10 *Depreciation and Income Taxes*

After this chapter you will be able to:

1. state which property is depreciable, and compute the cost basis I and the book value BV_n
2. build depreciation schedules by straight line, by declining balance with the salvage floor and the switch to straight line, and by units of production
3. compute MACRS tax depreciation from the published class percentages, $D_n = I\,p_n$, under the half-year convention
4. compute taxable income, income tax, and net income using the marginal tax rate t
5. compute the after-tax cash flow $(R - E)(1 - t) + t\,D_n$, and explain the depreciation tax shield $t\,D_n$
6. compute the net proceeds $S - t\,(S - BV)$ on disposal, and evaluate a project's present worth after tax

The evaluation toolbox is complete: present worth (chapter 5), annual equivalence (chapter 6), and the rate of return (chapter 7), all resting on the time-value machinery of Chapter 2. Everything so far, however, shared one simplification: every cash flow was *pre-tax*. We discounted revenues and costs as if no one taxed the profit. Real firms do pay tax on their income, and the tax depends on how the cost of equipment is spread over the years, because that spread is deducted from profit before the tax is computed. The spreading is called *depreciation*, and it is the accounting bridge between buying an asset and the taxes paid in each year the asset serves. This chapter first builds depreciation schedules four ways: straight line, declining balance, units of production, and the tabulated tax schedule MACRS. It then runs the tax pipeline, from taxable income to net income to the after-tax cash flow, quantifies the tax saved by each unit of depreciation, settles the gain or loss when an asset is sold, and closes with a complete after-tax project evaluation. Please note what does *not* change: present worth at the MARR still makes the decision, exactly as in chapter 5. Taxes change the cash flows that go into the tools, not the tools themselves. The two chapters that follow add the remaining layers of realism: estimates can be wrong (chapter 12), and assets age and must eventually be replaced (chapter 16).

What depreciation does to cash.

Q1. A bagging machine at a phosphate plant is bought for 27,000 SAR. Freight to the site costs 1,500 SAR and installation costs another 1,500 SAR. The machine has a 4-year depreciable life and a salvage value of 6,000 SAR, depreciated straight line. What is the first-year depreciation charge?

Q2. Depreciation is a non-cash charge: when it is recorded, no money leaves the firm. Can it still change the amount of cash the firm keeps in a year?

Q3. An asset with a book value of 9,000 SAR is sold for 13,000 SAR. Gains on disposal are taxed at 20%. How much cash does the seller keep?

Answers.

Q1. The cost basis is everything needed to place the machine in service: $27{,}000 + 1{,}500 + 1{,}500 = 30{,}000$ SAR. The straight-line charge is $(30{,}000 - 6{,}000)/4 = 6{,}000$ SAR per year. Freight and installation are part of the asset's cost, not expenses of the year.

Q2. *Yes.* Depreciation is deducted from taxable income, so each SAR of depreciation cuts the tax bill by t SAR, where t is the tax rate. The tax is very much cash. This saving, $t \times D$, is the depreciation tax shield, and it is the central mechanism of this chapter.

Q3. Only the gain above book value is taxed: gain $= 13{,}000 - 9{,}000 = 4{,}000$ SAR, tax $= 0.20 \times 4{,}000 = 800$ SAR, so the net proceeds are $13{,}000 - 800 = 12{,}200$ SAR. Taxing the full selling price is the common error.

Lecture 21. This section is covered by Lecture 21.

Reference. Park, 3rd ed., Sections 9.1–9.2 (pp. 386–402).

Depreciation. The systematic allocation of an asset's cost over its depreciable life. A non-cash accounting charge: no money leaves the firm when it is recorded.

10.1 Depreciable Property, Cost Basis, and Book Value

Depreciation is the systematic allocation of an asset's cost over its depreciable life. Please note the second half of that definition carefully: it is a **non-cash** charge. When a firm records depreciation on a machine, no money leaves the bank account. The purchase already happened, possibly years ago. Depreciation only spreads that old cost across the years the machine serves, so that each year's income statement carries its share of the cost of the services the machine provides.

Depreciable property. Property used in business, with a definite service life longer than one year, that wears out, decays, gets used up, or becomes obsolete.

Not everything a firm owns can be depreciated. *Depreciable property* must pass three tests, and it must pass all three:

1. it is used in business or held to produce income;
2. it has a definite service life, and that life is **longer than one year**;
3. it is something that wears out, decays, gets used up, or becomes obsolete.

Buildings, machinery, equipment, and vehicles all qualify, and so do some intangibles such as patents. Two things never qualify. **Land** has no definite service life, so land is never depreciated. **Inventories** are held for sale to customers, not used in the business, so they are not depreciated either.

Cost basis. I: the total cost of placing an asset in service — purchase price plus freight, site preparation, and installation.

The first quantity that anchors every schedule is the *cost basis*, written I. It is not just the sticker price. It is everything the firm had to spend to place the asset *in service*: the purchase price, plus freight, site preparation, and installation. The reasoning is the matching principle of accounting: all of these payments bought the services the asset will deliver, so all of them are allocated against the revenue those services will generate. One boundary case is worth memorizing: the cost of *training the operators* prepares the people, not the asset, so it is treated as an operating expense of the period and is *not* capitalized into the basis.

Example 10.1 — The cost basis of a flotation cell

Najd Copper, a copper concentrator near Hafar Al-Batin, installs a new flotation cell with a purchase price of 46,000 SAR. Freight costs 1,200 SAR, site preparation costs 1,300 SAR, and installation costs 1,500 SAR. The cell has an estimated salvage value of 5,000 SAR after a 5-year depreciable life. Find the cost basis.

Solution.

Purchase price	46,000 SAR
Freight	1,200
Site preparation	1,300
Installation	1,500
Cost basis I	**50,000 SAR**

Table 10.1. The cost basis of the flotation cell: everything required to place it in service.

All four payments were required to put the machine to work, so all four are capitalized:

$$I = 46{,}000 + 1{,}200 + 1{,}300 + 1{,}500 = \boxed{50{,}000 \text{ SAR}}.$$

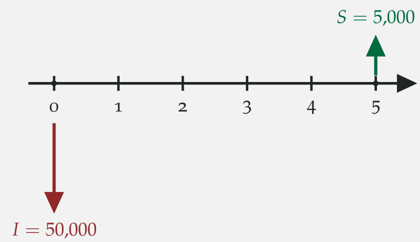

Figure 10.1. The flotation cell's actual cash flows: the full cost basis leaves at year 0 and the salvage value returns at year 5. The depreciation charges themselves never appear on a cash flow diagram, because they are not cash.

Please study Figure 10.1 for a moment. The only cash flows the flotation cell generates directly are the purchase at year 0 and the salvage at year 5. Every depreciation schedule in this chapter re-describes the same two arrows; none of them adds or removes cash. What a schedule changes is the *tax* paid in each year, and that is where this chapter is heading.

Book value. $BV_n = I - (D_1 + D_2 + \cdots + D_n)$: the cost basis minus the accumulated depreciation through year n.

The second anchoring quantity is the *book value*. After n years of charges $D_1, D_2, \ldots, D_n$, the book value is the cost basis minus the *accumulated* depreciation:

$$BV_n = I - \sum_{k=1}^{n} D_k. \tag{10.1}$$

It is the asset's worth on the accounting books at that moment, and it returns at the end of the chapter, when the asset is sold and the selling price is compared against it.

To build any schedule we need four inputs: the cost basis I, the salvage value

S (the estimate of what the asset sells for at the end), the depreciable life N in years, and a *method*. The methods are the next four sections.

Book depreciation. Depreciation computed for financial reports and pricing. The firm chooses the method (SL, DB, or UoP) that best matches the asset's actual loss of value.

Tax depreciation. Depreciation computed for the income-tax return, using the method the tax authority mandates. Usually accelerated: larger charges in the early years.

10.2 *Book Depreciation and Tax Depreciation*

Most firms compute depreciation twice, and both computations are legitimate. *Book depreciation* feeds the financial reports that go to stockholders and outsiders, and it supports pricing decisions. Its goal is matching: the charge each year should reflect the asset's actual loss of value, and the firm may choose among the straight-line, declining-balance, and units-of-production methods to achieve that. *Tax depreciation* exists for one purpose, computing income taxes, and the method is not a choice: the tax authority mandates it. In the United States the mandated system is **MACRS**, the Modified Accelerated Cost Recovery System; it is the system our textbook and our exams use as the standard example of an accelerated tax schedule, and the same mechanics apply to any authority's published schedule. The word *accelerated* is the point: the mandated schedules push larger charges into the early years.

In Saudi Arabia, the Zakat, Tax and Customs Authority (ZATCA) administers corporate income tax and zakat: corporate income tax at 20% applies to the taxable income attributable to non-Saudi ownership shares, while Saudi and GCC shares pay zakat instead. Whatever the authority, the depreciation and tax mechanics of this chapter are the same; the problems in this course state a single flat marginal tax rate. Section 10.8 sketches how the Saudi system actually organizes its tax depreciation.

> **Insight.** The total depreciation over an asset's life is the *same* under every method: the depreciable amount is written off in full, one way or another. Only the *timing* of the charges differs. Accelerated tax depreciation means larger deductions early, so taxes are deferred, and everything in Chapter 2 says that money kept earlier is worth more. This is why tax codes that want to encourage investment mandate accelerated schedules.

Straight-line (SL). $D_n = (I - S)/N$ every year: the depreciable amount spread evenly, at the rate $1/N$ per year.

10.3 *Straight-Line Depreciation*

The simplest method treats the asset as giving equal service every year, so it spreads the depreciable amount, cost basis minus salvage, evenly over the life:

$$D_n = \frac{I - S}{N} \qquad \text{for } n = 1, \ldots, N. \tag{10.2}$$

Equivalently, the depreciation rate is $1/N$ per year. The book value falls along a straight line, which gives the method its name, and it lands exactly on the salvage value at the end of the life.

Example 10.2 — A straight-line schedule for the flotation cell

Build the straight-line depreciation schedule for the flotation cell of Example 10.1: $I = 50{,}000$ SAR, $S = 5{,}000$ SAR, $N = 5$ years.

Solution.

The annual charge is

$$D_n = \frac{I - S}{N} = \frac{50{,}000 - 5{,}000}{5} = \boxed{9{,}000 \text{ SAR per year}}.$$

Year n	0	1	2	3	4	5
D_n	—	9,000	9,000	9,000	9,000	9,000
BV_n	50,000	41,000	32,000	**23,000**	14,000	5,000

Table 10.2. The straight-line schedule for the flotation cell. The book value steps down by 9,000 SAR each year and lands exactly on the salvage value.

Two observations. First, the book value falls in equal steps and ends exactly at $S = 5{,}000$ SAR: the schedule writes off the depreciable amount $I - S = 45{,}000$ SAR in full, no more and no less. Second, please note the book value after year 3, $BV_3 = 23{,}000$ SAR. We will reuse this number in Section 10.11, when the plant manager considers selling the cell. A common error on graded work is to forget the salvage value and divide 50,000 by 5; watch for it.

Declining balance (DB). $D_n = \alpha BV_{n-1}$: a fixed fraction $\alpha = \text{multiplier}/N$ of the *remaining* book value each year.

Double declining balance (DDB). Declining balance with multiplier 2.0, so $\alpha = 2/N$. The most common accelerated book method.

10.4 *Declining-Balance Depreciation*

The declining-balance method rests on the observation that most assets give their best service early, so the early charges should be larger. Each year it charges a fixed fraction α of the *remaining book value*, not of the original cost:

$$D_n = \alpha\, BV_{n-1}, \qquad \alpha = \frac{\text{multiplier}}{N}. \tag{10.3}$$

The common multipliers are 1.5 and 2.0. With multiplier 2.0 the method is called *double declining balance* (DDB), with rate $\alpha = 2/N$. Because each charge applies to the balance the previous charges left behind, declining-balance charges always chain through the book values one year at a time: there is no shortcut to the year-4 charge without computing years 1–3 first.

Salvage floor. The book value may never drop below the salvage value S. A declining-balance charge that would breach S is capped at $BV_{n-1} - S$.

Two complications come with the method, one for a high salvage value and one for a low one. The first is the **salvage floor**: an asset may never be depreciated below its salvage value, so a charge that would push the book value below S is cut back to exactly $BV_{n-1} - S$.

Example 10.3 — A DDB schedule with the salvage floor

Build the double-declining-balance schedule for the flotation cell (I = 50,000 SAR, S = 5,000 SAR, N = 5 years).

Solution.

The rate is $\alpha = 2/5 = 40\%$, applied to the prior book value each year:

$$\begin{aligned} D_1 &= 0.40 \times 50{,}000 = 20{,}000, & BV_1 &= 30{,}000,\\ D_2 &= 0.40 \times 30{,}000 = 12{,}000, & BV_2 &= 18{,}000,\\ D_3 &= 0.40 \times 18{,}000 = 7{,}200, & BV_3 &= 10{,}800,\\ D_4 &= 0.40 \times 10{,}800 = 4{,}320, & BV_4 &= 6{,}480. \end{aligned}$$

Year 5 needs care. The uncapped charge would be $0.40 \times 6{,}480 = 2{,}592$ SAR, which would push the book value to 3,888 SAR, below the salvage value of 5,000 SAR. The salvage floor caps the charge at

$$D_5 = BV_4 - S = 6{,}480 - 5{,}000 = \boxed{1{,}480 \text{ SAR}},$$

and the book value lands exactly on 5,000 SAR.

Year n	0	1	2	3	4	5
D_n	—	20,000	12,000	7,200	4,320	**1,480**
BV_n	50,000	30,000	18,000	10,800	6,480	5,000

Table 10.3. The DDB schedule for the flotation cell. The year-5 charge is capped by the salvage floor so that the book value never drops below S = 5,000 SAR.

Please pay attention to that final cap: forgetting the salvage floor and charging the full 2,592 SAR in year 5 is one of the most common errors in this chapter.

Switching rule. In each year, compare the DB charge with the SL charge on the remaining balance, $(BV_{n-1} - S)/(\text{remaining years})$. Once SL is larger, switch to it and stay.

The second complication is the opposite problem. If the salvage value is low, declining balance *never reaches* it, because the method always takes a fraction of what remains. The fix is the **switching rule**: in each year, compute two candidate charges, the declining-balance charge and the straight-line charge on what remains,

$$D_n^{SL} = \frac{BV_{n-1} - S}{\text{remaining years}},$$

and once the straight-line candidate is larger, switch to it and stay with it for the rest of the life.

Example 10.4 — Switching from DDB to straight line

An instrument bench costs 10,000 SAR, has a 5-year life, and no salvage value. Show that pure DDB fails to depreciate it fully, and build the DDB-with-switch schedule.

Solution.

Pure DDB at $\alpha = 2/5 = 40\%$ leaves $BV_5 = 10{,}000\,(0.60)^5 = 778$ SAR of cost that never becomes a deduction. Apply the switching rule year by year, comparing the DDB charge with straight line on the remaining balance:

n	DDB candidate	SL on remainder	taken	BV_n
1	4,000	$10{,}000/5 = 2{,}000$	4,000	6,000
2	2,400	$6{,}000/4 = 1{,}500$	2,400	3,600
3	1,440	$3{,}600/3 = 1{,}200$	1,440	2,160
4	864	$2{,}160/2 = \mathbf{1{,}080}$	**1,080 (switch)**	1,080
5	—	1,080	1,080	0

Table 10.4. The DDB-with-switch schedule. In year 4 the straight-line candidate first exceeds the DDB candidate, so the schedule switches and stays.

In years 1–3 the DDB candidate is larger, so it is taken. In year 4 the straight-line candidate, 1,080 SAR, exceeds the DDB candidate of 864 SAR, so the schedule switches and repeats 1,080 SAR in year 5. The total is

$$4{,}000 + 2{,}400 + 1{,}440 + 1{,}080 + 1{,}080 = \boxed{10{,}000 \text{ SAR}},$$

the full cost basis: the asset is fully depreciated. Remember this switch; the MACRS schedules of Section 10.7 build it in automatically.

Units of production (UoP). Depreciation proportional to usage: $D_n = (u_n/U)(I - S)$, where u_n is the service consumed in year n and U is the asset's total service capacity.

10.5 *Units-of-Production Depreciation*

The third method drops time altogether. Units of production treats the asset as a bundle of service units, kilometers, machine hours, or press cycles, and charges depreciation in proportion to the units consumed:

$$D_n = \frac{u_n}{U}(I - S), \tag{10.4}$$

where u_n is the service consumed in year n and U is the total service the asset is rated for. Choose this method when usage drives the wear, not the calendar: vehicles by kilometers, machines by hours, presses by cycles. A slow year means a small charge; a busy year means a large one.

Example 10.5— Depreciating an ore-transfer truck by kilometers

Najd Copper's ore-transfer truck has a cost basis of 80,000 SAR, a salvage value of 8,000 SAR, and is rated for 200,000 km of service. It was driven 35,000 km this year. Find this year's units-of-production charge.

Solution.

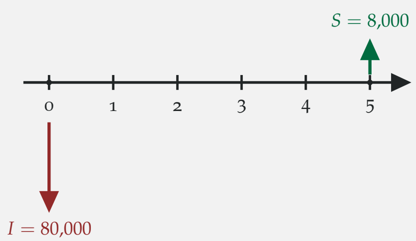

Figure 10.2. The truck's cash flows: purchase now, salvage at retirement. The UoP schedule allocates the difference across the years in proportion to the kilometers driven.

The depreciable amount is $80{,}000 - 8{,}000 = 72{,}000$ SAR, so each kilometer carries

$$\frac{72{,}000}{200{,}000} = 0.36 \text{ SAR per km.}$$

This year's charge follows this year's usage:

$$D = \frac{35{,}000}{200{,}000}(80{,}000 - 8{,}000) = 0.36 \times 35{,}000 = \boxed{12{,}600 \text{ SAR}}.$$

10.6 *Three Methods, One Asset*

The three book methods are easiest to compare on a single asset, side by side. The next example builds all three schedules for the flotation cell and plots the three book-value paths; this is exactly the exercise your team races through in the in-class schedule sprint.

Example 10.6 — Three schedules for one flotation cell

The flotation cell ($I = 50{,}000$ SAR, $S = 5{,}000$ SAR, $N = 5$ years) is expected to process 20, 30, 35, 40, and 25 thousand tonnes of ore over its five years, 150,000 tonnes in total. Build the SL, DDB, and UoP schedules and compare the book-value paths.

Solution.

The SL and DDB columns come from Example 10.2 and Example 10.3. For units of production, each tonne carries $45{,}000/150{,}000 = 0.30$ SAR of depreciation, so the annual charges follow the processing plan: $0.30 \times 20{,}000 = 6{,}000$ SAR in year 1, and so on.

	SL		**DDB**		**UoP**	
n	D_n	BV_n	D_n	BV_n	D_n	BV_n
1	9,000	41,000	20,000	30,000	6,000	44,000
2	9,000	32,000	12,000	18,000	9,000	35,000
3	9,000	23,000	7,200	10,800	10,500	24,500
4	9,000	14,000	4,320	6,480	12,000	12,500
5	9,000	5,000	1,480	5,000	7,500	5,000
Σ	**45,000**		**45,000**		**45,000**	

Table 10.5. Three depreciation schedules for the same flotation cell. Every column of charges

sums to the depreciable amount of 45,000 SAR.

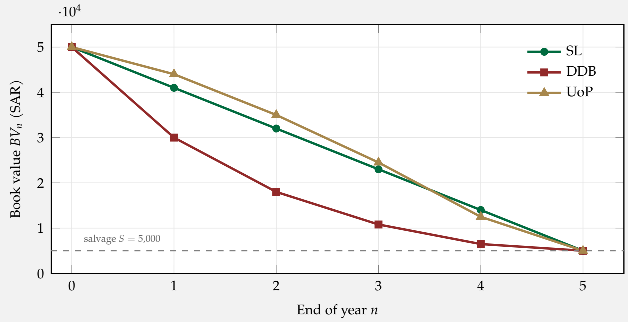

Figure 10.3. The three book-value paths for the flotation cell. All start at the cost basis and end on the salvage value; only the path between the endpoints differs.

Check the bottom row of Table 10.5: every column sums to $\boxed{45{,}000 \text{ SAR}}$, the depreciable amount. In Figure 10.3, all three paths start at 50,000 SAR and end at 5,000 SAR. Straight line is the straight diagonal. DDB dives early and flattens out. UoP sags slowly at first, then drops in the busy middle years. Same total, same endpoint, different path.

Insight. Same total, different timing: that is the entire difference between depreciation methods. Why should anyone care which path the book value takes? Because each year's charge is deducted from that year's taxable income. A method that front-loads the charges front-loads the tax deductions, and Section 10.10 shows that each deduction is worth real cash. Front-loaded deductions are the time value of money, Chapter 2, applied to the tax bill.

In practice. For a cash-constrained young firm this timing is not an accounting nicety. An accelerated schedule is, in effect, an interest-free loan from the tax authority: the same total tax, collected later, and later is exactly what a company burning cash needs.

Reference. Park, 3rd ed., Section 9.3.

10.7 *MACRS: Tax Depreciation in the United States*

The tax side abandons the idea of estimating each asset's useful life. Under MACRS, the tax depreciation system of the United States and the standard example used by our textbook and exams, every asset falls into a *property class* with a fixed recovery period. Table 10.6 lists the classes used in this course.

MACRS. The Modified Accelerated Cost Recovery System: the tax depreciation schedule of the United States, used in this book as the standard example of an accelerated tax schedule.

Property class. The fixed recovery period MACRS assigns to each kind of asset (3, 5, 7, 10 years for equipment; longer, straight-line-only periods for buildings).

Class	Examples
3-year	special manufacturing tools
5-year	automobiles, light trucks, computers, high-tech equipment
7-year	most manufacturing equipment, office furniture
10-year	vessels, barges, long-lived equipment
27.5 / 39-year	residential rental / commercial buildings (SL only)

Table 10.6. MACRS property classes. Real property (buildings) is depreciated straight line over the long periods; land is never depreciated.

In practice. The property class is fixed by regulation even when the engineer expects a different service life. A conveyor may be engineered for fifteen years of service yet recover its cost over a 7-year tax schedule; the physical life plan and the tax schedule are separate documents.

Half-year convention. Every asset is assumed to enter service at midyear: only a half-year charge is allowed in year 1, and the leftover half lands after the recovery period, so an N-year class spans $N+1$ tax years.

For personal property, meaning equipment rather than buildings, three rules define the system:

1. the salvage value is always treated as **zero**, with no exceptions: the full cost basis is recovered;

2. the method is DDB with the **switch to straight line** built in, the same switch as Example 10.4;

3. the **half-year convention**: every asset is assumed to enter service at midyear, so only half a year's depreciation is allowed in year 1, and the leftover half lands after the recovery period. An N-year class therefore spans $N+1$ tax years.

Because the recipe is fixed, the tax authority publishes the outcome once and for all, as *recovery allowance percentages* (Table 10.7). Using MACRS is then one multiplication per year:

$$D_n = I \times p_n.$$

Year n	3-year	5-year	7-year
1	33.33%	20.00%	14.29%
2	44.45%	32.00%	24.49%
3	14.81%	19.20%	17.49%
4	7.41%	11.52%	12.49%
5		11.52%	8.93%
6		5.76%	8.92%
7			8.93%
8			4.46%

Table 10.7. MACRS recovery allowance percentages under the half-year convention. Each column sums to 100%, so the full cost basis is recovered.

Insight. You can see the three rules sitting inside the numbers of Table 10.7. Each column sums to 100%: salvage is zero and the whole basis is recovered. The 5-year class uses DDB at $2/5 = 40\%$, but year 1 shows only 20%, exactly half: that is the midyear start. And the flat 11.52% entries in years 4–5 are the built-in switch to straight line. The same halving explains the small tail percentage in the final year of each column.

Example 10.7 — A full MACRS schedule for 5-year property

Najd Copper's bagging equipment has a cost basis of 25,000 SAR and is 5-year MACRS property. Build the full schedule, and state the allowed charge if the equipment were instead sold during year 2.

Solution.

Each charge is the cost basis times the table percentage, $D_n = 25{,}000 \times p_n$:

n	p_n	D_n	BV_n
1	20.00%	5,000	20,000
2	32.00%	8,000	12,000
3	19.20%	4,800	7,200
4	11.52%	2,880	4,320
5	11.52%	2,880	1,440
6	5.76%	1,440	0
	100%	**25,000**	

Table 10.8. The MACRS schedule for the bagging equipment: six tax years for a 5-year class, and the charges sum to the full cost basis.

The 5-year class spans six tax years, and the book value reaches exactly zero: the full basis of $\boxed{25{,}000 \text{ SAR}}$ is recovered.

Early disposal. One refinement completes the system. If the asset is disposed of *during* the recovery period, only **half** of that year's normal charge is allowed — the disposal-year twin of the half-year convention. If the firm sold this equipment in year 2, the year-2 deduction would be

$$D_2 = \frac{8{,}000}{2} = \boxed{4{,}000 \text{ SAR}},$$

and the book value for the disposal calculation would be $25{,}000 - 5{,}000 - 4{,}000 = 16{,}000$ SAR.

Reference. ZATCA, *Implementing Regulations of the Income Tax Law*, `https://zatca.gov.sa`; full reference at the end of the chapter.

10.8 *Depreciation and Tax in the Saudi System*

MACRS is this book's standard example of a mandated tax schedule, but it is the United States' answer to the question, not the Kingdom's. This short section is orientation for readers who will meet the Saudi system in practice: it shows how the same three design choices, method, life, and salvage, are made by the Zakat, Tax and Customs Authority (ZATCA) under the Saudi Income Tax Law and its Implementing Regulations. It is a sketch of the system's shape, not tax advice; rates and rules change, and the authoritative text is always the current law and regulations published at `https://zatca.gov.sa`.

Pooled-group depreciation. Tax depreciation computed on asset *groups* rather than single assets: each group's declining-balance rate applies to the group's remaining balance as a whole. Purchases add to the balance; disposal proceeds subtract from it.

Where MACRS assigns each asset a property class and a published percentage column, the Saudi Income Tax Law prescribes the *declining-balance* method, applied not asset by asset but to five pooled groups, each with a fixed annual rate: permanent buildings at 5%; movable industrial and agricultural buildings at 10%; factories, machinery, engines, computer hardware and software, and passenger and cargo vehicles at 25%; expenditures on geological surveying, drilling, and exploration for natural resources at 20%; and all other tangible and intangible depreciable assets, from furniture to ships to goodwill, at 10%. The mechanics are Section 10.4 with the bookkeeping moved up one level: a new crusher does not get its own schedule, it joins the 25% group, and the year's deduction is 25% of that group's remaining balance. A 200,000 SAR machine entering an otherwise empty group generates charges of 50,000, then 37,500, then 28,125 SAR, the group balance decaying geometrically with no salvage floor, no switch to straight line, and no half-year table; like all pure declining balance, the pool never reaches exactly zero, it just shrinks. Selling an asset works by the same pooled logic in reverse: the sale proceeds are subtracted from the group balance rather than compared with an individual book value, so the gain-or-loss settlement of Section 10.11 is, for most disposals, absorbed silently into the future deductions of the whole group.

The second Saudi particularity is who pays which levy. Corporate income tax, at a flat 20% for most activities, applies to taxable income in proportion to the *non-Saudi* ownership share of a resident company; the Saudi and GCC share pays *zakat* instead, computed at approximately 2.5% on a different base built from equity and working capital rather than from profit. A joint venture that is 60% Saudi-owned therefore runs both computations side by side, each on its share. The distinction matters for this chapter because the depreciation tax shield of Section 10.10.1 is an *income-tax* phenomenon: for the zakat-paying share, a bigger depreciation charge does not buy a $t\,D$ tax saving in the same way, so the value of accelerated depreciation in a Saudi project scales with the taxed ownership fraction.

How does the book's convention map onto all this? Our problems state a single flat marginal rate t, usually 20%, and let each asset carry its own schedule; that reproduces the corporate-income-tax side of the Saudi system faithfully in spirit, while simplifying the pooling to keep each asset's story visible. Every mechanism this chapter teaches, the deduction, the shield, the timing advantage of early charges, operates identically inside ZATCA's pools; only the bookkeeping unit changes. When a real filing is at stake, the pooled rules, the zakat base, and their current details belong to a tax professional working from the current regulations, not to a textbook.

10.9 Corporate Income Taxes

Lecture 22. This section is covered by Lecture 22.

Reference. Park, 3rd ed., Section 9.4 (pp. 402–410).

Taxable income. $TI = R - E - D$: revenues minus cash operating expenses minus depreciation.

Net income. $TI - \text{tax} = TI\,(1-t)$: the accounting measure of profit after the tax is paid.

Depreciation was built to feed a tax computation, and the computation is short. Taxable income is revenues minus expenses minus depreciation; the tax is the tax rate times taxable income; net income is what remains:

$$TI = R - E - D, \qquad \text{tax} = t \times TI, \qquad \text{net income} = TI\,(1-t). \tag{10.5}$$

The ingredients, one at a time:

- R: the gross revenues from products or services;
- E: the *cash* operating expenses — labor, material, rent, and interest on business debt — all deductible in the year they are paid;
- D: the depreciation allowance. Please pay attention here, because this is where the first half of the chapter plugs in. When a firm buys equipment, the purchase is *not* an expense of that year. The capital cost enters the tax computation only through depreciation, one allowance at a time, on the schedule you now know how to build;

Marginal tax rate. The rate applied to the last unit of income earned. The rate that matters for a new project, whose income lands on top of the income the firm already has.

Average tax rate. Total tax divided by total taxable income. Below the marginal rate whenever lower brackets exist.

In practice. A firm with negative taxable income pays no income tax now, but the loss is not wasted. Most tax codes let it carry forward to offset the first profitable years, so for a company that will not be profitable for some time, accumulated losses are a real asset in the after-tax arithmetic.

- t: the firm's tax rate. Our problems state a single flat marginal rate.

One distinction before the examples. Real corporate tax schedules are often *progressive*: the rate climbs in stair-step brackets as income grows. Two different rates can then be quoted for the same firm. The *marginal* rate is the rate on the last unit earned, and it is the rate that matters when we evaluate a new project, because the project's income lands on top of the income the firm already has. The *average* rate is the total tax divided by the total income. From now on, when a problem says "the tax rate is t," it means the marginal rate.

Example 10.8 — Marginal and average rates on a progressive schedule

The textbook illustrates the distinction with the United States corporate schedule of its era; the amounts below are in US dollars, and only the idea matters here. The first 50,000 of taxable income is taxed at 15%, the next 25,000 at 25%, and everything above 75,000 at 34%. A firm has a taxable income of 100,000. Find its tax, its marginal rate, and its average rate.

Solution.

Each bracket taxes only the income that falls inside it:

Bracket	Rate	Tax on $TI = 100{,}000$
first 50,000	15%	7,500
next 25,000	25%	6,250
above 75,000	34%	$0.34 \times 25{,}000 = 8{,}500$
		22,250

Table 10.9. A progressive schedule taxes each bracket separately; the total tax is the sum of the bracket taxes.

The total tax is $7{,}500 + 6{,}250 + 8{,}500 = 22{,}250$. The last unit of income fell in the top bracket, so the marginal rate is $\boxed{34\%}$. The average rate is

$$\frac{22{,}250}{100{,}000} = \boxed{22.25\%}.$$

The marginal rate is the one to use for a new project: every additional unit of project income would be taxed at 34%, not at the 22.25% average.

10.10 *After-Tax Cash Flow*

Net income is the accounting answer, the bottom line of the income statement. It is not the cash the firm banked, and the next example shows why.

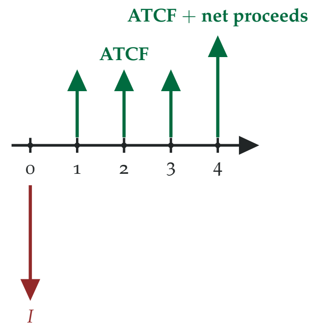

Figure 10.4. A typical after-tax project: the investment at year 0, the after-tax cash flow of each operating year, and the net proceeds of the final sale added to the last year.

Example 10.9 — One operating year at Najd Copper

In one operating year, Najd Copper earns revenues of 300,000 SAR, pays cash operating expenses (reagents, wages, power, and utilities) of 180,000 SAR, and records depreciation of 40,000 SAR on its flotation cell, ore-transfer truck, and bagging equipment. The marginal tax rate is 25%. Build the income statement, then find the cash the firm actually kept.

Solution.

Revenues R	300,000 SAR
− Cash operating expenses E	180,000
− Depreciation D	40,000
Taxable income TI	80,000
− Income tax (25%)	20,000
Net income	**60,000 SAR**
+ Depreciation (add back — not a cash payment)	40,000
After-tax cash flow	**100,000 SAR**

Table 10.10. The income statement of one operating year, extended by the depreciation add-back to reach the cash flow.

Look down the statement line by line. The revenues arrived as cash. The expenses and the tax left as cash. But the depreciation was only an accounting deduction: the 40,000 SAR never left the bank account. So to convert profit into cash, add the depreciation back:

$$\text{ATCF} = 60{,}000 + 40{,}000 = \boxed{100{,}000 \text{ SAR}}.$$

Net income says Najd Copper earned a profit of 60,000 SAR; the cash flow says it banked 100,000 SAR. Both are correct; they answer different questions, and project evaluation needs the second one.

After-tax cash flow (ATCF). Net income plus depreciation: $\text{ATCF} = (R - E - D)(1 - t) + D = (R - E)(1 - t) + t\,D$.

The example generalizes to the central formula of this chapter. The *after-tax* *cash flow* is net income plus depreciation, and it can be written three ways:

$$\text{ATCF} = \underbrace{(R - E - D)(1 - t) + D}_{\text{net income}+D} = \underbrace{(R - E)(1 - t)}_{\text{after-tax operating profit}} + \underbrace{t\,D}_{\text{tax shield}}. \qquad (10.6)$$

Memorize the first form as the pipeline (statement, tax, add-back) and the second as the split: the after-tax operating profit as if depreciation did not exist, plus the tax saved because it did. Check the split on Najd Copper's year: $120{,}000\,(0.75) + 0.25\,(40{,}000) = 90{,}000 + 10{,}000 = 100{,}000$ SAR, as before. When the two routes agree, you can trust the answer.

Depreciation tax shield. $t \times D$: the tax saved by each year's depreciation charge. Non-cash on its own, but the tax it saves is real cash.

10.10.1 *The depreciation tax shield*

Here is why a non-cash charge creates real cash. Every SAR of depreciation cuts taxable income by one SAR, and each SAR of taxable income would have been taxed at the rate t. So each SAR of depreciation saves t in tax. That saving is the *depreciation tax shield*, $t \times D$.

Example 10.10 — Najd Copper's tax bill, with and without depreciation

Recompute Najd Copper's year (Example 10.9) as if the tax authority did not allow depreciation as a deduction, and compare.

Solution.

Without the deduction, taxable income would be the full $R - E = 120{,}000$ SAR:

	Without D	With D
Taxable income	120,000	80,000
Income tax (25%)	30,000	20,000
After-tax cash flow	90,000	100,000

Table 10.11. The with-vs-without comparison. Depreciation moves no cash of its own, but it cuts the tax cheque by $t\,D$.

The tax falls from 30,000 SAR to 20,000 SAR, and the cash flow rises by the same amount:

$$\text{shield} = t\,D = 0.25 \times 40{,}000 = \boxed{10{,}000 \text{ SAR}}.$$

Depreciation shielded 10,000 SAR of real cash from tax.

Insight. Connect the shield to the timing point of Section 10.6. The total depreciation over an asset's life is the same under every method, so the *total* shield is the same too. What differs is when the shields arrive. Accelerated schedules, DDB and MACRS, deliver larger charges early, so the tax savings arrive early, and Chapter 2 says earlier cash is worth more. Note also the direction of the two effects: extra depreciation makes *net income fall* by $\Delta D\,(1-t)$ while the *cash flow rises* by $t\,\Delta D$. Accounting profit and cash move in opposite directions.

10.11 *Gains and Losses on Disposal*

Gain on disposal. $S - BV$ when an asset sells above its book value; called depreciation recapture, and taxed at the ordinary rate t.

Net proceeds. $S - t\,(S - BV)$: the cash kept from a sale after the gain is taxed or the loss is credited. One formula covers both directions.

One piece remains: the end of the asset's story. When an asset is sold, compare the selling price S with the book value BV on that date. Two cases:

- $S > BV$: the difference is a **taxable gain** of $S - BV$. Tax law calls it *depreciation recapture*, and it is taxed at the ordinary rate t;
- $S < BV$: the difference is a **deductible loss** of $BV - S$. It offsets other business income, so it *saves* $t\,(BV - S)$ — a tax credit in effect.

Both cases collapse into one formula for the cash the seller keeps:

$$\text{net proceeds} = S - t\,(S - BV). \tag{10.7}$$

When S exceeds the book value, the second term is a tax paid; when S falls short, the term flips sign and becomes a refund.

Insight. Depreciation was a running estimate of how fast the asset lost value; the sale reveals the truth. If depreciation claimed more loss than reality delivered, the firm deducted too much along the way, and the gain tax settles the account. If it claimed too little, the loss credit refunds the difference. Either way, the account is settled — which is why one formula covers both directions.

Example 10.11 — Two offers for the flotation cell

After three straight-line years, the flotation cell's book value is $BV_3 = 23{,}000$ SAR (Example 10.2). The tax rate is 25%, and the plant manager has two offers on the table: Offer A at 30,000 SAR and Offer B at 18,000 SAR. Find the net proceeds under each.

Solution.

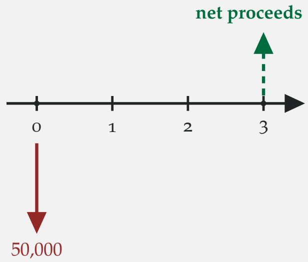

Figure 10.5. The sale at the end of year 3. The amount that enters the year-3 cash flow is the net proceeds, never the raw selling price.

Offer A: $S = 30{,}000$ SAR, above book value.

$$\text{gain} = 30{,}000 - 23{,}000 = 7{,}000; \qquad \text{tax} = 0.25\,(7{,}000) = 1{,}750;$$

$$\text{net proceeds} = 30{,}000 - 1{,}750 = \boxed{28{,}250\text{ SAR}}.$$

Offer B: $S = 18{,}000$ SAR, below book value.

$$\text{loss} = 23{,}000 - 18{,}000 = 5{,}000; \qquad \text{credit} = 0.25\,(5{,}000) = 1{,}250;$$

$$\text{net proceeds} = 18{,}000 + 1{,}250 = \boxed{19{,}250\text{ SAR}}.$$

Check both against Equation (10.7): for Offer B, $18{,}000 - 0.25\,(18{,}000 - 23{,}000) = 18{,}000 + 1{,}250 = 19{,}250$ SAR, with the sign doing the work. In any after-tax analysis, the number that enters the final year's cash flow is the net proceeds, never the raw selling price.

10.12 *An Integrated After-Tax Evaluation*

Everything now assembles into one workflow, the shape of the in-class activity and of every after-tax problem from here on: depreciate, tax, add the depreciation back, settle the disposal, then discount at the MARR exactly as in chapter 5.

Example 10.12 — A concentrate bagging line, from income statement to verdict

Najd Copper considers adding a concentrate bagging line. The cost basis is 60,000 SAR, depreciated straight line over 4 years to a salvage estimate of 12,000 SAR. The line adds revenue of 50,000 SAR per year and cash expenses of 20,000 SAR per year. At the end of year 4 the line actually sells for 15,000 SAR. The tax rate is 25% and the MARR is 10%. Should the firm accept the project?

Solution.

Step 1: the depreciation charge.

$$D = \frac{60{,}000 - 12{,}000}{4} = 12{,}000 \text{ SAR per year.}$$

Step 2: the operating ATCF, identical in years 1–4.

Revenues R	50,000 SAR
− Cash expenses E	20,000
− Depreciation D	12,000
Taxable income	18,000
− Income tax (25%)	4,500
Net income	13,500
+ Depreciation (add back)	12,000
ATCF (each of years 1–4)	**25,500 SAR**

Table 10.12. The operating pipeline of the bagging line. Every operating year contributes 25,500 SAR after tax.

Step 3: settle the disposal.

After four straight-line years the book value is down to the salvage estimate, $BV_4 = 12{,}000$ SAR, but the line sells for 15,000 SAR, 3,000 SAR above book value:

$$\text{net proceeds} = 15{,}000 - 0.25\,(15{,}000 - 12{,}000) = 15{,}000 - 750 = 14{,}250\,\text{SAR}.$$

Step 4: draw the after-tax cash flow diagram.

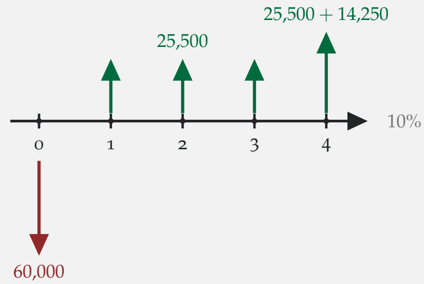

Figure 10.6. The bagging line after tax: the investment, four operating ATCFs, and the net proceeds of the sale added to the last year.

Step 5: present worth at the MARR.

From here it is chapter 5 again. Write the factor notation first:

$$\text{PW}(10\%) = -60{,}000 + 25{,}500\,(P/A,\,10\%,\,4) + 14{,}250\,(P/F,\,10\%,\,4).$$

N	(P/A)	(P/F)
3	2.4869	0.7513
4	**3.1699**	**0.6830**
5	3.7908	0.6209

Table 10.13. Excerpt from the $i = 10\%$ interest factor table used for Example 10.12.

On the 10% page, row $N = 4$ (Table 10.13) gives $(P/A) = 3.1699$ and $(P/F) = 0.6830$:

$$\begin{aligned}\text{PW}(10\%) &= -60{,}000 + 25{,}500\,(3.1699) + 14{,}250\,(0.6830)\\ &= -60{,}000 + 80{,}832.45 + 9{,}732.75 = \boxed{+30{,}565.20 \text{ SAR}}.\end{aligned}$$

The present worth is positive at the MARR, so **accept** the bagging line. Step back and see the whole chain: depreciate → tax → ATCF → net proceeds → PW. The last step is unchanged from chapter 5; what this chapter changed is every number that went into it.

10.13 *Case Study from the Literature*

Every serious techno-economic assessment must declare its depreciation schedule and its tax rate, because the after-tax cash flows depend on both. An open-access study by Smerigan and Shi (2025) makes an unusually clean exhibit: it evaluates a first-of-its-kind plant that recovers rare earth elements from *phosphogypsum*, the waste stockpiled wherever phosphate rock is processed into fertilizer, and its methods section states the full tax apparatus of this chapter explicitly: capital depreciated on the 7-year MACRS schedule, a 30-year plant life, a 28% income-tax rate, and a 15% interest rate for discounting. The study's amounts are in US dollars, the currency of the original paper, and we keep them.

Reference. Smerigan & Shi (2025), arXiv:2504.10495. Open access; full reference at the end of the chapter.

Example 10.13 — Case study: MACRS tax shields in a rare-earth recovery plant

In Smerigan and Shi's baseline design, the two largest capital items are the ion-exchange resin and the biomolecule ligand of the selective separation section, about \$730 million and \$240 million respectively, roughly \$970 million of capital between them. Using the paper's stated tax basis — 7-year MACRS, a 28% tax rate, and a 15% discount rate — build the MACRS schedule for this \$970 million, find the present worth of its depreciation tax shields at the start of operations, and compare with the same capital depreciated straight line over the plant's 30-year engineered life.

Solution.

Step 1: the MACRS schedule and the shields.

Each year's charge is $D_n = 970 \times p_n$ million dollars, and each charge shields $t\,D_n = 0.28\,D_n$ from tax (Table 10.14).

n	p_n	D_n ($M)	$t\,D_n$ ($M)
1	14.29%	138.6	38.8
2	24.49%	237.6	66.5
3	17.49%	169.7	47.5
4	12.49%	121.2	33.9
5	8.93%	86.6	24.3
6	8.92%	86.5	24.2
7	8.93%	86.6	24.3
8	4.46%	43.3	12.1
	100%	**970.0**	**271.6**

Table 10.14. The 7-year MACRS schedule on the $970 million of adsorption capital, with the 28% tax shield of each charge. Entries are rounded to $0.1 million; the totals are exact.

The whole basis is recovered across eight tax years (the half-year convention at work), and the nominal shields total $0.28 \times 970 = \$271.6$ million.

Step 2: present worth of the shields at 15%.

Discount each shield one year at a time:

$$PW_{\text{MACRS}} = \frac{38.8}{1.15} + \frac{66.5}{(1.15)^2} + \cdots + \frac{12.1}{(1.15)^8} = \boxed{\approx \$170 \text{ million}}.$$

Step 3: the same capital, spread over the engineered life.

The plant is designed to run for 30 years, but that is not the tax life. If the tax authority instead required straight line over 30 years, the annual charge would be $970/30 = \$32.3$ million, the annual shield $0.28 \times 32.3 = \$9.05$ million, and with $(P/A, 15\%, 30) = 6.5660$:

$$PW_{\text{SL-30}} = 9.05\,(6.5660) \approx \$59 \text{ million}.$$

Step 4: read the comparison.

Same capital, same 28% rate, same $271.6 million of nominal shields either way — yet the mandated 7-year accelerated schedule delivers about $170 million of present worth against $59 million for the 30-year spread, a difference of roughly $\boxed{\$111 \text{ million}}$ created purely by *timing*. On a project whose total reported NPV is $570 million, the depreciation schedule is not a bookkeeping detail; it is a fifth of the project's value. This is the insight of Section 10.6 measured in field data, and it is why the paper's cost breakdown lists capital recovery, "composed of capital depreciation, average income tax, and average return on investment,'' at $20 of the $44 minimum selling price per kilogram of rare-earth oxide.

What did the researchers conclude? That the baseline plant is profitable after tax: an NPV of $570 million at the 15% rate, an IRR of 20%, a return on investment of 23%, and a payback of 4 years on the 30-year investment (Smerigan & Shi, 2025). Their Monte Carlo analysis, three thousand samples over uncertain prices, costs, and performance, found the IRR above the 15% hurdle in 87% of baseline simulations, and their scenario analysis found the system profitable only at large capacities and for phosphogypsum containing more than about 0.5% rare earths by weight, with the most dilute sources out of reach of the current process. The environmental verdict was mixed: better than conventional mining on ecosystem quality and resource depletion, worse on human health. Treating uncertainty as a distribution rather than a single number is exactly where this book goes next, in chapter 12.

Insight. Note what the case study verified and what it took on trust. Given the paper's stated basis, schedule, tax rate, and discount rate, the shields and their present worth follow mechanically from Table 10.7 — arithmetic anyone can check, and the check exposed how much of the project's value lives in the tax timing. What cannot be checked from outside are the estimates themselves: the $970 million of adsorption capital rests on cost correlations and a Lang factor, and the revenue on an uncertain rare-earth basket price. The tax pipeline is exact; its inputs are engineering judgment.

10.14 Chapter Summary

- **Depreciation** allocates an asset's cost over its depreciable life. It is a non-cash charge: the cash left at purchase, and the schedule only spreads that old cost. Depreciable property is used in business, has a definite life longer than one year, and wears out; **land and inventories never qualify**.

- The **cost basis** I is everything needed to place the asset in service: purchase price plus freight, site preparation, and installation (operator training is expensed, not capitalized). The **book value** is $BV_n = I - \sum_{k \leq n} D_k$.

- **Book** depreciation (SL, DB, UoP, chosen by the firm) serves financial reporting; **tax** depreciation is mandated by the tax authority — MACRS in the United States, the standard example in this book. Every method charges the same total; only the *timing* differs.

- **SL**: $D_n = (I - S)/N$, the same every year. **UoP**: $D_n = (u_n/U)(I - S)$, proportional to usage. **DB**: $D_n = \alpha\, BV_{n-1}$ with $\alpha = \text{multiplier}/N$ (DDB: $\alpha = 2/N$), subject to the **salvage floor** (never below S) and the **switch to SL** on the remainder when S is low.

- **MACRS**: find the property class, multiply the cost basis by the published percentage ($D_n = I\, p_n$); salvage is zero; the **half-year convention** stretches an N-year class over $N + 1$ tax years and allows only a **half charge in a disposal year**.

- The **Saudi system** (ZATCA) mandates declining balance on five **pooled asset groups** (5% to 25% by group) rather than per-asset schedules, and splits the levy by ownership: corporate income tax at 20% on non-Saudi shares, zakat on Saudi and GCC shares. The book's flat rate t approximates the income-tax side; the mechanics are unchanged.

- The tax pipeline: $TI = R - E - D$; tax $= t\,TI$; net income $= TI\,(1 - t)$. Use the **marginal** rate for project decisions; the **average** rate is total tax over total income.

- **ATCF** $=$ net income $+ D = (R - E - D)(1 - t) + D = (R - E)(1 - t) + t\,D$. The **depreciation tax shield** $t\,D$ is the cash each year's charge saves; its total is fixed, but accelerated schedules deliver it earlier. Extra depreciation lowers net income and *raises* cash flow.

- At **disposal**, compare price with book value: a gain $S - BV$ is taxed, a loss is credited, and the **net proceeds** $= S - t\,(S - BV)$ in both directions. The net proceeds, never the raw price, enter the final year's cash flow.

- The integrated workflow is: depreciate $\rightarrow$ tax $\rightarrow$ ATCF $\rightarrow$ net proceeds $\rightarrow$ PW at the MARR. The decision tools of Chapters 5–7 are unchanged; this chapter changed the cash flows that go into them. The next two chapters add the remaining layers: uncertain estimates (chapter 12) and aging assets (chapter 16).

10.15 *Exercises*

These problems begin with single depreciation schedules — straight line, declining balance, units of production, and MACRS — and build toward the full after-tax workflow, including disposal gains and the marginal rate.

Building depreciation schedules

Exercise 10.1. A filling machine has a cost basis of 90,000 SAR, a salvage value of 10,000 SAR, and an 8-year depreciable life, depreciated straight line. Find the annual charge D and the book value at the end of year 5.

Exercise 10.2. Lab equipment has a cost basis of 60,000 SAR and a 5-year life, depreciated by double declining balance. Find the year-2 charge D_2.

Exercise 10.3. An asset has a cost basis of 40,000 SAR, a salvage value of 4,000 SAR, and a 6-year life, depreciated straight line. Find the annual charge D and the book value at the end of year 2.

Taxes and after-tax cash flow

Exercise 10.4. A firm reports revenues of 180,000 SAR, cash expenses of 100,000 SAR, and depreciation of 25,000 SAR; its marginal tax rate is 30%. Run the pipeline: taxable income $\rightarrow$ tax $\rightarrow$ net income $\rightarrow$ ATCF, then check the result with the closed form $(R - E)(1 - t) + t\,D$.

Exercise 10.5. A firm projects revenues of 140,000 SAR, cash expenses of 65,000 SAR, and depreciation of 30,000 SAR, with a tax rate of 20%. Find the after-tax cash flow.

Exercise 10.6. An asset with a book value of 14,000 SAR is sold for 22,000 SAR; gains are taxed at 25%. Find the net proceeds from the sale.

Exercise 10.7. A firm's depreciation charge doubles this year; revenues, expenses, and the tax rate are unchanged. What happens?

(A) Net income falls and ATCF falls.

(B) Net income falls and ATCF rises.
(C) Net income rises and ATCF rises.
(D) Neither changes — depreciation is non-cash.

Depreciation schedule sprint (in-class activity)

Saudi Arabia's push to make mining the Kingdom's third industrial pillar has *Najd Copper* expanding its concentrator near Hafar Al-Batin. The firm has just installed a vibrating screening unit with a cost basis of $I = 100{,}000$ SAR (purchase plus installation), an estimated salvage value of $S = 10{,}000$ SAR, a depreciable life of $N = 5$ years, and a rated lifetime throughput of 90,000 tonnes of ore. The accountant asks for the full schedule under three methods, a comparison, a taste of the tax table, and a recommendation.

Exercise 10.8. Straight line. For the screening unit ($I = 100{,}000$ SAR, $S = 10{,}000$ SAR, $N = 5$ years), compute the annual straight-line charge and build the schedule: D_n and BV_n for $n = 1$ to 5.

Exercise 10.9. Double declining balance. For the same screening unit, state the DDB rate α, build the schedule for $n = 1$ to 5, and watch for the salvage floor. In which year does the floor bind, and what would the (wrong) uncapped charge have been that year?

Exercise 10.10. Units of production. The screening unit's actual throughput is 24,000, 20,000, 18,000, 16,000, and 12,000 tonnes in years 1–5. Compute the depreciation rate per tonne and build the UoP schedule.

Exercise 10.11. Compare the three schedules. Using the results of Exercise 10.8–Exercise 10.10: (a) copy the book value at the end of year 2 from each schedule; (b) which method front-loads depreciation, and why does its BV_2 confirm this? (c) all three schedules end at $BV_5 = 10{,}000$ SAR — in one sentence, what do the methods change, and what do they *not* change?

Exercise 10.12. MACRS taster. For tax purposes, suppose the screening unit is treated as 5-year MACRS property (percentages 20.00, 32.00, 19.20, 11.52, 11.52, 5.76). Compute the MACRS charges D_1 and D_2, and note in one phrase how D_2 compares with the DDB year-2 charge of Exercise 10.9.

Exercise 10.13. Decision sentence. Najd Copper pays income tax on its profits and wants the largest depreciation charges in the early years, because bigger early deductions are worth more. Complete for the accountant's file: "We recommend the ______________ method because _________________________________."

After-tax mini-case (in-class activity)

The project engineer at *Wadi Phosphate* runs a mobile crushing unit serving the company's satellite pits. The operation is a registered establishment that pays income tax at a flat 20% on its profits. The unit and its screening fit-out have a cost basis of $I = 160{,}000$ SAR, a salvage value of $S = 10{,}000$ SAR, and a 5-year depreciable life, depreciated straight line. In year 3, revenues were 180,000 SAR and cash operating expenses (ore feed, fuel, wages, permits) were 100,000 SAR.

Exercise 10.14. Year 3 with depreciation. Compute the annual straight-line charge for the crushing unit, then run the year-3 pipeline: taxable income, income tax, net income, and the after-tax cash flow.

Exercise 10.15. Year 3 without depreciation. Suppose the tax authority did *not* allow depreciation as a deduction, everything else unchanged. Run the pipeline again. Then: (a) by how much is the ATCF of Exercise 10.14 larger than here? (b) verify that this difference equals $t \times D$, the depreciation tax shield, and state in one sentence how a non-cash charge can increase cash flow.

Exercise 10.16. Disposal. At the end of year 5, the company actually sells the unit for 22,000 SAR; gains on disposal are taxed at the same 20%. (a) Find the book value at the end of year 5 from the straight-line schedule. (b) Find the gain on disposal, the tax on the gain, and the net proceeds from the sale.

Exercise 10.17. Decision sentence. Complete for the accountant's file: "Depreciation ____________ (is / is not) itself a cash flow, but it raises the after-tax cash flow by ____________ SAR per year because _________________________________."

Exam-level practice

Exercise 10.18. The plant manager at *Wadi Lithium* installs a carbonate screening line at the refinery: equipment price 70,000 SAR, site preparation 6,000 SAR, and installation 4,000 SAR. The line is 5-year MACRS property (percentages 20, 32, 19.2, 11.52, 11.52, 5.76), and the marginal tax rate is $t = 20\%$. (a) State the cost basis; compute D_1, D_2, D_3 and the book value at the end of year 3. (b) In year 2, the line generated 120,000 SAR of revenue with 58,000 SAR of cash operating expenses. Run the pipeline: taxable income → tax → net income → ATCF. (c) The line is sold *during* year 4 for 30,000 SAR. Apply the half-year convention to the year-4 charge, then compute the book value at disposal, the gains tax, and the net proceeds. (d) State the year-2 depreciation tax shield, and explain in one sentence how a non-cash charge increases cash flow.

Exercise 10.19. A machine ($I = 95{,}000$ SAR, $S = 15{,}000$ SAR, $N = 8$ years) is depreciated by the straight-line method. Its book value at the end of year 5 is closest to:

(A) 35,625 SAR
(B) 45,000 SAR
(C) 50,000 SAR
(D) 55,000 SAR

Exercise 10.20. A firm reports revenues of 220,000 SAR, cash expenses of 130,000 SAR, and depreciation of 40,000 SAR; its marginal tax rate is 20%. Its after-tax cash flow is closest to:

(A) 40,000 SAR
(B) 72,000 SAR
(C) 80,000 SAR
(D) 90,000 SAR

Exercise 10.21. A concentrate pump at a phosphate mine has a book value of 26,000 SAR. Gains and losses on disposal are taxed and credited at $t = 25\%$. Two buyers make offers: 18,000 SAR and 31,000 SAR. Compute the net proceeds under each offer, and verify that both follow the single formula $S - t\,(S - BV)$.

Exam-style problems

The problems below follow the format, length, and point weights of the final exam. Work each part in exam order: state the inputs, build the schedule or the pipeline line by line, box each answer with its units, and close with the check or the decision sentence.

Exercise 10.22. The project engineer at *Wadi Phosphate* installs a screening-and-washing line at a satellite pit: equipment price 84,000 SAR, site preparation 6,000 SAR, installation 10,000 SAR. The line is 5-year MACRS property (percentages 20, 32, 19.2, 11.52, 11.52, 5.76), and the marginal tax rate is $t = 20\%$. (a) (*8 pts*) State the cost basis; compute D_1, D_2, D_3 and the book value at the end of year 3. (b) (*10 pts*) In year 3, the line generated 160,000 SAR of revenue with 88,000 SAR of cash operating expenses. Run the pipeline: taxable income $\rightarrow$ tax $\rightarrow$ net income $\rightarrow$ ATCF. (c) (*8 pts*) The line is sold *during* year 4 for 36,000 SAR. Apply the half-year convention to the year-4 charge, then compute the book value at disposal, the gains tax, and the net proceeds. (d) (*4 pts*) State the year-3 depreciation tax shield, and explain in one sentence how a non-cash charge increases cash flow.

Exercise 10.23. *Najd Copper* considers a reagent-mixing line. The cost basis is 90,000 SAR, depreciated straight line over 5 years to a salvage estimate of 15,000 SAR. The line adds revenue of 70,000 SAR per year and cash expenses of 30,000 SAR per year. At the end of year 5 the line actually sells for 18,000 SAR. The tax rate is 25% and the MARR is 12%. (a) (*6 pts*) Compute the annual depreciation charge and the operating ATCF of each of years 1–5. (b) (*6 pts*) Settle the disposal: book value at year 5, gains tax, and net proceeds. (c) (*9 pts*) Draw the after-tax cash flow diagram, compute the present worth at the MARR (factor notation first), and state the decision. (d) (*4 pts*) State the net income of one operating year, and explain in one sentence why it differs from the ATCF of part (a).

Exercise 10.24. *Wadi Lithium* buys a carbonate compactor with a cost basis of 50,000 SAR, a salvage value of 10,000 SAR, and a 4-year depreciable life, depreciated by double declining balance. (a) (*8 pts*) State the DB rate α and build the full schedule, D_n and BV_n for $n = 1$ to 4. (b) (*4 pts*) In which year does the salvage floor bind, and what would the (wrong) uncapped charge have been that year? (c) (*6 pts*) The compactor is sold at the end of year 3 for 13,500 SAR; gains are taxed at $t = 20\%$. Compute the gain, the tax, and the net proceeds.

Assessing outside recommendations

Each problem below quotes a finished analysis produced by someone else: an AI assistant, a vendor, or a consultant. The work looks professional and its arithmetic is internally consistent, but each analysis contains one or two genuine method errors. Decide whether to accept or reject the recommendation, name each flaw precisely, and produce the corrected number.

Exercise 10.25. *Wadi Lithium* plans to sell its conveyor stacker *during* year 3 of its recovery period for 26,000 SAR. The stacker has a cost basis of 60,000 SAR and is 5-year MACRS property (percentages 20, 32, 19.2, 11.52, 11.52, 5.76); the tax rate is 20%. An AI assistant prepared the tax memo:

> "Year-1 charge: $0.20 \times 60{,}000 = 12{,}000$ SAR. Year-2 charge: $0.32 \times 60{,}000 = 19{,}200$ SAR, so the book value after year 2 is $60{,}000 - 31{,}200 = 28{,}800$ SAR. Year-3 charge: $0.192 \times 60{,}000 = 11{,}520$ SAR, so the book value at the sale is $28{,}800 - 11{,}520 = 17{,}280$ SAR. Tax on the sale: $0.20 \times 26{,}000 = 5{,}200$ SAR. The firm keeps $26{,}000 - 5{,}200 = 20{,}800$ SAR."

Accept or reject the memo? Name each flaw and produce the corrected net proceeds.

Exercise 10.26. A consultant evaluates a filter press for *Najd Copper*: cost basis 80,000 SAR, depreciated straight line over 5 years to a salvage estimate of 20,000 SAR; the press adds $R - E = 30{,}000$ SAR per year of pre-tax operating profit; it will actually sell for 26,000 SAR at the end of year 5. The tax rate is 25% and the MARR is 10%. The consultant's memo reads:

> "Depreciation: $(80{,}000 - 20{,}000)/5 = 12{,}000$ SAR per year. Taxable income: $30{,}000 - 12{,}000 = 18{,}000$ SAR; tax at 25%: 4,500 SAR; so the annual after-tax cash flow is $18{,}000 - 4{,}500 = 13{,}500$ SAR. With the 26,000 SAR sale at year 5: $PW(10\%) = -80{,}000 + 13{,}500\,(P/A, 10\%, 5) + 26{,}000\,(P/F, 10\%, 5) = -80{,}000 + 13{,}500\,(3.7908) + 26{,}000\,(0.6209) = -12{,}681$ SAR. The present worth is negative: reject the press."

Accept or reject the memo? Name each flaw and produce the corrected present worth and decision.

References

Park, C. S. (2012). *Fundamentals of Engineering Economics* (3rd ed.). Pearson.

Smerigan, A., & Shi, R. (2025). Advancing the economic and environmental sustainability of rare earth element recovery from phosphogypsum. *arXiv preprint*, arXiv:2504.10495. https://arxiv.org/abs/2504.10495

Zakat, Tax and Customs Authority (ZATCA) (2019). *Implementing Regulations of the Income Tax Law* (Ministerial Resolution No. 1535 of 1425H, as amended). Riyadh: ZATCA. https://zatca.gov.sa/en/RulesRegulations/Taxes/Pages/IncomeTaxlaw.aspx