6 *Annual-Equivalence Analysis*

After this chapter you will be able to:

1. convert a present worth into an annual equivalent with $AE(i) = PW(i)\,(A/P, i, N)$, and state why the two criteria always give the same decision
2. separate capital costs from operating costs, and compute the capital recovery cost $CR(i) = (I - S)(A/P, i, N) + iS$ of owning an asset
3. build the full annual equivalent cost of a project, including a perpetual asset whose capital cost is $I \times i$ per year
4. compute a unit cost, a minimum sustainable price, and the levelized cost of energy from an annual equivalent cost
5. decide a make-or-buy question, and find the break-even volume or usage at which two options cost the same
6. compare alternatives with unequal lives using their one-cycle annual equivalents

In chapter 5, every project was judged by its present worth: all cash flows were moved to time zero and added there. The present worth is a single number, expressed in money at time zero. This chapter expresses the same information in a different unit: money per year. The *annual equivalent worth* of a project is the uniform annual amount that is economically equivalent to the project's whole cash flow series. The two criteria never disagree, because one is a fixed positive multiple of the other. But the per-year unit is often the more natural one. Budgets are annual. Rents, salaries, and lease payments are annual. A production manager asks what a machine costs per year, not what it costs at time zero. A sales manager asks what one unit must sell for. This chapter develops the tools that answer such questions: the AE criterion; the separation of capital costs from operating costs; the capital recovery cost, which is the annual cost of owning an asset; unit costs and minimum prices; make-or-buy decisions and their break-even volume; and the comparison of alternatives with unequal lives, where annual equivalence removes the replication step that present worth required.

Thinking in cost per year.

Q1. A plant manager asks, "What does this compressor cost us per year?" Which criterion answers the question in the manager's own unit: present worth or annual equivalence?

Q2. A project has $PW = +12{,}000$ SAR at the MARR over 6 years. Can its annual equivalent at the same rate be negative?

Q3. A machine has a purchase price and an annual electricity bill. Which of the two must be converted before it can enter a per-year comparison?

Answers.

Q1. *Annual equivalence.* Present worth answers in money at time zero. Annual equivalence restates the same information as a uniform amount per year, which is the unit the manager is using.

Q2. *No.* $AE = PW \times (A/P, i, N)$, and the factor (A/P) is always positive. Multiplying a positive number by a positive number cannot change the sign.

Q3. *The purchase price.* It is a one-time capital cost, so it must be annualized. The electricity bill is a recurring operating cost and is already stated per year.

Lecture 15. This section is covered by Lecture 15.

Reference. Park, 3rd ed., Sections 6.1–6.2.

Annual equivalent worth. $AE(i) = PW(i)\,(A/P, i, N)$: the uniform annual amount, over the project life, that is economically equivalent to the project's entire cash flow series.

6.1 *The Annual Equivalence Criterion*

The annual equivalent worth of a project is its present worth spread evenly over the project life:

$$AE(i) = PW(i) \times (A/P,\ i,\ N). \tag{6.1}$$

The factor $(A/P, i, N)$ is the capital-recovery factor from chapter 2. There it converted a loan principal into equal installments. Here it converts a present worth into an equal yearly worth. It is the same factor doing the same job: spreading a single amount evenly over N periods.

The decision rule is unchanged from present worth. For a single project at the MARR, accept if $AE > 0$ and reject if $AE < 0$.

Insight. Why AE and PW always give the same decision. The factor $(A/P, i, N)$ is positive for every positive rate and every N. Equation (6.1) therefore multiplies the present worth by a positive number, and multiplying by a positive number cannot change the sign. If $PW > 0$ then $AE > 0$; if $PW < 0$ then $AE < 0$; if $PW = 0$ then $AE = 0$. The two criteria are two statements of the same fact, one at time zero and one per year. An exam statement claiming that PW and AE can give different verdicts on the same project is false, always.

Example 6.1 — From present worth to annual equivalent

The haul-truck autonomy retrofit of chapter 5 has $PW = 7{,}216$ SAR at the MARR of 12% over a 5-year life. Restate the project's value as an annual equivalent.

Solution. The present worth is a single amount at time zero. The (A/P) factor spreads it into five equal annual amounts.

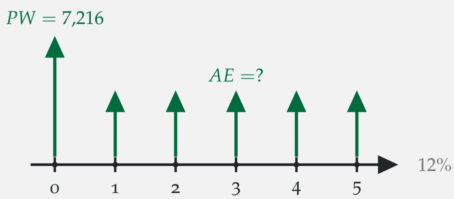

Figure 6.1. The present worth at year 0 and its equivalent uniform series over years 1–5.

$$AE(12\%) = PW\,(A/P,\,12\%,\,5) = 7{,}216\,(0.27741) = \boxed{2{,}002 \text{ SAR per year}}.$$

Both numbers value the same project. The retrofit creates 7,216 SAR of value stated at time zero, or 2,002 SAR of value in each year of its life. The verdict is accept either way.

Use annual equivalence whenever the audience thinks per year. The common cases are annual budgets, lease and rental pricing, cost per unit of output, and the comparison of assets with unequal lives, which closes this chapter.

6.2 *Capital Costs and Operating Costs*

Capital cost. A one-time ownership cost: the purchase price I at year 0 and the salvage value S recovered at year N.

Salvage value. The amount S recovered when the asset is sold or retired at the end of its service life.

Operating cost. A recurring cost of using the asset: energy, labor, maintenance. When uniform, it is already stated per year.

A project carries two kinds of cost, and they enter a per-year analysis by different routes.

- *Capital (ownership) costs* are one-time. The asset is purchased for I at year 0, and the salvage value S is recovered at year N. These amounts must be annualized before they can enter a per-year comparison, because a lump sum and a yearly amount are different units.

- *Operating costs* are recurring: energy, labor, maintenance. When they are uniform, they are already on a per-year basis and need no conversion.

Please keep the two categories separate in every setup. Problems in this chapter nearly always give a mix of both, and mixing them up is the most common structural error. The annualized capital cost has its own name, the *capital recovery cost*, and it is the subject of the next section.

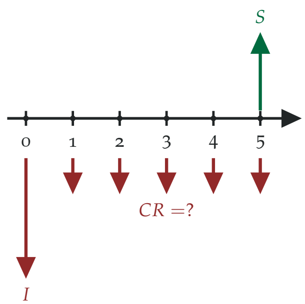

Figure 6.2. The capital flows of ownership: pay I now, recover S at year N. $CR(i)$ is their equivalent annual cost.

Capital recovery cost. $CR(i) = (I-S)(A/P,i,N)+iS$: the annual cost of owning an asset bought for I and resold for S after N years.

6.3 *The Capital Recovery Cost*

The capital recovery cost answers one question: what does owning this asset cost per year? Ownership involves two capital flows, the purchase I at year 0 and the salvage S at year N. Their combined annual equivalent cost is

$$CR(i) = (I - S)\,(A/P,\,i,\,N) + i\,S. \tag{6.2}$$

Insight. Where the formula comes from. Annualize each capital flow directly: the purchase costs $I\,(A/P,i,N)$ per year, and the salvage returns $S\,(A/F,i,N)$ per year, so $CR(i) = I\,(A/P,i,N) - S\,(A/F,i,N)$. Now use the identity from chapter 2, $(A/F,i,N) = (A/P,i,N) - i$, and substitute:

$$CR(i) = I\,(A/P) - S\,[(A/P) - i] = (I-S)\,(A/P,i,N) + iS.$$

The rearranged form has a direct reading. The asset loses the amount $(I - S)$ over its life, and this depreciating portion is repaid in installments through (A/P). The remaining portion S is capital that is eventually returned, but while it is parked in the asset it earns nothing, so its annual cost is the interest iS.

Two special cases mark the ends of the range. If the salvage is zero, the formula collapses to $I\,(A/P, i, N)$, the plain capital-recovery pattern of chapter 2. If the asset never loses value, so $S = I$, the formula collapses to iI: pure interest on the parked capital. Every real asset sits between these two ends.

Example 6.2 — Capital recovery cost of a site pickup truck

A site pickup truck is bought for 78,000 SAR and will be resold for 30,000 SAR after 4 years. At $i = 8\%$, find the annual cost of owning the truck.

Solution.

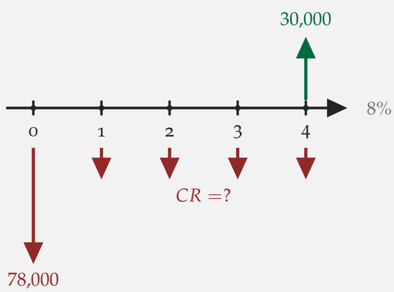

Figure 6.3. Purchase at year 0, resale at year 4, and the unknown equivalent annual ownership cost.

Step 1: factor notation.

$$CR(8\%) = (I - S)\,(A/P,\,8\%,\,4) + i\,S.$$

N	(A/P)	(P/A)
3	0.38803	2.5771
4	**0.30192**	3.3121
5	0.25046	3.9927
6	0.21632	4.6229

Table 6.1. Excerpt from the $i = 8\%$ interest factor table used for Example 6.2.

Step 2: read the table. On the 8% page, row $N = 4$, the (A/P) column (Table 6.1) gives 0.30192.

Step 3: substitute.

$$\begin{aligned} CR(8\%) &= (78{,}000 - 30{,}000)\,(0.30192) + 0.08\,(30{,}000) \\ &= 48{,}000\,(0.30192) + 2{,}400 = 14{,}492 + 2{,}400 \\ &= \boxed{16{,}892 \text{ SAR per year}}. \end{aligned}$$

Owning the truck costs about 16,900 SAR per year before fuel, insurance, or maintenance. Vehicle and equipment leases are priced on exactly this structure: the lessor recovers the value loss in installments and charges interest on the capital tied up in the vehicle.

In practice. A startup deciding whether to buy or lease equipment should compute exactly this number. $CR(i)$ at the firm's own hurdle rate is the fair ownership cost to hold against the vendor's lease line, and because a young firm's capital is expensive, its i is high and the lease often wins.

Example 6.3— Justifying a purchase by annual savings

A fabrication shop considers a pipe-threading machine. It costs 110,000 SAR, has a salvage value of 10,000 SAR after 5 years, and saves 33,000 SAR per year in outsourced work. The MARR is 15%. Should the shop buy the machine?

Solution. Compare the annual savings against the annual cost of ownership.

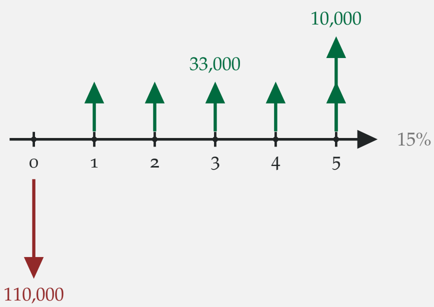

Figure 6.4. The machine's full cash flow: purchase, five years of savings, and salvage.

Step 1: the ownership cost per year.

$$\begin{aligned} CR(15\%) &= (110{,}000 - 10{,}000)\,(A/P,\, 15\%,\, 5) + 0.15\,(10{,}000) \\ &= 100{,}000\,(0.29832) + 1{,}500 = \boxed{31{,}332 \text{ SAR per year}}. \end{aligned}$$

Step 2: the verdict per year.

$$AE(15\%) = 33{,}000 - 31{,}332 = \boxed{+1{,}668 \text{ SAR per year}} > 0 \;\Rightarrow\; \text{accept.}$$

The machine covers its own cost: owning it costs about 31,300 SAR per year, and it brings in 33,000 SAR per year. This is the same verdict that present worth would give, stated in the language a shop manager uses. This savings-versus-ownership comparison is the most common pattern in the chapter; most problems are this pattern with more pieces attached.

6.4 *Building a Full Annual Equivalent Cost*

Annual equivalent cost (AEC). The total cost of a project per year: the capital recovery cost plus the annual equivalent of all operating costs.

When a problem asks for the total cost of an asset per year, both cost categories must be included. The pattern is: capital through $CR(i)$, and operating costs through their annual equivalent, then add. When the operating costs are uniform, they are already annual. When they are not uniform, collapse them to a present worth first and then apply (A/P).

Example 6.4 — Annual equivalent cost of a standby generator

A standby generator costs 90,000 SAR, has a salvage value of 9,000 SAR after 5 years, and costs 15,000 SAR per year to operate in years 1–2 and 18,000 SAR per year in years 3–5. The MARR is 10%. Find the total annual equivalent cost.

Solution.

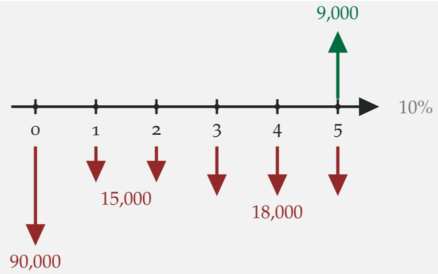

Figure 6.5. Purchase, stepped operating costs, and salvage for the standby generator.

Step 1: the capital, through $CR(i)$.

$$\begin{aligned} CR(10\%) &= (90{,}000 - 9{,}000)\,(A/P,\,10\%,\,5) + 0.10\,(9{,}000) \\ &= 81{,}000\,(0.26380) + 900 = 22{,}268 \text{ SAR per year.} \end{aligned}$$

Step 2: the operating costs, through present worth.

The costs step up at year 3, so one factor cannot handle them. Collapse them to a present worth first. The second block is a uniform series over years 3–5; $(P/A, 10\%, 3)$ values it at year 2, and $(P/F, 10\%, 2)$ carries that value to year 0.

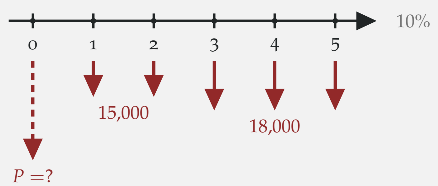

Figure 6.6. The stepped operating costs collapsed to a single present worth at year 0.

$$\begin{aligned} P_{\text{opr}} &= 15{,}000\,(P/A,\,10\%,\,2) + 18{,}000\,(P/A,\,10\%,\,3)\,(P/F,\,10\%,\,2) \\ &= 15{,}000\,(1.7355) + 18{,}000\,(2.4869)(0.8264) \\ &= 26{,}033 + 36{,}993 = 63{,}026 \text{ SAR.} \end{aligned}$$

Then spread this present worth over the 5-year life:

$$AE_{\text{opr}} = 63{,}026\,(A/P,\,10\%,\,5) = 63{,}026\,(0.26380) = 16{,}626 \text{ SAR per year.}$$

Step 3: add the two pieces.

$$AEC = 22{,}268 + 16{,}626 = \boxed{38{,}894 \text{ SAR per year}}.$$

Please pay attention here, because this is a common trap. Averaging the operating costs, $(15 + 15 + 18 + 18 + 18)/5 = 16.8$ thousand SAR, ignores when each cost occurs. The cheap years come first, and early amounts count for more at 10%, so the true annual equivalent of the operating costs (16,626 SAR) is below the plain average (16,800 SAR). The present-worth route gets the timing right.

6.5 *Perpetual Assets*

Perpetual asset. An asset with an infinite service life, such as a dam, a tunnel, or a permanent station. Its capital is never recovered, so its annual capital cost is the interest $I \times i$.

Some assets serve indefinitely. When the life is infinite, the capital is never recovered, so its annual cost is simply the interest on it, forever: $I \times i$. Annual operating and maintenance costs are added directly. A repair that recurs every k years is spread into an annual equivalent with $(A/F, i, k)$ over one cycle, because the same cycle repeats forever.

Insight. This is the exact dual of the capitalized equivalent $CE = A/i$ in chapter 5. There, an annual amount was converted into the single deposit that funds it forever. Here, a capital amount is converted into the annual interest it costs forever. The bridge runs both ways: dividing a perpetual asset's annual equivalent cost by i gives the single deposit that funds the asset forever.

Example 6.5 — A perpetual water-treatment station

A mine-site water-treatment station costs 3,000,000 SAR to build, costs 80,000 SAR per year to operate and maintain, and needs a 1,000,000 SAR overhaul every 25 years, indefinitely. At $i = 8\%$, find the annual equivalent cost.

Solution.

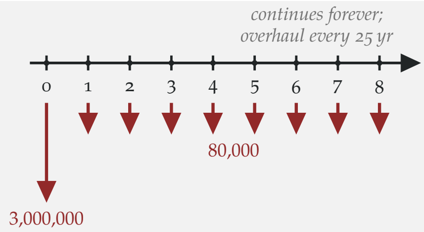

Figure 6.7. The station's cash flows: construction, perpetual upkeep, and a repeating overhaul (only the first years are drawn).

Three pieces, all per year:

$$\begin{aligned} \text{Capital (forever):} \quad & 3{,}000{,}000 \times 0.08 = 240{,}000, \\ \text{O\&M:} \quad & 80{,}000 \text{ (already annual)}, \\ \text{Overhaul:} \quad & 1{,}000{,}000\,(A/F, 8\%, 25) = 1{,}000{,}000\,(0.01368) = 13{,}680. \end{aligned}$$

$$AEC = 240{,}000 + 80{,}000 + 13{,}680 = \boxed{333{,}680 \text{ SAR per year}}.$$

Please note the two different factor jobs. The (A/F) factor spreads a repeating future cost across its cycle. The interest-only term handles capital that is never paid back. As a check on the dual relationship, $333{,}680/0.08 \approx 4.17$ million SAR is the single deposit at 8% that would fund the station forever.

In practice. The 25-year overhaul cycle is a life estimate for liners, membranes, and structures, not a financial choice. If corrosion shortens the cycle to 20 years, the (A/F) term must be recomputed. The finance is only as good as the life estimate under it.

Lecture 16. This section is covered by Lecture 16.

Reference. Park, 3rd ed., Sections 6.2–6.3.

Unit cost. The annual equivalent cost divided by the annual volume of output: $AEC/(\text{units per year})$.

Minimum sustainable price. The unit cost computed at the MARR. At this price the operation earns exactly its required return; below it, the operation destroys value.

6.6 Unit Cost and Unit Profit

Annual equivalence becomes operational when it is divided by volume. Cost per kilogram, price per unit, cost per machine hour: these are the numbers a plant negotiates with. The recipe has three steps: compute the capital recovery cost, add the annual operating cost to get the total annual equivalent cost, and divide by the annual volume.

Example 6.6 — Minimum price for a reagent-bagging line

A line bags 40,000 kg of flotation reagent per year. The line costs 300,000 SAR, has a salvage value of 30,000 SAR after 8 years, and costs 90,000 SAR per year to operate. The MARR is 15%. Find the minimum price per kilogram.

Solution.

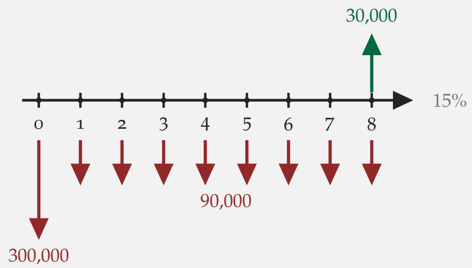

Figure 6.8. The bagging line: investment, eight years of operating cost, and salvage.

Step 1: the capital.

$$\begin{aligned} CR(15\%) &= (300{,}000 - 30{,}000)\,(A/P,\,15\%,\,8) + 0.15\,(30{,}000) \\ &= 270{,}000\,(0.22285) + 4{,}500 = 60{,}170 + 4{,}500 = 64{,}670 \text{ SAR per year.} \end{aligned}$$

Step 2: the total annual equivalent cost.

$$AEC = 64{,}670 + 90{,}000 = 154{,}670 \text{ SAR per year.}$$

Step 3: divide by the volume.

$$\text{minimum price} = \frac{154{,}670}{40{,}000} = \boxed{3.87 \text{ SAR per kg}}.$$

Below 3.87 SAR per kg the line destroys value at a 15% MARR; above it, every kilogram contributes surplus.

Insight. What sits inside a minimum price. It is not only the out-of-pocket costs. The 15% return on the invested capital is built into the capital recovery term. A plant selling exactly at the minimum price is therefore not breaking even in the accounting sense; it is earning its full hurdle rate. This is why the number is called the minimum *sustainable* price. Please also note the volume assumption: the computation spreads the fixed costs over the full 40,000 kg. If demand falls short, the fixed costs spread over fewer units and the true unit cost rises.

Unit profit. The selling price minus the unit cost. It is surplus per unit beyond the MARR return on capital.

When the product has a selling price, the *unit profit* is the price minus the unit cost. Because the unit cost already carries the MARR return on capital, the unit profit is surplus beyond the required return. Multiplying the unit profit by the annual volume gives the project's annual equivalent worth, which is a useful cross-check: if unit profit times volume disagrees with the annual equivalent computed directly, one of the two has an error.

Levelized cost of energy. the AE unit cost of a power plant, stated in money per kilowatt-hour: the annualized capital cost plus the annual O&M, divided by the annual energy delivered.

6.7 *Levelized Cost: The Annual-Equivalence Idea at Grid Scale*

The unit-cost recipe of section 6.6 runs the world's electricity investment decisions under an industry name: the *levelized cost of energy* (LCOE). A power plant is a machine whose product is the kilowatt-hour. Its capital cost is annualized with $(A/P, i, N)$, its annual operating cost is added, and the total is divided by the energy it delivers in a year:

$$\mathrm{LCOE}(i) = \frac{I\,(A/P,\, i,\, N) + C_{\mathrm{O\&M}}}{E}, \tag{6.3}$$

where I is the installed capital cost, $C_{\mathrm{O\&M}}$ is the annual operating and maintenance cost, and E is the annual energy delivered. Nothing in Equation (6.3) is new: it is the minimum sustainable price of section 6.6, with kilowatt-hours as the product. A plant selling its energy at exactly its LCOE earns exactly the rate i on the capital invested in it.

Capacity factor. The fraction of the year's 8,760 hours, weighted at full rating, that a plant actually delivers: annual energy $E = 8{,}760 \times CF$ per kW of capacity.

The convenient scale is one kilowatt of capacity. A year has 8,760 hours, but no plant runs at full rating around the clock: clouds cover the sun, wind calms, machines stand down for maintenance. The *capacity factor CF* is the delivered fraction, so each kilowatt of capacity delivers $E = 8{,}760 \times CF$ kilowatt-hours per year. A solar plant with $CF = 0.20$ delivers 1,752 kWh per kW-year; a base-load plant with $CF = 0.80$ delivers 7,008.

Insight. An LCOE is not a new criterion, and it is not a market price. It is an annual equivalent cost divided by an annual output, so it embeds a discount rate and a lifetime exactly the way every AE number in this chapter does. Two LCOE figures computed at different rates, or over different lifetimes, are not comparable, and a published LCOE with no stated rate cannot be checked at all. When you read an LCOE, ask the same two questions you would ask of any annual equivalent: what i, and what N?

Example 6.7 — Diesel against solar at a remote mine

A remote mine generates its own electricity. Diesel generator sets are already the default; a developer proposes a solar-plus-storage plant. The mine's MARR is 8%, and the course-style numbers below are in SAR.

- **Solar plus storage**: installed cost 6,000 SAR per kW, fixed O&M 60 SAR per kW per year, capacity factor 25% (the storage shifts part of the midday output into the evening), life 25 years.
- **Diesel**: generator sets cost 1,200 SAR per kW, run at a capacity factor of 60% over a 15-year life, and burn fuel and consumables worth 0.95 SAR per delivered kWh.

Find the levelized cost of each option.

Solution.

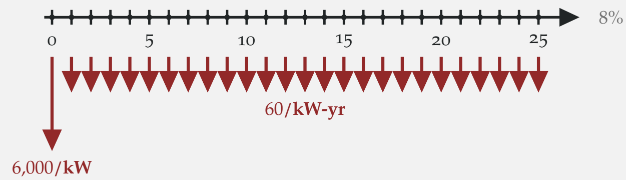

Figure 6.9. One kilowatt of the solar option: installed cost at year 0 and fixed O&M each year. The product is the stream of kilowatt-hours.

Step 1: annual energy per kilowatt.

$$E_{\text{solar}} = 8{,}760 \times 0.25 = 2{,}190 \text{ kWh}, \qquad E_{\text{diesel}} = 8{,}760 \times 0.60 = 5{,}256 \text{ kWh}.$$

Step 2: the solar LCOE.

Annualize the capital with $(A/P, 8\%, 25) = 0.09368$, add the O&M, and divide:

$$\text{LCOE}_{\text{solar}} = \frac{6{,}000\,(0.09368) + 60}{2{,}190} = \frac{562.1 + 60}{2{,}190} = \boxed{0.284 \text{ SAR per kWh}}.$$

Step 3: the diesel LCOE.

The capital part uses $(A/P, 8\%, 15) = 0.11683$; the fuel is already per kWh:

$$\text{LCOE}_{\text{diesel}} = \frac{1{,}200\,(0.11683)}{5{,}256} + 0.95 = 0.027 + 0.95 = \boxed{0.977 \text{ SAR per kWh}}.$$

Every solar kilowatt-hour costs about 0.28 SAR against 0.98 SAR for diesel, so each kilowatt-hour the solar plant displaces saves the mine roughly 0.69 SAR. The cost structures are opposites: solar is nearly all capital, priced through (A/P), while diesel is nearly all fuel, priced per kWh. That is why the discount rate barely moves the diesel number but governs the solar one. Note also what the comparison does *not* say: at a 25% capacity factor the solar plant cannot carry the mine alone, so the gensets stay for the hours the storage cannot cover, and the real decision is what fraction of the load to shift.

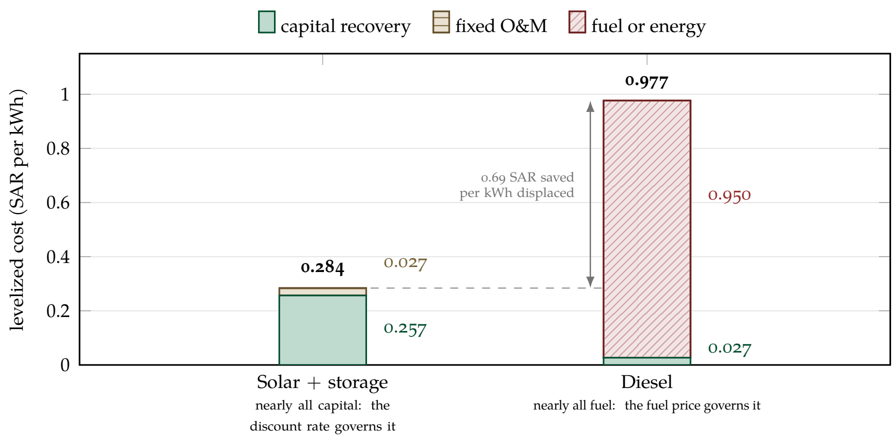

Figure 6.10. What a levelized cost is made of, for the remote-mine options of Example 6.7 at a MARR of 8%. The two totals are far apart, 0.284 against 0.977 SAR per kWh, but the compositions are opposites: solar is almost entirely capital recovery through (A/P), diesel almost entirely fuel, so the discount rate governs one bar and the fuel price governs the other.

Figure 6.10 breaks both totals into their three parts; read the segments rather than the bar heights, because the composition is what tells you whether the discount rate or the fuel price governs a given option.

One caution belongs next to every levelized-cost comparison. A vendor's LCOE brochure is an advertisement, and the softest number in Equation (6.3) is the capacity factor, which enters the denominator. A brochure computed at an irradiance the site does not have, or with no allowance for dust, soiling, and downtime, divides the same annual cost by kilowatt-hours that will never be delivered, and every one of those missing kilowatt-hours makes the quoted cost per kWh look cheaper. The arithmetic in such a brochure is usually correct; the assumption is where the sale is made. Recompute any vendor LCOE at your own site's capacity factor, your own MARR, and your own lifetime before comparing it with anything.

6.8 Make-or-Buy Decisions

Make-or-buy. The choice between producing a good or service in-house (own the asset) and purchasing it from an outside vendor (outsource).

A make-or-buy decision compares the in-house unit cost against a vendor's price for the same good or service. The comparison is fair only under three requirements.

1. The same volume and quality must sit on both sides.
2. The in-house side must carry *all* ownership costs through $CR(i)$, not only the operating costs. Counting only operating costs is the standard way these comparisons are done wrong, and it makes the make option look artificially cheap.
3. The decision can flip with volume. The vendor charges a purely variable price, so the buy side scales down when volume drops. The in-house side carries fixed capital whether or not the line runs. At low volumes the vendor wins.

In practice. The volume effect explains a standard pattern in young firms: outsource first, bring the work in-house later. A young firm sits far to the left of Q^*, where the vendor's variable price beats the fixed cost of owning a line; growth is what moves it across the break-even volume.

Break-even volume. The annual volume at which two alternatives have the same annual cost. Above it one alternative is cheaper; below it, the other.

Example 6.8 — Make or buy: the reagent line

A tolling contractor offers to bag the reagent of Example 6.6 for 4.50 SAR per kg. The in-house cost is 3.87 SAR per kg at the planned 40,000 kg per year. (a) Should the firm make or buy at the planned volume? (b) Treating the in-house annual cost of 154,670 SAR as fixed, find the annual volume at which the two options tie.

Solution.

(a) *The per-unit comparison at planned volume.*

The vendor option has no investment; it is a pure annual payment of $4.50 \times 40{,}000 = 180{,}000$ SAR per year.

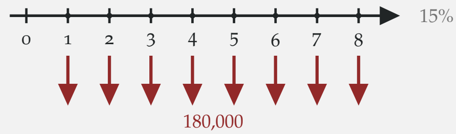

Figure 6.11. The buy option at full volume: a vendor fee with no investment and no salvage.

The in-house cost is 3.87 SAR per kg against the vendor's 4.50 SAR per kg, so the firm should *make*, and save

$$(4.50 - 3.87) \times 40{,}000 = \boxed{25{,}200 \text{ SAR per year}}.$$

Watch the size of the error if the capital is omitted. Without $CR(i)$, the in-house side would show only the 90,000 SAR of operating cost, which is $90{,}000/40{,}000 = 2.25$ SAR per kg, and the make option would look far cheaper than it is. Including the capital brings it to 3.87 SAR per kg, and the comparison becomes honest.

(b) *The break-even volume.*

Let Q be the annual volume in kg. The in-house cost is fixed at 154,670 SAR per year; the vendor cost is $4.50\,Q$. Set them equal:

$$4.50\,Q = 154{,}670 \quad \Longrightarrow \quad Q^* = \frac{154{,}670}{4.50} = \boxed{34{,}371 \text{ kg per year}}.$$

Figure 6.12 shows the two cost lines. The in-house line is flat: the plant pays its capital and operating costs whether or not the line runs. The vendor line starts at zero and climbs with every kilogram. Above about 34,400 kg per year, owning the line is cheaper; below it, the vendor wins.

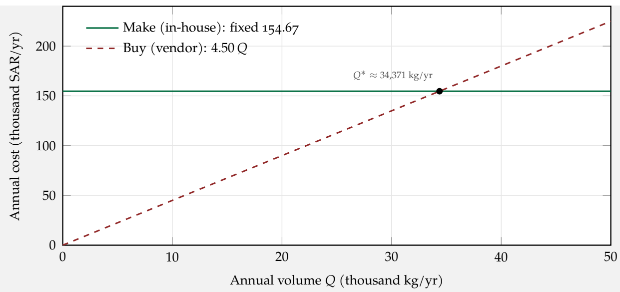

Figure 6.12. Annual cost against volume for the make and buy options. The lines cross at the break-even volume Q^*; above it the in-house line is lower, below it the vendor line is lower.

The mirror error exists on the buy side. If outsourcing lets the firm sell the machine, the freed capital must be credited to the buy option. Fairness cuts both ways.

6.9 *Break-Even Usage Between Two Machines*

The same fixed-versus-variable logic settles a choice between two machines: an expensive machine that is cheap to run, and a cheap machine that is expensive to run. Write each machine's annual cost as a function of usage, equate, and solve. The answer is the minimum usage that justifies the more expensive machine.

Example 6.9 — Break-even hours for two sample grinders

An assay lab compares two sample grinders. Both last 10 years with no salvage, and annual maintenance is 15% of first cost. G1 costs 4,500 SAR and uses energy at 0.52 SAR per hour. G2 costs 3,600 SAR and uses energy at 0.60 SAR per hour. At $i = 6\%$, find the operating hours per year at which the two machines cost the same.

Solution. Let h be the operating hours per year. Each machine's annual cost has a fixed part, which does not depend on h, and a variable part, which grows with every hour of use.

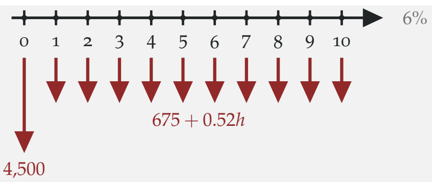

Figure 6.13. G1's cash flows: first cost at year 0, then a yearly cost with a fixed maintenance part and a usage part. G2 has the same shape with its own numbers.

$$\begin{aligned} AE_1(h) &= 4{,}500\,(A/P,\,6\%,\,10) + 675 + 0.52h \\ &= 611.4 + 675 + 0.52h = 1{,}286.4 + 0.52h, \\ AE_2(h) &= 3{,}600\,(A/P,\,6\%,\,10) + 540 + 0.60h \\ &= 489.1 + 540 + 0.60h = 1{,}029.1 + 0.60h. \end{aligned}$$

Set the two equal and solve:

$$1{,}286.4 + 0.52h = 1{,}029.1 + 0.60h \implies 0.08h = 257.3,$$

$$h^* = \boxed{3{,}216 \text{ hours per year}}.$$

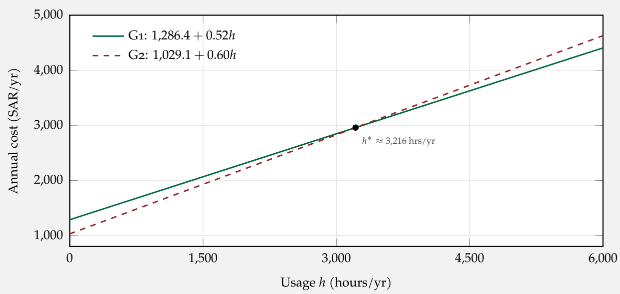

Figure 6.14. The two annual-cost lines. G1 starts higher but climbs more slowly; G2 starts lower but climbs faster. They cross once, at the break-even usage.

Finish with the decision sentence the problem is really asking for. If the lab expects to grind more than about 3,216 hours per year, buy G1; otherwise buy G2.

Repeatability assumption. Each alternative can be repeated with identical costs and life, cycle after cycle, over the whole service period. Under this assumption, one-cycle AE comparisons are valid.

6.10 *Comparing Alternatives with Unequal Lives*

In chapter 5, alternatives with unequal lives were compared by replicating each one out to a common horizon, the least common multiple of the lives, and then comparing present worths. Annual equivalence skips that step. Annualize each alternative over *its own* life and compare the annual amounts directly.

Insight. Why one cycle is enough. If an alternative is repeated identically, every cycle has the same cash flows and the same length, so every cycle has the same annual equivalent. The AE over one cycle is therefore the AE over two cycles, ten cycles, or any horizon made of whole cycles. Comparing one-cycle AEs is the same comparison as the least-common-multiple replication, without building the replication.

Two conditions apply. First, the repetition must be identical: same costs, same life, cycle after cycle. Second, a stated study period still overrides. When the problem imposes a horizon, go back to present worth over that period, as in chapter 5.

Example 6.10 — Two tank coatings with unequal lives

A storage tank can be protected with an epoxy coating that costs 5,000 SAR and lasts 10 years, or a standard coating that costs 3,000 SAR and lasts 5 years. Either coating would be renewed identically when it wears out. At $i = 10\%$, which coating is cheaper per year?

Solution. Annualize each coating over its own life.

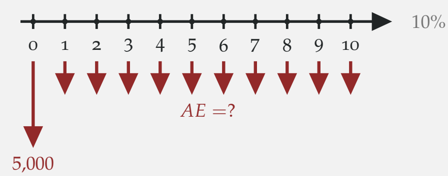

Figure 6.15. The epoxy coating: one payment spread over its 10-year life.

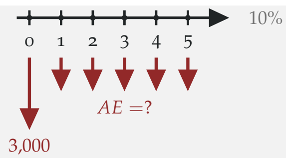

Figure 6.16. The standard coating: one payment spread over its 5-year life.

$$AE_{\text{epoxy}} = 5{,}000\,(A/P,\,10\%,\,10) = 5{,}000\,(0.16275) = \boxed{813.7 \text{ SAR per year}},$$
$$AE_{\text{std}} = 3{,}000\,(A/P,\,10\%,\,5) = 3{,}000\,(0.26380) = \boxed{791.4 \text{ SAR per year}}.$$

The standard coating is cheaper, by about 22 SAR per year.

Check against the replication method. Over a 10-year common horizon, the standard coating costs 3,000 now plus 3,000 at year 5: $PW = 3{,}000 + 3{,}000\,(P/F, 10\%, 5) = 4{,}863$ SAR, against 5,000 SAR for the epoxy. Annualizing the replication, $4{,}863\,(A/P, 10\%, 10) = 791.4$ SAR per year, reproduces the one-cycle AE exactly. The two methods give the same verdict; AE simply skips the replication.

This is the standard exam shape for unequal-life comparisons: two options, different first costs, different lives, and a question asking which annual cost is cheaper and by how much. Annualize each over its own life and subtract.

6.11 *Case Study from the Literature*

The levelized cost of section 6.7 is not a textbook artifact; it is the number on which the world's power-plant investment is reported. The International Renewable Energy Agency publishes an annual open-access survey of what newly built plants actually cost, drawn from its database of real projects. Its report for 2023 (IRENA, 2024) states that newly commissioned utility-scale solar photovoltaic plants had a global weighted-average installed cost of 758 USD per kW, a capacity factor of 16.2%, and a levelized cost of 0.044 USD per kWh. The case study below checks that published LCOE with nothing but this chapter's machinery.

Reference. IRENA (2024), *Renewable power generation costs in 2023*. Open access; full reference at the end of the chapter.

Example 6.11 — Case study: reproducing IRENA's solar LCOE

Take IRENA's published 2023 global averages for utility-scale solar PV: installed cost $I = 758$ USD per kW and capacity factor $CF = 16.2\%$. All amounts are in US dollars, the currency used in the report. IRENA's project-level discount rates are country-specific, so adopt a working assumption of $i = 5\%$ (real), a 25-year life, and a fixed O&M of 10 USD per kW-year, and compute the LCOE. Then recompute it at $i = 10\%$.

Solution.

Step 1: annual energy per kilowatt.

$$E = 8{,}760 \times 0.162 = 1{,}419 \text{ kWh per kW-year.}$$

Step 2: the annual equivalent cost per kilowatt.

With $(A/P, 5\%, 25) = 0.07095$:

$$AEC = 758\,(0.07095) + 10 = 53.78 + 10 = 63.78 \text{ USD per kW-year.}$$

Step 3: divide by the energy.

$$\text{LCOE}(5\%) = \frac{63.78}{1{,}419} = \boxed{0.045 \text{ USD per kWh}},$$

which reproduces IRENA's published 0.044 USD per kWh within rounding. The chapter's unit-cost recipe and the agency's methodology are the same calculation.

Step 4: the same plant at a 10% rate.

With $(A/P, 10\%, 25) = 0.11017$:

$$\text{LCOE}(10\%) = \frac{758\,(0.11017) + 10}{1{,}419} = \frac{93.51}{1{,}419} = \boxed{0.066 \text{ USD per kWh}}.$$

Nothing physical changed, yet the cost per kilowatt-hour rose by almost half. In a capital-heavy technology, the discount rate is one of the largest cost components, and it never appears on any invoice.

What did the researchers conclude? IRENA (2024) reports that the global weighted-average LCOE of utility-scale solar PV fell from 0.460 USD per kWh in 2010 to 0.044 USD per kWh in 2023, a decline of 90%, and that solar in 2023 cost 56% less than the weighted-average fossil-fuel alternative, which stood at 0.100 USD per kWh. By its count, 81% of the utility-scale renewable capacity commissioned in 2023 produced electricity more cheaply than the fossil average. The same report tracks battery storage, whose project costs fell from 2,511 to 273 USD per kWh of storage capacity between 2010 and 2023, which is why the solar-plus-storage option of Example 6.7 has become a serious competitor for remote mine power. At residential scale, the Jeddah rooftop case study of chapter 5 (Imam, Al-Turki, & Kumar, 2020) reported an LCOE of 0.031 USD per kWh against a 0.048 USD per kWh utility tariff, which is the same comparison at household size.

Insight. Note what the case study did and did not verify. Given IRENA's inputs and a stated rate and lifetime, the LCOE follows mechanically from (A/P); that part is arithmetic, and anyone can check it. What cannot be checked from the outside are the inputs themselves: the installed cost, the capacity factor, and the discount rate embedded in the published number. This is the general lesson of the chapter applied to the energy industry: an annual equivalent is only as good as the estimates and the rate under it.

6.12 Chapter Summary

- $AE(i) = PW(i)\,(A/P, i, N)$: the project's present worth spread evenly over its life. Because $(A/P) > 0$, **AE and PW always carry the same sign** and the same decision.

- **Capital costs** (purchase I, salvage S) are one-time and must be annualized. **Operating costs** recur and, when uniform, are already annual.

- The **capital recovery cost**, $CR(i) = (I - S)(A/P, i, N) + iS$, is the annual cost of ownership: repay the value loss $(I - S)$ in installments, and pay interest on the capital parked in S. Special cases: $S = 0$ gives $I\,(A/P)$; $S = I$ gives iI.

- A purchase is justified when its annual savings exceed $CR(i)$.

- Full **AEC**: capital through $CR(i)$; irregular operating costs through present worth, then (A/P); then add.

- **Perpetual assets**: $I \times i$ per year for the capital, plus annual O&M, plus every-k-year repairs spread with $(A/F, i, k)$. This is the dual of the capitalized equivalent $CE = A/i$.

- **Unit cost** = AEC ÷ annual volume. The **minimum sustainable price** is the unit cost at the MARR; it already includes the required return on capital. **Unit profit** = price − unit cost.

- The **levelized cost of energy** is the AE unit cost of a power plant: LCOE $= [I\,(A/P, i, N) + C_{\text{O\&M}}]/E$, with $E = 8{,}760 \times CF$ per kW. It embeds a discount rate, a lifetime, and a capacity factor; recompute any quoted LCOE with your own three before comparing.

- **Make-or-buy**: compare per unit at equal volume and quality; the in-house side must carry the full $CR(i)$. The verdict can flip with volume at the **break-even volume**.

- **Break-even usage**: write each option's annual cost as a function of usage, equate, and solve.

- **Unequal lives**: compare one-cycle AEs under the repeatability assumption. A stated study period overrides; then compare PW over that period.

6.13 *Exercises*

These problems follow the chapter's own sequence: capital recovery cost, full annual equivalent cost, unit cost and minimum sustainable price, then make-or-buy, break-even usage, and assets with unequal lives.

Capital recovery cost

Exercise 6.1. A machine costs 60,000 SAR and has a salvage value of 10,000 SAR after 5 years. At $i = 10\%$, find the capital recovery cost.

Exercise 6.2. A CNC machine for a mine's maintenance shop costs 50,000 SAR and has a salvage value of 8,000 SAR after 6 years. At $i = 12\%$, find the capital recovery cost.

Perpetual assets

Exercise 6.3. A tailings pump station will serve its site indefinitely. Construction costs 4,000,000 SAR, operations and maintenance cost 150,000 SAR per year, and a major repair costs 1,500,000 SAR every 20 years. The MARR is 8%. Find the annual equivalent cost of the station.

Unit cost and minimum price

Exercise 6.4. A lubricant-bottling machine at a mine's supply depot costs 150,000 SAR, has a salvage value of 15,000 SAR after 6 years, and costs 40,000 SAR per year to operate. It fills 20,000 units per year, and each unit sells at 4.50 SAR. The MARR is 12%. Find the unit cost and the unit profit.

Exercise 6.5. A water-bottling line will produce 3,000,000 bottles per year for 10 years. The investment is 2,000,000 SAR, the salvage value is 400,000 SAR, and operating costs are 700,000 SAR per year. The MARR is 15%. Determine the minimum price per bottle the company should charge.

Quick checks (multiple choice)

Exercise 6.6. A project has $PW(15\%) = -3{,}400$ SAR over 8 years. Without computing, what can you say about its $AE(15\%)$?

(A) Positive; AE and PW can disagree.

(B) Zero; AE ignores the initial investment.

(C) Negative; $AE = PW \times (A/P)$ and $(A/P) > 0$.

(D) It cannot be determined without the cash flows.

Exercise 6.7. In a make-or-buy analysis, the most common error that makes "make" look artificially cheap is:

(A) using the same volume on both sides.

(B) omitting the capital recovery cost from the in-house side.

(C) including the vendor's price per unit.

(D) discounting at the MARR.

From present worth to per-year language (in-class activity)

Exercise 6.8. *Najd Copper* must dewater its copper concentrate before rail shipment. An automated filter press costs 70,000 SAR today, cuts net moisture-penalty and labor costs by 22,000 SAR per year for 5 years, and can be resold for 10,000 SAR at the end of year 5. The MARR is 10%. (a) Compute the NPW of buying and operating the press. (b) A vendor offers to run the same dewatering as a service for 18,000 SAR per year (the 22,000 SAR per year of savings are earned either way). Find the capital recovery cost of the press and decide: own or outsource? (c) Show that the press's annual equivalent carries the same sign as the NPW in part (a), and explain why that must be true.

What your car really costs (in-class activity)

Exercise 6.9. In class, each team prices the ownership of a real car model and defends its assumptions. Work the same analysis with the following data. A sedan costs 95,000 SAR new. If kept for 4 years, its resale value will be about 52,000 SAR; if kept for 6 years, about 34,000 SAR. Money is worth 10%. Insurance costs 4,800 SAR per year and fuel costs 6,000 SAR per year in either case. Maintenance averages 2,400 SAR per year over a 4-year ownership and 3,400 SAR per year over a 6-year ownership. (a) Find the capital recovery cost for the 4-year window. (b) Find the total annual cost of ownership for the 4-year window. (c) Repeat for the 6-year window. (d) Which ownership window is cheaper per year? Write the decision in one sentence.

The outsourcing lab (in-class activity)

Exercise 6.10. *Wajd* must grind coffee for its roastery over a 6-year horizon at $i = 12\%$. **Make:** buy a burr mill for 84,000 SAR (salvage 12,000 SAR at year 6) and grind at an operating cost of 1.10 SAR per kg. **Buy:** a toll-roaster grinds for 2.05 SAR per kg, with no investment. (a) Compute the mill's capital recovery cost. (b) At Wajd's planned volume of 30,000 kg per year, compute the in-house cost per kg and decide make-or-buy. (c) Find the annual volume Q^* at which the two options tie, and state which option wins on each side of Q^*. (d) Write the decision in one sentence.

PW/AE mixed workshop (in-class activity)

The workshop rule: for each scenario, first name the tool (PW, capitalized equivalent, AE, or project balance), then justify the choice in one line, then solve.

Exercise 6.11. A municipality must light a road *forever*: installation 900,000 SAR, energy 70,000 SAR per year, and a lamp overhaul of 150,000 SAR every 10 years, at $i = 8\%$. What single budget figure funds the road lighting forever?

Exercise 6.12. Two compressors provide the same service: C1 costs 40,000 SAR, lasts 8 years, and costs 5,000 SAR per year to run; C2 costs 26,000 SAR, lasts 4 years, and costs 7,500 SAR per year to run. Either would be replaced identically at the end of its life. At $i = 10\%$, which is cheaper per year? Also state, in one sentence, why the one-cycle AE comparison is valid here even though the lives differ.

Exercise 6.13. A 4-year project invests 100,000 SAR at year 0 and returns 38,000 SAR per year, at $i = 12\%$. The CFO asks: "How exposed are we at the end of year 2?" Compute the relevant quantity.

Levelized cost of energy

Exercise 6.14. A mining company plans a 20 MW solar plant to displace part of its diesel generation. The installed cost is 2,900 SAR per kW, fixed O&M is 45 SAR per kW per year, the site's expected capacity factor is 22%, the life is 25 years, and the MARR is 9%. (a) Find the annual energy delivered per kW of capacity. (b) Compute the levelized cost of energy. (c) The vendor's brochure quotes 0.139 SAR per kWh for the same plant, computed at a capacity factor of 28%. Check the vendor's arithmetic and state, in one sentence, what the brochure hides.

Exam-style problems

The problems below follow the format, length, and point weights of Major Exam 2. Work each part in exam order: draw the diagram, write the factor notation, substitute, box the answer with its units, and close with the decision sentence.

Exercise 6.15. *Najd Copper* installs a concentrate drying unit at its rail-loading yard. The unit costs 120,000 SAR and can be resold for 30,000 SAR after 6 years. Operating costs are 16,000 SAR per year in years 1–3 and 20,000 SAR per year in years 4–6. The unit dries 5,000 tonnes of concentrate per year, and the MARR is 10%. (a) (*8 pts*) Compute the capital recovery cost $CR(10\%)$. (b) (*9 pts*) Compute the total annual equivalent cost of the unit. (c) (*8 pts*) A tolling contractor offers the same drying service at 11.00 SAR per tonne at equal quality. Treating the in-house annual cost as fixed, find the break-even yearly volume, and decide make-or-buy at the planned 5,000 tonnes per year. (d) (*4 pts*) In the long run the company will replace the unit with an identical one forever, and in addition a major overhaul of 25,000 SAR is required every 10 years. Compute the capitalized (perpetual) cost of the whole operation at the MARR.

Exercise 6.16. *Wadi Phosphate* must keep one slurry pump running at its washing plant. Two models do the identical job and would be replaced with identical units when they wear out. The MARR is 12%. Model P1 costs 26,000 SAR, costs 7,000 SAR per year to operate, lasts 4 years, and salvages for 6,000 SAR. Model P2 costs 40,000 SAR, costs 5,200 SAR per year to operate, lasts 8 years, and salvages for 8,000 SAR. (a) (*9 pts*) Compute the total annual equivalent cost of P1. (b) (*9 pts*) Compute the total annual equivalent cost of P2 and state the recommendation in one sentence. (c) (*7 pts*) The lives differ, 4 years against 8. State the assumption that makes the one-cycle comparison of parts (a)–(b) fair, and state what you would do instead if management imposed a fixed 5-year study period.

Assessing outside recommendations

Each problem below quotes a finished analysis produced by someone else: an AI assistant, a vendor, or a consultant. The work looks professional and its arithmetic is internally consistent, but each analysis contains one or two genuine method errors. Decide whether to accept or reject the recommendation, name each flaw precisely, and produce the corrected number.

Exercise 6.17. *Najd Copper* must choose between two liner-handling machines for its ball mills; either would be replaced identically when it wears out, and the MARR is 10%. H1 costs 60,000 SAR, costs 9,000 SAR per year to operate, lasts 6 years, and salvages for 6,000 SAR. H2 costs 36,000 SAR, costs 12,000 SAR per year to operate, lasts 3 years, and salvages for 4,000 SAR. A consultant's memo concludes:

> "Life-cycle cost of H1: $PW = 60{,}000 + 9{,}000\,(P/A, 10\%, 6) - 6{,}000\,(P/F, 10\%, 6) = 60{,}000 + 9{,}000\,(4.3553) - 6{,}000\,(0.5645) = 95{,}811$ SAR. Life-cycle cost of H2: $PW = 36{,}000 + 12{,}000\,(P/A, 10\%, 3) - 4{,}000\,(P/F, 10\%, 3) = 36{,}000 + 12{,}000\,(2.4869) - 4{,}000\,(0.7513) = 62{,}838$ SAR. H2 costs 62,838 SAR against 95,811 SAR for H1. We recommend H2, a saving of 32,973 SAR."

Accept or reject the memo? Name the flaw and produce the corrected comparison and decision.

Exercise 6.18. *Najd Copper*'s warehouse team asks an AI assistant whether to buy a concentrate forklift for 150,000 SAR (resale 45,000 SAR after 5 years) or lease the same capability for 31,500 SAR per year, at $i = 12\%$. The assistant replies:

> "The forklift loses $150{,}000 - 45{,}000 = 105{,}000$ SAR of value over five years. Spreading this loss over the life with the sinking-fund factor: annual ownership cost $= 105{,}000\,(A/F, 12\%, 5) = 105{,}000\,(0.15741) = 16{,}528$ SAR per year. Owning at 16,528 SAR per year is far cheaper than leasing at 31,500 SAR per year. Recommendation: buy."

Accept or reject? Name each flaw and produce the corrected ownership cost and decision.

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