8 Benefit–Cost Analysis and Public Projects

After this chapter you will be able to:

1. distinguish the users' benefits and disbenefits from the sponsor's capital and operating costs
2. compute the benefit–cost ratio $\mathrm{BC}(i) = B/(I + C')$, and decide whether a single public project is acceptable
3. compute the profitability index $\mathrm{PI}(i) = (B - C')/I$, and explain why it agrees with the present worth
4. choose among mutually exclusive public projects using incremental ratios $\Delta B/(\Delta I + \Delta C')$
5. convert engineering quantities into money values for benefits and disbenefits, and recognize double counting
6. explain how the choice of social discount rate changes the verdict on a long-lived project

The decision rules built so far serve a firm. Chapter 5 discounted a project's cash flows at the MARR and accepted when the present worth was positive; chapter 7 reached the same verdicts through the rate of return, and warned that ratios and rates must never rank mutually exclusive alternatives directly. This chapter carries the same machinery into the public sector, where the investor is a government and the returns accrue to the public: a desalination plant, a rural road, a flood-control channel. Three things change. The benefits belong to one party (the users) while the costs fall on another (the sponsor); the verdict is usually stated as a *benefit–cost ratio* rather than a present worth; and the discount rate is no longer a company's MARR but a *social discount rate*, whose choice is one of the oldest debates in public economics. Nothing changes in the underlying equivalence arithmetic: every number in this chapter is a present worth from Chapter 5, arranged into a ratio.

Ratios, scale, and the discount rate.

Q1. A public project's benefits have a present worth of 60 million SAR, and its costs (capital plus operating) have a present worth of 48 million SAR, both at the sponsor's discount rate. What is the benefit–cost ratio, and should the project be accepted?

Q2. Design X has a benefit–cost ratio of 2.1; design Y, for the same river crossing, has 1.4. Only one can be built. Can you conclude that X is the better choice?

Q3. A wetland restoration will deliver benefits of 1 million SAR per year, indefinitely. What is the present worth of those benefits at a discount rate of 2%? At 7%?

Answers.

Q1. $\text{BC} = 60/48 = 1.25 > 1$, so *accept*. The ratio above one says the users' benefits exceed the sponsor's costs in present-worth terms. The same data give $\text{PW} = 60 - 48 = 12$ million SAR > 0: the ratio and the present worth always agree on accept or reject.

Q2. *No.* A ratio ignores scale. Y may be a much larger project whose extra benefits are worth far more than X's, even though each of Y's SAR works less hard. Mutually exclusive alternatives are compared by the *incremental* ratio, exactly as rates of return were in chapter 7.

Q3. Use the capitalized equivalent of Chapter 5: $CE = A/i$. At 2%, $1/0.02 = 50$ million SAR; at 7%, $1/0.07 = 14.29$ million SAR. The choice of rate changes the valuation by a factor of 3.5. For long-lived public projects, the discount rate is not a technical detail; it often decides the verdict.

Reference. Park, 3rd ed., Section 8.1 (pp. 352–355).

8.1 Public Projects: Users and Sponsors

A firm evaluating a crusher owns both sides of the ledger: it pays the investment and it keeps the returns. A government evaluating a highway does not. The ministry pays for the road, but the travel-time savings belong to the drivers, the safety gains belong to the passengers, and the lost trade on the old route belongs to the shopkeepers there. Benefit–cost analysis keeps these parties separate.

The *users* are the public. Their favorable outcomes are *benefits*; their unfavorable outcomes are *disbenefits*, and the net users' benefit is $B = \text{benefits} - \text{disbenefits}$. The *sponsor* is the government body that builds and runs the project. Its side of the ledger nets the same way:

$$\text{sponsor's costs} = \text{capital costs} + \text{operating and maintenance costs} - \text{revenues},$$

where revenues are any tolls, tariffs, or fees the project collects. The general framework then has five steps.

1. Identify all users' benefits and disbenefits expected from the project.

2. Quantify these benefits and disbenefits in money terms, as well as the data allow.

3. Identify and quantify the sponsor's costs and revenues.

4. Convert every item to an equivalent value at a common date, using a discount rate appropriate for a public project.

5. Accept the project if the equivalent users' net benefits exceed the equivalent sponsor's net costs.

Steps 4 and 5 are Chapter 5 exactly. Steps 1 to 3 are where public projects earn their reputation for difficulty, because pricing a saved hour or an avoided crash is harder than reading an invoice.

Benefits are classified as *primary*, directly attributable to the project, or *secondary*, indirectly attributable. The distinction is practical: if the primary benefits alone justify the costs, the secondary ones need not be priced at all. It also guards against a classic error, double counting, which returns in the exercises: when a road saves commuting time, nearby land values rise *because* of that saved time, so counting both the time savings and the land-value rise counts the same benefit twice.

Users. The public: everyone who receives the favorable and unfavorable outcomes of a public project.

Sponsor. The government body that pays for and operates the project.

Disbenefit. An unfavorable outcome borne by the users, such as lost business on a bypassed road. Disbenefits reduce the users' benefit: $B = \text{benefits} - \text{disbenefits}$.

In practice. Benefit estimates rest on engineering models, not on wishes. A road's travel-time saving comes from traffic assignment models calibrated with counted vehicles; a flood channel's avoided damage comes from hydrological return periods; a desalination plant's supply benefit comes from a demand forecast. The economics inherits the error bars of the engineering underneath it.

Reference. Park, 3rd ed., Section 8.2 (pp. 355–362).

Benefit–cost ratio. $\mathrm{BC}(i) = B/(I + C')$: the present worth of users' net benefits divided by the present worth of the sponsor's net costs. Accept a single project when $\mathrm{BC}(i) \geq 1$.

8.2 The Benefit–Cost Ratio

Let b_n be the users' net benefit and c_n the sponsor's net cost at the end of period n, both written as positive amounts. Discount each series at the sponsor's rate i:

$$B = \sum_{n=0}^{N} b_n(1+i)^{-n}, \qquad C = \sum_{n=0}^{N} c_n(1+i)^{-n}. \tag{8.1}$$

The sponsor's cost splits into the capital spent during the first K periods, I, and the operating costs of the remaining periods, C', so that $C = I + C'$. The *benefit–cost ratio* is

$$\mathrm{BC}(i) = \frac{B}{C} = \frac{B}{I+C'}, \qquad I + C' > 0, \tag{8.2}$$

and the decision rule for a single project is: **accept** if $\mathrm{BC}(i) \geq 1$, **reject** otherwise.

Insight. The ratio adds no information to the present worth for a single project. $B/(I+C') > 1$ holds exactly when $B - (I+C') > 0$, and $B - (I+C')$ *is* the project's present worth. The ratio is a reporting convention, standard in public decision documents because ''each SAR of public money returns 1.51 SAR of public benefit'' communicates well. The danger begins only when ratios are used to *rank* projects, as Section 8.4 shows. Note also one bookkeeping change: all b_n and c_n enter as positive amounts, unlike the signed cash flows of Chapter 5.

Because both B and C are equivalent worths, they may be stated as present worths or as annual equivalents; the ratio is the same either way, since the conversion factor cancels. Governments pay no income tax, so depreciation and tax cash flows, which chapter 11 develops for firms, play no role here.

Example 8.1 — Benefit–cost ratio of a stormwater channel

A municipality considers a stormwater channel to protect a low-lying district. Construction takes two years: 30 million SAR now and 20 million SAR at the end of year 1. From year 2 through year 10, the channel prevents flood damage valued at 14 million SAR per year, and maintenance costs the city 2 million SAR per year over the same years. The social discount rate is 5%. Compute B, I, C', and $\mathrm{BC}(5\%)$, and state the decision.

Solution.

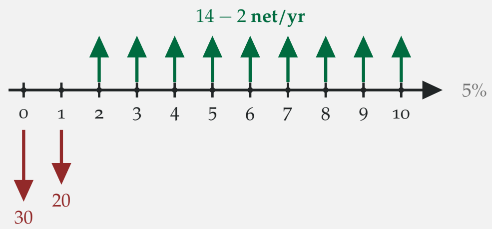

Figure 8.1. The channel in millions of SAR: two years of construction, then nine years of benefits and maintenance. For the ratio, the benefit and cost series are kept separate rather than netted.

Step 1: the users' benefits.

The 14 million SAR series runs from year 2 to year 10, nine payments. Value it at year 1 with $(P/A, 5\%, 9)$, then move it to year 0 with $(P/F, 5\%, 1)$:

$$B = 14\,(P/A,\ 5\%,\ 9)(P/F,\ 5\%,\ 1) = 14\,(7.1078)(0.9524) = 94.77 \text{ million SAR.}$$

Step 2: the sponsor's capital and operating costs.

The capital is spent in years 0 and 1 ($K = 1$):

$$I = 30 + 20\,(P/F,\ 5\%,\ 1) = 30 + 19.05 = 49.05 \text{ million SAR,}$$

and the maintenance series mirrors the benefit series:

$$C' = 2\,(7.1078)(0.9524) = 13.54 \text{ million SAR.}$$

Step 3: the ratio.

$$\mathrm{BC}(5\%) = \frac{94.77}{49.05 + 13.54} = \frac{94.77}{62.59} = \boxed{1.51} > 1 \;\Rightarrow\; \textbf{accept}.$$

Each SAR the city sinks into the channel returns about 1.51 SAR of flood protection in present-worth terms. The present-worth check gives the same verdict from the same numbers: $\mathrm{PW}(5\%) = 94.77 - 62.59 = +32.19$ million SAR.

Reference. Park, 3rd ed., Section 8.3 (pp. 362–366).

Profitability index. $\mathrm{PI}(i) = (B - C')/I$: net benefit after operating costs per unit of invested capital. Accept a single project when $\mathrm{PI}(i) \geq 1$.

8.3 The Profitability Index

The benefit–cost ratio asks whether benefits outweigh *all* costs. A close relative, the *profitability index*, nets the operating costs against the benefits first and asks how much net value each unit of *capital* creates:

$$\mathrm{PI}(i) = \frac{B - C'}{I}, \qquad I > 0. \tag{8.3}$$

Accept a single project when $\mathrm{PI}(i) \geq 1$. The acceptance boundary is again the same: $\mathrm{PI} > 1$ exactly when $B - C' - I > 0$, which is the present worth once more. The two ratios answer different managerial questions. BC asks "do the benefits cover everything?"; PI asks "how hard does each SAR of capital work?"

In practice. The profitability index is how a capital-rationed sponsor triages: rank independent candidates by value created per unit of scarce capital, exactly what PI measures, and fund down the list until the budget runs out. The same logic applies inside a firm in a capital-rationed year. Triage by PI is sound for *independent* candidates under a budget; it is still wrong for choosing among mutually exclusive versions of one project.

Example 8.2 — Profitability index of the stormwater channel

For the channel of Example 8.1 ($B = 94.77$, $I = 49.05$, $C' = 13.54$, in millions of SAR), compute the profitability index at 5% and interpret it next to the benefit–cost ratio.

Solution.

No new discounting is needed; the three components are already present worths.

$$\mathrm{PI}(5\%) = \frac{94.77 - 13.54}{49.05} = \frac{81.23}{49.05} = \boxed{1.66} > 1 \;\Rightarrow\; \textbf{accept}.$$

The interpretation: after paying its own upkeep, the channel returns about 1.66 SAR of net benefit for every SAR of construction capital. The same project gave $\mathrm{BC} = 1.51$; the numbers differ because the operating cost sits in the denominator of BC but is netted from the numerator of PI. Both exceed one together, and both fail together, because both are rearrangements of the same present worth: $94.77 - 62.59 = +32.19$ million SAR either way.

Incremental B/C ratio. $\mathrm{BC}(i)_{k-j} = \Delta B/(\Delta I + \Delta C')$, computed on the differences between a higher-cost alternative k and the current best lower-cost alternative j. If it exceeds 1, the extra spending is justified and k replaces j.

8.4 Mutually Exclusive Alternatives: The Incremental Ratio

Ratios measure quality per SAR; they do not measure size. chapter 7 established the principle for rates of return, and it transfers here without change: **never rank mutually exclusive alternatives by their raw ratios**. A small project can carry a splendid ratio on a small denominator while a larger rival creates more total value. The remedy is also the same: examine the *increment* of spending, and ask whether the extra money earns its keep. The procedure:

1. Discard any alternative with $\mathrm{BC}(i) < 1$ (it loses to do-nothing).
2. Order the survivors by increasing denominator $(I + C')$.
3. Compare the two cheapest, j then k, through the increment:

$$\mathrm{BC}(i)_{k-j} = \frac{B_k - B_j}{(I_k - I_j) + (C'_k - C'_j)} = \frac{\Delta B}{\Delta I + \Delta C'}.$$

   If the incremental ratio exceeds 1, the extra spending is justified: k becomes the current best. Otherwise j survives.

4. Repeat against the next alternative on the list until none remain. The survivor of the final pairing is the choice.

Two special cases. If $\Delta I + \Delta C' = 0$, the alternatives cost the same, and the one with the larger B is chosen directly. If lives are unequal and the projects can be repeated, state B, I, and C' as annual equivalents before differencing, the same repair used in Chapter 5.

Example 8.3 — Choosing one interchange design

A transport directorate must build exactly one of three interchange designs. The present worths of each component at the 5% social discount rate are given below, in millions of SAR.

	A1 (signals)	A2 (full flyover)	A3 (partial flyover)
B	26	72	45
I	12	44	27
C'	8	14	5

Which design should be built? Show that ranking by raw ratio picks a different, and poorer, design.

Solution.

	A1	A2	A3
$I + C'$	20	58	32
BC	1.30	1.24	**1.41**
PW	6	**14**	13

Table 8.1. Raw ratios and present worths for the three designs. The largest ratio (A3) and the largest present worth (A2) disagree.

Step 1: screen against do-nothing.

All three ratios exceed 1 (Table 8.1), so all three survive. Note the trap immediately: A3 has the best ratio (1.41), but A2 has the largest present worth (14 million SAR). If ratios could rank, A3 would win; the present worth says otherwise.

Step 2: order by denominator.

$I + C'$ is 20, 32, and 58 for A1, A3, and A2, so the comparison order is A1, then A3, then A2.

Step 3: A3 versus A1.

$$\mathrm{BC}(5\%)_{3-1} = \frac{45 - 26}{(27 - 12) + (5 - 8)} = \frac{19}{12} = 1.58 > 1,$$

so the extra 12 million SAR of net cost earns more than it costs: A3 replaces A1 as the current best.

Step 4: A2 versus A3.

$$\mathrm{BC}(5\%)_{2-3} = \frac{72 - 45}{(44 - 27) + (14 - 5)} = \frac{27}{26} = 1.04 > 1,$$

just above 1: the further increment to the full flyover is also justified. Choose $\boxed{\text{A2, the full flyover}}$. The verdict agrees with the present-worth ranking $(14 > 13 > 6)$, as it must. The raw-ratio ranking would have stopped at A3 and quietly given up 1 million SAR of net public value. Decision sentence: we recommend A2, noting that the final increment clears the bar narrowly, so its benefit estimate deserves a sensitivity check before commitment.

Figure 8.2 draws the three designs side by side: the tallest bar is A3, but both increments tested along the way clear 1, so the walk ends one bar further to the right, at A2.

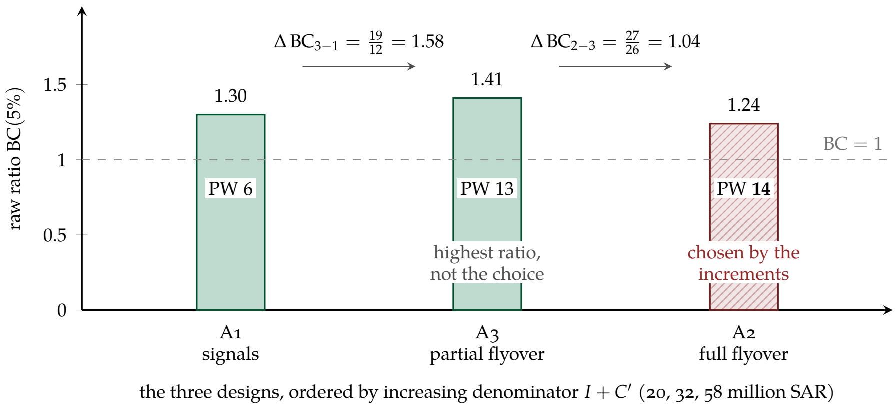

Figure 8.2. Raw benefit–cost ratios for the three interchange designs of Example 8.3, ordered by increasing denominator. Ranking by the ratio would pick A3, the tallest bar; the incremental ratios written above the arrows are both above 1, so the correct choice is A2, the shortest bar and the largest present worth.

Insight. Why does the incremental method work? Accepting increment after increment, each with $\Delta B > \Delta I + \Delta C'$, adds only positive present-worth steps, so the walk ends at the alternative with the largest total present worth. The ratio is used only to *test* each step, never to rank the field. This is the same repair chapter 7 applied to the IRR, because both flaws have one cause: a ratio strips away scale.

Reference. Park, 3rd ed., Section 8.4 (pp. 366–371), on highway user benefits.

8.5 *Valuing Benefits and Disbenefits*

The arithmetic of this chapter is easy; the inputs are not. Users' benefits must be expressed in SAR even when no market prices them directly. Standard practice builds them from engineering quantities multiplied by unit values. For a highway, the benefit of an alternative is measured against the base case through three channels: travel time (vehicle-hours saved, times a value of time), vehicle operating costs (vehicle-kilometers, times a cost per kilometer), and safety (expected crashes avoided, times a cost per crash). For a water project, the channels are supplied volume, reliability, and quality. Disbenefits enter the same way with the opposite sign: traffic diverted from an old route, a fishery degraded by brine discharge, land severed by an alignment.

One bookkeeping caution. A disbenefit is an unfavorable outcome for *users* and belongs in the numerator as a subtraction from B; it is not a sponsor cost. Moving an item between numerator and denominator does not change whether the ratio is above or below 1 (the accept boundary $B > I + C'$ is unchanged), but it *does* change the ratio's value, so two careful analysts can report different ratios for the same project. Present worth has no such ambiguity, which is one more reason to report it alongside the ratio.

Social discount rate. The interest rate used to discount the benefits and costs of public projects. It plays the role the MARR plays for a firm.

8.6 The Social Discount Rate

What rate should i be? For a firm, Chapter 5 answered: the MARR, anchored to the cost of capital and opportunity cost. For a public project the answer has been argued for decades, because the choice embodies a value judgment about the future. Two traditional views frame the debate.

1. *Projects without private counterparts* (flood control, rural access roads, community water): discount at the government's borrowing rate, the actual cost of the public funds.

2. *Projects with private counterparts* (power generation, toll roads): discount at the rate the displaced private capital would have earned, so that public investment does not pull capital from higher-yielding private uses.

A lower rate weights distant benefits more heavily and therefore favors long-lived projects; the third pre-class question showed a factor of 3.5 between 2% and 7% for a perpetual benefit. Because many public benefits, climate protection, groundwater recharge, road safety, run for generations, the rate choice is often *the* decisive input.

Reference. U.S. Office of Management and Budget, Circular No. A-94, revised November 9, 2023, and its Appendix D. Figures verified against the published circular in July 2026; Appendix D states it remains in force until a scheduled 2026 update.

Current U.S. federal guidance illustrates where the debate has settled for now. OMB Circular A-94, the government-wide standard for benefit–cost analysis of federal programs, was revised in November 2023. Its Appendix D estimates the risk-free real discount rate as the 30-year average real return on long-term government debt, currently 2.0% per year, and directs agencies to discount at this 2.0% real rate when benefits and costs are stated as certainty equivalents, or at 3.1% (adding a 1.1% default risk premium) when they are not. The previous guidance had used a 7% real base rate for decades, so the 2023 revision roughly halved-and-halved-again the rate at which future public benefits are shrunk. The appendix is updated every three years. For coursework in this book, the social discount rate is given in each problem, but you should recognize the orders of magnitude: a public 2–3% real rate is far below a private MARR of 10–15%, and Section 8.7 shows how much that gap matters.

Insight. Why did the 2023 revision lower the rate? Not by changing philosophy but by re-measuring: A-94's Appendix D computes the average *real* yield on 10-year Treasury debt over the trailing 30 years, and three decades of falling real interest rates pulled that average down to 2.0%. The lesson for any analysis: the social discount rate is an empirical, dated quantity. A benefit–cost ratio is meaningful only next to its rate and its vintage, the same lesson Chapter 5 taught for a published NPV and its MARR.

8.7 *A Full Public Example: Coastal Desalination*

The pieces now assemble into one complete analysis, in the format expected on graded work: classify the parties, draw the flows, compute B, I, and C', report both ratios and the present worth, and test the sensitivity to the discount rate.

Example 8.4 — Benefit–cost analysis of a coastal desalination plant

A coastal governorate proposes a seawater reverse-osmosis plant to end chronic summer shortages. Construction takes two years: 400 million SAR now and 250 million SAR at the end of year 1. The plant then operates from year 2 through year 25. Each operating year: the reliable supply is valued by the users at 95 million SAR; brine discharge reduces the local fishery's catch by 6 million SAR (a disbenefit); the plant costs the governorate 45 million SAR to run, of which 18 million SAR is recovered through water tariffs. The social discount rate is 3%. (a) Compute B, I, C', BC(3%), PI(3%), and the present worth. (b) Recompute the ratio at 2% and at 7% and comment.

Solution.

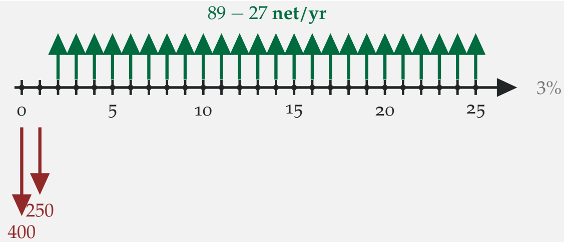

Figure 8.3. The desalination plant in millions of SAR: two construction years, then 24 operating years. The users' net benefit is $95 - 6 = 89$ per year; the sponsor's net operating cost is $45 - 18 = 27$ per year.

Step 1: classify and net each side.

Users: benefits 95, disbenefit 6, so $b_n = 95 - 6 = 89$ million SAR per year for years 2–25. Sponsor: operating cost 45 less tariff revenue 18, so $c_n = 27$ million SAR per year over the same years, plus the two capital payments $(K = 1)$.

Step 2: discount each component.

The operating window is 24 years (years 2 through 25). Value each series at year 1 with $(P/A, 3\%, 24) = 16.9355$, then move to year 0 with $(P/F, 3\%, 1) = 0.9709$; the product is 16.4427:

$$B = 89\,(16.4427) = 1{,}463.40 \text{ million SAR},$$
$$I = 400 + 250\,(0.9709) = 642.73 \text{ million SAR},$$
$$C' = 27\,(16.4427) = 443.95 \text{ million SAR}.$$

Step 3: the verdict at 3%.

$$\mathrm{BC}(3\%) = \frac{1{,}463.40}{642.73 + 443.95} = \frac{1{,}463.40}{1{,}086.68} = \boxed{1.35} > 1,$$
$$\mathrm{PI}(3\%) = \frac{1{,}463.40 - 443.95}{642.73} = \boxed{1.59} > 1,$$
$$\mathrm{PW}(3\%) = 1{,}463.40 - 1{,}086.68 = \boxed{+376.72 \text{ million SAR}}.$$

Accept: the plant returns about 1.35 SAR of net public benefit per SAR of public cost, a surplus of roughly 377 million SAR in today's money.

Step 4: sensitivity to the rate.

Repeating steps 2–3 with the factors at 2% and at 7%:

Rate	2%	3%	7%
B	1,650.34	1,463.40	954.01
$I + C'$	1,145.77	1,086.68	923.07
BC	1.44	1.35	1.03
PW	+504.58	+376.72	+30.94

Table 8.2. The same plant at three social discount rates (millions of SAR). The verdict survives, but the margin shrinks by a factor of sixteen between 2% and 7%.

The project passes at all three rates, but not equally: at 7%, the old U.S. federal base rate, the surplus is a thin 31 million SAR and the ratio is 1.03; at 2%, the current risk-free real rate of Circular A-94, the surplus is 505 million SAR. A reviewer applying the older, higher rate would call this project marginal; a reviewer applying the newer rate would call it comfortable. The engineering is identical in both reviews. Decision sentence: we recommend the plant, and we report the ratio alongside its rate, because the rate is doing much of the work.

8.8 A Distributional Remark

The benefit–cost ratio adds SAR across people. A saved hour of a wealthy commuter and a saved hour of a poor farmer enter B at their monetized values, and the ratio is indifferent to who receives them, and to who bears the disbenefits. The desalination plant of Example 8.4 passes comfortably at 3%, yet its fishery disbenefit falls entirely on one small community while the water benefits spread across a governorate. Two projects with identical ratios can therefore embody very different social bargains.

Insight. BC > 1 says the winners gain more SAR than the losers lose, so the winners *could* compensate the losers and still come out ahead. It does not say the compensation happens. A complete public recommendation therefore reports the ratio *and* the incidence: who gains, who loses, and whether compensation (resettlement payments, fishery restoration funds) is part of the design.

Formal tools exist for weighing efficiency against distribution, service coverage, and other objectives that resist a single SAR figure; chapter 18 develops them. For this chapter, the discipline is simpler: state the incidence next to the ratio, and never let a strong ratio silence the question of who pays.

8.9 Case Study from the Literature

Published benefit–cost studies use exactly the ratio of this chapter, and reading one closes the loop between the classroom formula and a policy document. Hurtado and Berbel (2024) evaluated two responses to the drought in Axarquia, a farming region of southern Spain whose reservoir can no longer meet demand: reusing treated wastewater (reclaimed water) for irrigation, and desalinating seawater. Their analysis prices the users' benefits (added agricultural value, employment welfare, avoided fertilizer, CO_2 effects) and the sponsors' costs (production, storage and pumping, CO_2 emissions) as uniform annual amounts, and evaluates both programs over 25 years at a 4% discount rate. All amounts below are in thousands of euros per year, the currency and form used in the paper.

Reference. Hurtado and Berbel (2024), *Land*, 13(12):2156. Open access; full reference at the end of the chapter. Figures read from the published tables in July 2026.

Example 8.5 — Case study: reclaimed water and desalination in Axarquia

Hurtado and Berbel (2024) report the following annual benefit and cost components (thousands of EUR per year).

	Reclaimed water ($19.5\ hm^3/yr$)	Desalinated water ($25\ hm^3/yr$)
Added agricultural value	37,398	47,946
Employment welfare (agric.)	8,780	11,257
Fertilizer savings	39	—
Avoided discharge to water bodies	27	—
Avoided fertilizer CO_2	1	—
CO_2 capture by new crops	945	1,211
Induced value, other sectors	29,919	38,357
Employment welfare (other)	8,846	11,341
Production cost	2,137	21,200
Storage and pumping	2,672	3,426
CO_2 emissions cost	240	2,542

Compute the benefit–cost ratio of each program and compare with the published values (17.02 and 4.05). Explain why no (P/A) factor is needed, and state what the two ratios may and may not be used to conclude.

Solution.

Step 1: sum each side.

For reclaimed water,

$$B = 37{,}398 + 8{,}780 + 39 + 27 + 1 + 945 + 29{,}919 + 8{,}846 = 85{,}955,$$

$$C = 2{,}137 + 2{,}672 + 240 = 5{,}049$$

thousand EUR per year. For desalinated water,

$$B = 47{,}946 + 11{,}257 + 1{,}211 + 38{,}357 + 11{,}341 = 110{,}112,$$
$$C = 21{,}200 + 3{,}426 + 2{,}542 = 27{,}168.$$

Step 2: the ratios.

Every component is a uniform annual amount over the same 25 years, so numerator and denominator would each be multiplied by the same $(P/A, 4\%, 25) = 15.6221$; the factor cancels, and the ratio of annual equivalents equals the ratio of present worths, as Section 8.2 noted:

$$\text{BC}_{\text{reclaimed}} = \frac{85{,}955}{5{,}049} = \boxed{17.02}, \qquad \text{BC}_{\text{desalinated}} = \frac{110{,}112}{27{,}168} = \boxed{4.05},$$

matching the published figures. In present-worth terms the desalination program's net benefit is $(110{,}112 - 27{,}168)(15.6221) \approx 1{,}296$ million EUR over the 25 years.

Step 3: what the ratios mean.

Both programs pass the BC ≥ 1 test decisively, and the paper evaluates them as *complementary* water sources, not as rivals, so both can be recommended. What the raw ratios must *not* be used for is a mutually exclusive choice. If the region could build only one program, the comparison would run through the increment (Section 8.4), and the answer could favor desalination despite its far smaller ratio, because it mobilizes more water and more total value. A ratio of 17 describes efficiency per euro, not total contribution.

What did the researchers conclude? That reclaimed water is the highly cost-effective first move, largely because its production cost is about 0.11 EUR per cubic meter against 0.848 EUR per cubic meter for desalinated water, and that desalination remains economically viable (BC $= 4.05 > 1$) as a supplementary source where reuse capacity runs out. Their policy reading matches the arithmetic: fund both, sequence by efficiency.

In practice. The gap between the two ratios is thermodynamics before it is economics. Desalination must push seawater across membranes at high pressure, and the paper's cost of 0.848 EUR/m^3 carries that energy bill; reclaimed water is already fresh and needs mainly polishing and pumping (the paper uses 0.426 kWh/m^3 for a 100 m lift). When an energy price or a treatment technology changes, these engineering coefficients, not the benefit–cost formula, move the verdict.

Insight. Note what the case study quietly assumed: the induced value in other sectors (a multiplier of 1.80 on agricultural value) and the employment welfare that follows from it together account for about 45% of each B. Multiplier-based secondary benefits are accepted in some guidelines and excluded in others precisely because they sit close to double counting. Strip them out and the ratios drop to about 9.3 and 2.2: still above 1, so here the verdict is robust, but a reviewer should always recompute a published ratio without its most debatable benefit class.

8.10 Chapter Summary

- Public projects separate the **users** (who receive benefits and disbenefits) from the **sponsor** (who pays capital and operating costs, net of any revenues). The users' side nets to B = benefits − disbenefits; the sponsor's side is $C = I + C'$.

- The **benefit–cost ratio** is $\mathrm{BC}(i) = B/(I + C')$; accept a single project when it is at least 1. The **profitability index** $\mathrm{PI}(i) = (B - C')/I$ measures net benefit per unit of capital. Both cross 1 exactly where the present worth crosses 0, so for a single project all three criteria agree.

- Where an item is booked (numerator or denominator) changes the value of a ratio but never the accept/reject verdict; present worth is free of this ambiguity, so report it alongside the ratio.

- **Never rank mutually exclusive alternatives by raw ratios.** Order by denominator and step through **incremental ratios** $\Delta B/(\Delta I + \Delta C')$, keeping the higher-cost alternative when the increment exceeds 1. The result always matches the present-worth ranking; the raw-ratio ranking often does not. This is the same scale blindness, and the same repair, as the IRR in chapter 7.

- Benefits and disbenefits are engineering quantities times unit values: vehicle-hours, crashes avoided, cubic meters supplied. Guard against **double counting**, especially land values that capitalize benefits already counted, and label multiplier-based secondary benefits.

- The **social discount rate** is the public MARR, and its choice is a debate: the government borrowing rate for projects without private counterparts, the displaced private return for projects with them. Current U.S. guidance (OMB Circular A-94, revised 2023) uses a 2.0% risk-free real rate, or 3.1% with a default risk premium; the earlier standard of 7% stood for decades. Low rates favor long-lived projects, so state the rate next to every ratio.

- A ratio above 1 proves the winners could compensate the losers, not that they do. Report the **distribution** of benefits and burdens next to the ratio; chapter 18 gives formal tools when several objectives must be weighed.

8.11 Exercises

These problems screen single public projects with the benefit–cost ratio and the profitability index, compare mutually exclusive alternatives incrementally, and test how much the social discount rate moves the verdict.

Screening a single public project

Exercise 8.1. A wadi-crossing culvert program has, at the sponsor's discount rate, a benefit present worth of 18.2 million SAR, a capital present worth of 9.5 million SAR, and an operating-cost present worth of 4.1 million SAR. Compute the benefit–cost ratio, the profitability index, and the present worth, and state the decision.

Exercise 8.2. A rural clinic-upgrade project costs 40 million SAR at year 0. From year 1 to year 8 it delivers health benefits valued at 12 million SAR per year, imposes a disbenefit of 1 million SAR per year (longer travel during reconstruction detours), and costs 3 million SAR per year to operate. The social discount rate is 6%. Compute B, I, C', and BC(6%), and state the decision.

Exercise 8.3. An analyst moves a project's disbenefit from the numerator (subtracted from B) to the denominator (added to the sponsor's costs). Which statement is correct?

(A) The accept/reject verdict can flip, so the convention must be fixed by regulation.

(B) The ratio's value changes, but the accept/reject verdict cannot flip; the present worth is unchanged entirely.
(C) Both the ratio's value and the present worth change.
(D) Nothing changes, because the ratio is invariant to bookkeeping.

Ratios under a capital budget

Exercise 8.4. A directorate has four *independent* proposals, all with $\mathrm{BC} > 1$, and a capital budget that cannot fund them all. The components (millions of SAR, present worths) are: W ($B = 30$, $I = 10$, $C' = 12$), X ($B = 30$, $I = 22$, $C' = 0$), Y ($B = 14$, $I = 5$, $C' = 6$), Z ($B = 44$, $I = 40$, $C' = 1$). Compute each proposal's profitability index and rank the proposals for funding. Why is PI, rather than BC, the right triage tool here, and why would neither ratio be acceptable if the proposals were mutually exclusive?

Mutually exclusive public projects

Exercise 8.5. Two designs for the same flood-protection berm are mutually exclusive. At the social discount rate, the present worths (millions of SAR) are: D1 ($B = 95$, $I = 60$, $C' = 10$) and D2 ($B = 140$, $I = 95$, $C' = 15$). Compute both raw ratios, then make the correct choice by incremental analysis, and verify against present worths.

The discount rate decides

Exercise 8.6. A village water system requires capital with a present worth of 45 million SAR and will then run *indefinitely*: users' benefits of 4 million SAR per year, sponsor operating costs of 1.5 million SAR per year. Compute the benefit–cost ratio at a social discount rate of 2% and again at 7%. What decides this project's fate?

Exercise 8.7. Classify each item for the benefit–cost analysis of a toll highway: (i) travel-time savings to drivers; (ii) toll receipts; (iii) pavement maintenance; (iv) noise near the alignment; (v) construction cost; (vi) lost sales at bypassed roadside shops.

Exam-style problems

The following problems are written in the format of a major exam: a scenario, lettered parts with point values, and full work expected. Classify the parties before computing, write each component (B, I, C') before forming any ratio, box each final answer with its units, and end with a decision sentence.

Exercise 8.8. Upgrading a rural access road. A ministry will upgrade a gravel road serving a phosphate-mining district and its villages. Construction costs 120 million SAR now and 80 million SAR at the end of year 1; the upgraded road then serves years 2 through 20. Each service year: travel-time and vehicle-operating savings are 32 million SAR; avoided crashes are worth 6 million SAR; severed grazing access imposes a disbenefit of 3 million SAR; and the ministry spends 4 million SAR on maintenance, with no tolls. The social discount rate is 4%. (a) (*8 pts*) Compute the users' net annual benefit and the present worths B, I, and C'. (b) (*8 pts*) Compute the benefit–cost ratio and the profitability index, and state the decision. (c) (*4 pts*) The mining company offers to pay half of the maintenance. Without recomputing, state which components change, and in which direction the two ratios move.

Exercise 8.9. Choosing one flood-control scheme for a wadi. Exactly one of three schemes will be built; do-nothing is excluded by a royal directive after last year's flooding. At the 5% social discount rate the component present worths (millions of SAR) are:

	S1 (levees)	S2 (dam)	S3 (channel + levees)
B	180	310	225
I	95	200	140
C'	25	40	20

(a) (*4 pts*) Compute the three raw ratios and state why they must not be used to rank the schemes. (b) (*12 pts*) Carry out the incremental analysis in the correct order and state the choice. (c) (*4 pts*) Verify the choice against the present worths.

Assessing outside recommendations

The two problems below present finished analyses produced by others. In each, the arithmetic is correct; the flaw is in the method. For each problem, (a) decide whether to accept the recommendation, (b) identify each error or omission precisely, and (c) produce the corrected numbers.

Exercise 8.10. A consultant ranks park-and-ride sites by raw B/C. A transit authority must build exactly one of three park-and-ride facilities. A consultant's memo reports, at the approved discount rate (present worths in millions of SAR): P1 ($B = 54$, $I = 30$, $C' = 6$), P2 ($B = 100$, $I = 70$, $C' = 10$), P3 ($B = 76$, $I = 49$, $C' = 6$), and concludes:

> *The benefit–cost ratios are P1:* $54/36 = 1.50$*, P2:* $100/80 = 1.25$*, and P3:* $76/55 = 1.38$*. All are acceptable, and P1 delivers the most benefit per riyal. Recommendation: build P1, the highest-ratio site.*

(a) (*4 pts*) Should the authority accept the recommendation? (b) (*8 pts*) Identify the error precisely. (c) (*8 pts*) Produce the corrected analysis and the correct choice.

Exercise 8.11. An AI assistant counts a benefit twice. A governorate asked an AI assistant to evaluate a commuter-rail extension. At the approved rate, the assistant reports (present worths, millions of SAR): travel-time and operating savings to riders, 40; rise in land values along the corridor, 25; capital, 35; operating cost, 17, offset by fare revenue of 5. Its analysis:

> $B = 40 + 25 = 65$. *Sponsor cost* $= 35 + (17 - 5) = 47$. $\text{BC} = 65/47 = 1.38 > 1$. *The extension pays for itself in public value. Recommendation: proceed.*

(a) (*4 pts*) Should the governorate accept the recommendation? (b) (*8 pts*) Identify each error or omission precisely. (c) (*8 pts*) Produce the corrected ratio and the correct decision.

References

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