209) = 0,$$

$$A^* = \frac{55{,}000 - 3{,}104.50}{3.7908} = \frac{51{,}895.50}{3.7908} = \boxed{13{,}690 \text{ SAR per year}}.$$

Step 2: the cushion.

The base estimate is 16,000 SAR per year, so

$$\text{cushion} = \frac{16{,}000 - 13{,}690}{16{,}000} = \boxed{14.4\%}.$$

The annual net can slip 14.4% before the accept verdict flips.

Step 3: state it in revenue terms.

The operating cost is held at base, so the break-even revenue is

$$R^* = 13{,}690 + 26{,}000 = 39{,}690 \text{ SAR per year},$$

which is only $(42{,}000 - 39{,}690)/42{,}000 = \boxed{5.5\%}$ below the forecast. That is a thin cushion on the revenue side, and it matches the graph of Example 12.2, where the revenue line crosses PW $= 0$ near -6%.

Step 4: verify.

Substitute A^* back: $-55{,}000 + 13{,}690\,(3.7908) + 3{,}104.50 \approx 0$. The solve checks out.

Insight. Report the cushion, not just the raw break-even value. ''The break-even net is 13,690 SAR'' forces the reader to do arithmetic; ''the net can slip 14.4% before the verdict flips, but the revenue alone only 5.5%'' is a sentence a decision maker can act on.

In practice. A break-even value also needs a physical check. If the break-even throughput sits above the plant's nameplate capacity, the project cannot reach it at any price, and the algebra alone will not say so.

Scenario analysis. Bundle the inputs into coherent stories, pessimistic, most likely, and optimistic, and compute one PW per story.

12.5 *Scenario Analysis*

One-way analysis has a blind spot: it moves each input *alone*, but in a bad year the inputs move *together*. Lower demand means less revenue and, often, a higher cost per tonne. Scenario analysis repairs this by bundling the inputs into coherent stories and computing one present worth per story.

Example 12.5 — Three scenarios for the crushing unit

Wadi Phosphate's planning team writes three stories for the crushing unit's five years:

	pessimistic	most likely	optimistic
annual revenue	36,000	42,000	46,000
operating cost	28,000	26,000	24,000
salvage	3,000	5,000	7,000

The investment (55,000 SAR), the life (5 years), and the MARR (10%) are common to all three. Compute the present worth of each scenario.

Solution.

Net the flows first: the annual nets are 8,000, 16,000, and 22,000 SAR per year.

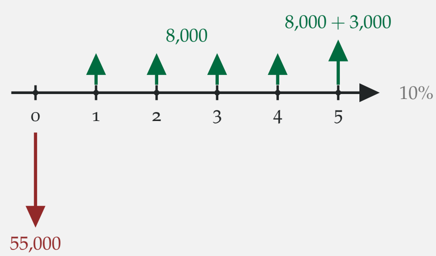

Figure 12.7. The pessimistic story: the same investment, but a thinner net and a smaller salvage. The optimistic story has the same shape with 22,000 SAR nets and a 7,000 SAR salvage.

One present worth per story, with the same two factors as before:

$$\begin{aligned} \mathrm{PW}_{\text{pess}} &= -55{,}000 + 8{,}000\,(3.7908) + 3{,}000\,(0.6209) = \boxed{-22{,}811 \text{ SAR}}, \\ \mathrm{PW}_{\text{likely}} &= -55{,}000 + 16{,}000\,(3.7908) + 5{,}000\,(0.6209) = \boxed{+8{,}757 \text{ SAR}}, \\ \mathrm{PW}_{\text{opt}} &= -55{,}000 + 22{,}000\,(3.7908) + 7{,}000\,(0.6209) = \boxed{+32{,}744 \text{ SAR}}. \end{aligned}$$

	pessimistic	most likely	optimistic
annual net (SAR/yr)	8,000	16,000	22,000
PW(10%) (SAR)	**−22,811**	**+8,757**	**+32,744**

Table 12.4. The scenario table for the crushing unit: one present worth per coherent story.

The spread, from about −22,800 to +32,700 SAR, *is* the project's risk picture: three numbers, one per story, instead of a single point estimate.

Insight. Each scenario column must be a *coherent story*. Do not pair optimistic revenue with pessimistic cost in the same column unless there is a reason they would happen together. The value of scenario analysis over one-way sensitivity is exactly that it respects how inputs move together; an incoherent column throws that value away.

12.6 *Expected Present Worth*

Reference. Park, 3rd ed., Section 11.3 (pp. 493–495).

Expected present worth. The probability-weighted average of the scenario present worths: $E[\mathrm{PW}] = \sum_j p_j \, \mathrm{PW}_j$.

The first three tools are deterministic: no probabilities anywhere. When the planning team can attach probabilities to the scenarios, the analysis becomes probabilistic, and the scenarios become *states* with weights p_j that sum to one. The center of the risk picture is then the *expected present worth*,

$$E[\mathrm{PW}] = \sum_j p_j \, \mathrm{PW}_j. \tag{12.1}$$

In practice. The probabilities are estimates too, and usually the softest numbers in the table. Before reporting $E[\mathrm{PW}]$, move them within a plausible range and check whether the verdict survives.

Example 12.6 — Expected present worth of the crushing unit

The planning team reads the demand outlook as one chance in four of the pessimistic state, one half for the most likely, and one in four for the optimistic: $p = (0.25,\ 0.50,\ 0.25)$. Using the scenario worths of Example 12.5, compute the expected present worth and describe the risk that remains.

Solution.

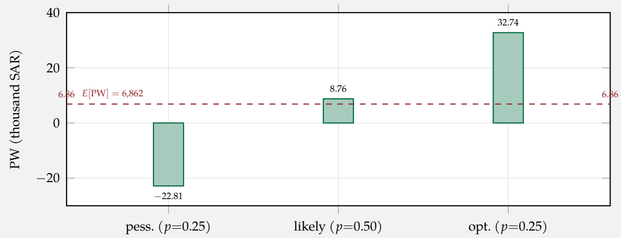

Figure 12.8. The three state worths of the crushing unit and their probability-weighted center. The average is positive, but one of the three bars is deeply negative.

Weight each state worth by its probability and add, term by term:

$$\begin{aligned} E[\mathrm{PW}] &= 0.25\,(-22{,}811) + 0.50\,(8{,}757) + 0.25\,(32{,}744) \\ &= -5{,}703 + 4{,}379 + 8{,}186 = \boxed{+6{,}862 \text{ SAR}}. \end{aligned}$$

The expected present worth is positive, so the project is acceptable *on average*. But the verdict needs a second sentence: there is a 25% chance of losing about 22,800 SAR, the pessimistic state. A risk-averse owner may demand more than a positive average before committing.

Insight. Report *both* numbers: the center ($E[\mathrm{PW}]$) and the risk (the probability and size of the losing state). When an exam question asks for a verdict, write one sentence that carries the numbers: "accept; the expected present worth is +6,862 SAR, the revenue cushion is 5.5%, and there is a 25% chance of a loss near 22,800 SAR."

Reference. Park, 3rd ed., Section 11.4 (pp. 504–511).

12.7 *Where the MARR Comes From: Cost of Capital and Risk*

Every present worth in this chapter was discounted at "the MARR," and since Chapter 5 the MARR has simply been given in each problem, with a note that setting it involves the cost of capital, risk, and opportunity cost. This section opens that box. A firm builds its MARR in two steps: first it prices the money it invests, then it adds a premium for how uncertain the project's cash flows are. Both steps belong in this chapter, because the second one is a standing, averaged answer to the question the rest of the chapter answers explicitly.

Cost of capital. The weighted average rate a firm pays for its investment money: each financing source's share times that source's rate. Abbreviated WACC, for weighted average cost of capital.

12.7.1 *The cost of capital*

Project money comes from somewhere, and none of it is free. Firms finance investments from two sources. *Debt*, bank loans and bonds, must be paid its interest rate. *Equity*, the owners' money, carries no invoice, but the owners expect the return they could earn on other investments of similar risk, and a firm that persistently pays them less loses them. The price of the firm's money as a whole is the weighted average of the two:

$$k = \underbrace{\frac{D}{D+E}\, i_D}_{\text{debt share}\times\text{debt rate}} + \underbrace{\frac{E}{D+E}\, i_E}_{\text{equity share}\times\text{equity return}}, \tag{12.2}$$

where D and E are the amounts financed by debt and equity. One refinement is worth a margin of memory: interest on debt is tax-deductible, so in an after-tax analysis the debt rate enters as $i_D\,(1-t)$ with the tools of chapter 10; the examples here stay before tax.

Example 12.7 — The blended price of Wadi Phosphate's money

Wadi Phosphate funds its capital program with 60% debt and 40% equity. The banks charge 6% interest, and the owners expect 15% on their money. The firm is raising 20 million SAR for new processing equipment. Find the cost of capital.

Solution.

Price each source's share of the 20 million SAR, then blend.

source	amount (SAR)	rate	charge (SAR/yr)
debt	12,000,000	6%	720,000
equity	8,000,000	15%	1,200,000
total	20,000,000		1,920,000

Table 12.5. The financing mix behind the 20 million SAR raise. The blended charge is the firm's cost of capital.

The money as a whole costs 1,920,000/20,000,000 per year, or directly from Equation (12.2):

$$k = 0.60\,(6\%) + 0.40\,(15\%) = 3.6\% + 6.0\% = \boxed{9.6\%}.$$

A project financed from this pool must earn at least 9.6% just to pay the bank its interest and the owners their expected return. Below 9.6%, someone who supplied the money is not being paid; that is why the MARR can never sit below the cost of capital.

Risk-adjusted MARR. A hurdle rate set above the cost of capital by a premium that grows with the risk class of the project.

12.7.2 *The risk-adjusted MARR*

The cost of capital is the floor, not the MARR itself. The cash flows this chapter has been stress-testing are estimates, and a project whose estimates are shakier should have to promise more. Most firms encode this as policy: a table of premiums by project class, added on top of the cost of capital, so that riskier classes face higher hurdles without a bespoke risk study for every small proposal.

project class	risk premium	MARR
cost reduction in an existing operation	≈ 0	10%
capacity expansion, known market	+2 to 3	12%
new product or new market	+6 to 7	16%
unproven process or technology	+10 or more	20%

Table 12.6. A firm-policy table of risk-adjusted MARRs for Wadi Phosphate, whose cost of capital is 9.6%. The premium is a standing charge for uncertainty, graded by how far the project strays from what the firm already knows how to do.

Read Table 12.6 from the top. Replacing a worn screen in a plant the firm has run for years has cash flows it can estimate within a few percent, so the hurdle barely clears the cost of capital. A plant built on an unproven process can miss its estimates by half, so the same firm demands 20%. Two cautions belong next to any such table. First, a premium buried in the discount rate compounds: it punishes year-10 cash flows far more than year-1 cash flows, which is right when risk grows with distance and wrong when it does not. Second, the premium is a blunt instrument, an *average* charge for a class, where the tools of this chapter measure the risk of *this* project.

Insight. Chapter 5 defined the MARR in the margin as the lowest return the firm will accept, set by "the cost of capital, risk, and opportunity cost." Those words now have numbers: the cost of capital is the floor (Equation (12.2)), and risk enters as the premium above it. The premium and this chapter's tools are substitutes for one another. A team that has measured a project's fragility, its steepest sensitivity line, its break-even cushion, its scenario spread, can argue for a premium fitted to that project; a team that has not gets the class average.

12.8 *Case Study from the Literature*

The tools of this chapter are the standard risk section of a published techno-economic assessment. This section reads one such study with the chapter's eyes: an open-access analysis of hydrogen production economics (Curcio, 2025) whose headline findings are a base-case cost model, a sensitivity ranking, and a set of break-even prices, exactly the sequence of Sections 12.2 through 12.4.

Example 12.8 — Case study: sensitivity and break-even in a hydrogen techno-economic assessment

Curcio (2025) compares three hydrogen production routes, gray (from natural gas), blue (natural gas with carbon capture), and green (renewable electrolysis), using a net-present-value model with a 7% weighted average cost of capital and a 20-year project life; the paper's currency is US dollars. Its sensitivity analysis reports that the hydrogen selling price is the dominant input: a $1-per-kg increase in the selling price improves the NPV of a 10 MW plant by over $10 million. (a) Verify that claim with this chapter's per-unit sensitivity arithmetic. (b) Read the paper's break-even prices as this chapter's break-even solve.

Solution.

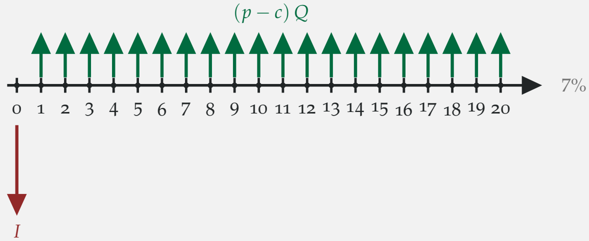

Figure 12.9. The paper's model in this book's shape: a plant investment I followed by 20 years of net revenue $(p - c)\,Q$, where p is the selling price, c the production cost, and Q the annual output in kg, discounted at the 7% cost of capital.

Step 1: the factor that carries the claim.

At the paper's 7% cost of capital over 20 years,

$$(P/A,\,7\%,\,20) = \frac{(1.07)^{20} - 1}{0.07\,(1.07)^{20}} = \frac{2.8697}{0.2709} = 10.594.$$

Step 2: invert the sensitivity claim.

The linearity insight of Section 12.3 applies unchanged: a price change of Δp on an annual output Q moves the present worth by $\Delta p \times Q \times (P/A,\,7\%,\,20)$. For the paper's claim,

$$1 \times Q \times 10.594 \geq 10{,}000{,}000 \quad \Rightarrow \quad Q \geq \frac{10{,}000{,}000}{10.594} = \boxed{944{,}000 \text{ kg per year}}.$$

Step 3: check the implied output against the plant.

The paper puts green electrolyzer efficiency at 60–70%; with hydrogen's energy content of about 33.3 kWh per kg, that is 47.6–55.5 kWh of electricity per kg. A 10 MW plant running every hour of the year consumes $10{,}000 \times 8{,}760 = 87{,}600$ MWh, so its full-load output is 1.58–1.84 million kg per year. An output of 944,000 kg per year is a capacity factor of 51–60%, which is how an electrolyzer fed by solar and wind actually runs. The paper's "over \$10 million per \$1/kg" is the chapter's per-percent arithmetic at industrial scale, and it checks out.

Step 4: the break-even prices.

pathway	LCOH ($/kg)	break-even ($/kg)
gray	1.50–2.50	1.50–2.00
blue	2.00–3.50	2.50–3.50
green	3.50–6.00	4.50–6.00

Table 12.7. Levelized cost of hydrogen and break-even selling price by production pathway, from Curcio (2025), in US dollars. Each break-even price is a PW = 0 solve with the selling price as the unknown.

The paper's break-even selling prices are the solve of Section 12.4 with the price as the uncertain input: the selling price at which the 20-year NPV crosses zero. The cushion reading carries over too. A gray-hydrogen producer selling at $2.50 against a $2.00 break-even has a 20% cushion, and the paper then shows what eats it: a carbon price above $100 per tonne of CO_2 makes gray hydrogen uncompetitive, and at $200 per tonne its cost reaches nearly $4 per kg. For green hydrogen the paper runs a two-input break-even of the kind in Exercise 12.6: electricity below $20–30 per MWh is required for cost parity with the fossil routes, while at $60 per MWh the route needs a selling price above $6 per kg.

Step 5: the ranking, and what it is for.

The paper's tornado ranks the selling price first, the feedstock second (electricity is 50–70% of green operating cost), and the CAPEX last: even a 30% cut in electrolyzer cost lowers the production cost by only about $1 per kg. Exactly as in Example 12.3, the ranking directs the effort. The paper's own recommendation is the one the ranking implies: secure long-term electricity procurement contracts, because the top of the tornado, not the equipment quote, decides the project.

12.9 *Chapter Summary*

- A **point estimate** gives one PW and no measure of risk. There is no guarantee the estimates match the actual values, and a single PW cannot say how likely the project is to lose money. The repair is **what-if analysis**.

- Every tool starts from the **base case**: the PW computed from the most-likely estimate of every input.

- **One-way sensitivity analysis** varies one input at a time by $\pm 10\%$ and $\pm 20\%$, holding the rest at base. On the sensitivity graph, all lines pass through the base case; **the steeper the line, the more dangerous the input**. Where a line crosses $\text{PW} = 0$ is that input's break-even point.

- Because the PW is linear in each input, each table cell is the base PW plus the input change times its factor; no full recomputation is needed.

- Ranking inputs by their total **PW swing** (a tornado chart) tells the team where estimation effort belongs: firm up the top of the ranking, do not refine the bottom.

- **Break-even analysis** sets $\text{PW} = 0$ and solves for the uncertain input, the same algebra as solving for i^* in Chapter 7 but with the MARR held fixed. Report the **cushion**: the percentage gap between the base estimate and the break-even value.

- **Scenario analysis** bundles the inputs into coherent pessimistic, most-likely, and optimistic stories, one PW per story; the spread of the three worths is the risk picture. Each column must be a coherent story.

- With probabilities attached, $E[\text{PW}] = \sum_j p_j \text{PW}_j$. Accept on average when $E[\text{PW}] > 0$, but always report the **center and the risk**: the probability and size of the losing state.

- The MARR is built in two steps. The **cost of capital** is the blended price of the firm's money, $k = (\text{debt share})\, i_D + (\text{equity share})\, i_E$, and no MARR can sit below it. The **risk-adjusted MARR** adds a premium by project class; the premium is a standing, averaged substitute for the explicit risk measurement of this chapter.

- Match the tool to the question. *Which input matters* is sensitivity; *how far it can slip* is break-even; *what a bad year looks like* is a scenario; *what happens on average* is the expectation.

12.10 Exercises

These problems work through the chapter's four tools in the order they were introduced: break-even values and cushions, one-way sensitivity tables, coherent scenario bundles, and expected present worth, with the risk-adjusted MARR appearing in the later items. They start from short warm-ups on a single input and build to full risk sections read as a reviewer would read them.

Break-even warm-ups and lecture practice

Exercise 12.1. A project requires 30,000 SAR today and returns a uniform net amount A per year for 4 years, with no salvage. The MARR is 10%. Find the break-even value A^* and interpret it.

Exercise 12.2. *Najd Copper* studies a satellite ore-testing lab near its new pit. The investment is 60,000 SAR, the salvage is 6,000 SAR after 5 years, the most-likely annual net cash flow is 18,000 SAR per year, and the MARR is 10%. (a) Compute the base-case PW. (b) Find the break-even annual net A^* and the percentage cushion. (c) If the net comes in 15% below the estimate, is the lab still acceptable?

Quick checks (multiple choice)

Exercise 12.3. From the crushing unit's sensitivity analysis (Example 12.2), which statement is **incorrect**?

(A) The PW responds more to revenue than to any other input.
(B) The salvage line is nearly flat, so refining that estimate adds little.
(C) The project stays acceptable if revenue falls 10%.
(D) The operating-cost line slopes downward: higher cost, lower PW.

Exercise 12.4. A project's PW depends on the demand state: PW $= -30{,}000$ SAR with $p = 0.25$, PW $= +12{,}000$ SAR with $p = 0.45$, and PW $= +40{,}000$ SAR with $p = 0.30$. The expected PW is closest to:

(A) 7,333 SAR
(B) 9,900 SAR
(C) 12,000 SAR
(D) 22,000 SAR

Exercise 12.5. An investment of 90,000 SAR returns a uniform net amount A for 5 years, with no salvage, at a MARR of 10%. The break-even value of A is closest to:

(A) 18,000 SAR
(B) 21,000 SAR
(C) 23,742 SAR
(D) 26,000 SAR

How much is the old truck worth? (in-class activity)

The in-class activity for this part of the course combines the uncertainty tools of this chapter with the replacement decision of chapter 16. At *Najd Copper*, keeping the current ore-transfer truck for 3 more years has an annual equivalent cost that depends on the truck's resale value MV today:

$$AEC_D(MV) = (MV - 5{,}000)\,(A/P,\ 10\%,\ 3) + 0.10\,(5{,}000) + 9{,}000,$$

with $(A/P, 10\%, 3) = 0.4021$. The best challenger's minimum annual equivalent cost is $AEC^* = 21{,}237.80$ SAR per year (computed in the replacement part of the activity, Exercise 16.7). The current resale estimate is $MV = 20{,}000$ SAR, which gives $AEC_D = 15{,}531.50$ SAR per year, so the recommendation is to keep the truck. The two exercises below are the uncertainty part of the activity.

Exercise 12.6. A dealer hints he might pay *more* than 20,000 SAR for the old truck. A higher offer raises the defender's opportunity cost, and could flip the verdict. Find the break-even market value MV^* at which the two options tie.

Exercise 12.7. Today's resale estimate of 20,000 SAR is uncertain by a few thousand SAR either way. In one line: is the keep decision robust to that uncertainty? (Compare the size of the plausible error with the gap to MV^* from Exercise 12.6.)

Exam-level practice

Exercise 12.8. *Najd Copper* studies a portable ore-sorting unit for its satellite pit: investment 80,000 SAR; life 5 years; salvage 8,000 SAR; MARR $= 10\%$. The annual net cash flow depends on how much ore the pit delivers:

state	pessimistic	most likely	optimistic
probability	0.30	0.50	0.20
annual net (SAR/yr)	16,000	22,000	27,000

(a) Compute the PW of each state (factor notation first). (b) Compute the expected PW. (c) Compute the break-even annual net A^* and the percentage cushion below the most-likely estimate. (d) Write the verdict sentence, carrying $E[\text{PW}]$, the cushion, and the probability of a negative-PW outcome.

Exercise 12.9. The plant manager at the *Wadi Phosphate* concentrator studies an automated reagent-dosing line for the flotation circuit. Base-case estimates: first cost 130,000 SAR; annual revenue 82,000 SAR; annual operating cost 46,000 SAR; life 5 years; salvage 15,000 SAR; MARR $= 12\%$. Work on a before-tax basis. (a) Draw the cash flow diagram and compute the base-case PW. Should the manager accept at the base case? (b) *One-way sensitivity.* Recompute the PW when the annual revenue varies by $\pm 10\%$ (cost at base), and when the annual operating cost varies by $\pm 10\%$ (revenue at base). Which input is more critical, and how can you tell? (c) *Break-even revenue.* Keeping every other estimate at base, set PW $= 0$ and solve for the annual revenue R^*; report the cushion as a percentage. (d) *Worst-case scenario.* Revenue falls 10% *and* operating cost rises 10% in the same years. Compute the scenario PW, and state in one sentence what scenario analysis captures that one-way sensitivity does not.

Exercise 12.10. *Wadi Lithium* finances its projects half with debt at 6% interest and half with equity on which the owners expect 14%. Company policy adds a 5-percentage-point risk premium to the cost of capital for new-market projects. A proposed spodumene trading terminal, a new market for the firm, requires 50,000 SAR and returns a net 14,000 SAR per year for 5 years, with no salvage. (a) Compute the firm's cost of capital. (b) Compute the terminal's PW at the cost of capital and at the risk-adjusted MARR. (c) In one sentence: what did the premium change, and why does the firm charge it?

Exam-style problems

Exercise 12.11. *Wadi Lithium* plans a booster station on the brine transfer line from its wellfield to the evaporation ponds: investment 120,000 SAR; life 5 years; salvage 10,000 SAR; MARR $= 10\%$. The annual net cash flow depends on how much brine the wellfield delivers:

state	pessimistic	most likely	optimistic
probability	0.20	0.55	0.25
annual net (SAR/yr)	24,000	32,000	38,000

(a) (*10 pts*) Compute the PW of each state (factor notation first). (b) (*5 pts*) Compute the expected PW. (c) (*6 pts*) Compute the break-even annual net A^* and the percentage cushion below the most-likely estimate. (d) (*4 pts*) Write the verdict sentence, carrying $E[\text{PW}]$, the cushion, and the probability of a negative-PW outcome.

Exercise 12.12. The utilities engineer at *Najd Copper* studies a water-recycling plant that supplies dust suppression and mill make-up water. Base-case estimates: first cost 95,000 SAR; annual saving in purchased water 61,000 SAR; annual operating cost 33,000 SAR; life 5 years; salvage 9,000 SAR; MARR $= 12\%$. Work on a before-tax basis. (a) (*6 pts*) Draw the cash flow diagram and compute the base-case PW. Should the engineer accept at the base case? (b) (*8 pts*) *One-way sensitivity.* Recompute the PW when the annual saving varies by $\pm 10\%$ (cost at base), and when the operating cost varies by $\pm 10\%$ (saving at base). Which input is more critical, and how can you tell? (c) (*6 pts*) *Break-even saving.* Keeping every other estimate at base, set PW $= 0$ and solve for the annual saving R^*; report the cushion as a percentage. (d) (*5 pts*) *Worst-case scenario.* The saving falls 10% *and* the operating cost rises 10% in the same years. Compute the scenario PW, and state in one sentence what the one-way table missed.

Assessing outside recommendations

Real recommendations arrive finished: a memo, a sheet, a verdict. The two problems below are audits. Decide whether to accept the recommendation, name every flaw, and correct the numbers. The flawed work is quoted exactly as received.

Exercise 12.13. The planning team at *Wadi Lithium* evaluated a reagent-recovery unit for its carbonate circuit: investment 75,000 SAR; life 5 years; MARR $= 10\%$. The team's scenario estimates, and its demand memo, read:

	pessimistic	most likely	optimistic
annual net (SAR/yr)	13,000	21,000	26,000
salvage (SAR)	4,000	6,000	8,000
probability (demand memo)	0.30	0.40	0.30

An AI assistant summarized the analysis as follows:

> *"Scenario present worths at 10%: pessimistic* $-23{,}236$ *SAR, most likely* $+8{,}332$ *SAR, optimistic* $+28{,}528$ *SAR. Expected present worth:* $E[\text{PW}] = 0.30\,(-23{,}236) + 0.50\,(8{,}332) + 0.30\,(28{,}528) = +5{,}754$ *SAR. The expected present worth is positive, so the unit should be accepted."*

Accept or reject the summary? Name each flaw and correct the numbers.

Exercise 12.14. A vendor proposes a stacker-reclaimer for *Najd Copper*'s concentrate stockyard: first cost 120,000 SAR; added annual revenue 88,000 SAR; annual operating cost 52,000 SAR; life 5 years; salvage 12,000 SAR; MARR $= 12\%$. The proposal's risk section reads:

> *"Base case: the present worth at 12% is* $+16{,}582$ *SAR. One-way sensitivity on demand: with demand 10% lower, revenue falls to 79,200 SAR and, since handling costs scale with tonnage, operating cost falls to 46,800 SAR; the PW is* $+3{,}604$ *SAR, so the project survives a 10% demand slip. Break-even: the annual net can fall from 36,000 to 31,400 SAR before the PW reaches zero, a cushion of 12.8%, so the revenue forecast can slip 12.8% before the verdict changes."*

Accept or reject the risk section? Name each flaw and correct the numbers.

References

Curcio, E. (2025). Techno-economic analysis of hydrogen production: Costs, policies, and scalability in the transition to net-zero. *arXiv:2502.12211* [*econ.GN*]. `https://arxiv.org/abs/2502.12211`

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