5 Present-Worth Analysis

After this chapter you will be able to:

1. state what the MARR is, and distinguish project cash flows from financing cash flows
2. compute the conventional payback period and the discounted payback period, and explain why neither one is a decision criterion
3. compute the net present worth at the MARR, and decide to accept a single project when PW > 0
4. build a project balance table from $PB_n = PB_{n-1}(1+i) + A_n$, and read the discounted payback and the terminal net future worth from it
5. compute the capitalized equivalent $CE = A/i$ of a service that continues forever
6. choose among mutually exclusive alternatives, replicating to the least common multiple of the lives when a study period is not stated

Chapter 2 built the machinery for moving money through time: the cash flow diagram, the interest factors, and the idea of equivalence. This chapter uses that machinery to answer the question engineers are paid to answer: should we do this project? The chapter first separates project cash flows from loan cash flows and defines the MARR, the hurdle rate that every project must beat. It then develops the screening tools in order of strength: the payback period, the discounted payback period, and the net present worth criterion, which is the main decision rule of the whole course. Two close relatives of present worth follow: the net future worth, which states the same verdict at the end of the project, and the project balance, which tracks the project year by year like a bank statement. The capitalized equivalent extends present worth to projects that never end. The chapter closes with the comparison of mutually exclusive alternatives, including the do-nothing option, service and revenue projects, and the treatment of alternatives with unequal lives.

What a zero present worth means.

Q1. A firm's MARR is 12%. A proposed project has a present worth of exactly zero at 12%. Is this a losing project?

Q2. A machine costs 20,000 SAR and saves 5,000 SAR per year. How long until the savings repay the cost, if interest is ignored?

Q3. What single deposit funds a withdrawal of 10,000 SAR per year, forever, if the account earns 10%?

Answers.

Q1. *No.* A present worth of zero means the project repays its capital and earns interest at exactly 12% on every SAR while it is invested. There is no surplus beyond the hurdle rate, but there is no loss.

Q2. $20{,}000/5{,}000 = 4$ years. This is the conventional payback period. Note what was ignored to get it: the time value of money, and everything the machine does after year 4.

Q3. $P = A/i = 10{,}000/0.10 = 100{,}000$ SAR. The deposit earns exactly 10,000 SAR of interest each year, the withdrawal takes only the interest, and the principal never shrinks. This is the perpetuity from Chapter 2; in this chapter it becomes the capitalized equivalent.

Lecture 12. This section is covered by Lecture 12.

Reference. Park, 3rd ed., Sections 5.1–5.3 (pp. 194–209).

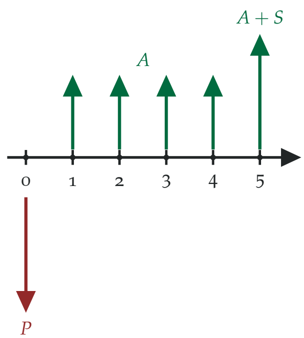

Figure 5.1. A typical project: investment down at year 0, net returns up each year, and a salvage value S added to the last return.

MARR. Minimum attractive rate of return: the lowest rate of return a firm accepts on a new project.

5.1 *From Loans to Projects: The MARR*

In Chapter 2, most transactions were loans and deposits: money is received now and paid back later, so the interest rate is the *cost* of money. In a project the direction flips. The firm pays money now, the investment, and receives returns later, so the rate describes what the money *earns*.

Every project competes with the other uses the firm has for its capital. The *minimum attractive rate of return*, the MARR, is the lowest return the firm will accept on a new project, because its capital could earn that much elsewhere at similar risk. Every project must clear this hurdle. Setting the MARR is a management decision that involves the cost of capital, risk, and opportunity cost; in this course the MARR is given in each problem.

Two remarks connect the MARR to earlier material. First, the MARR plays the same role for a project that the loan rate played for a loan: it is the price of capital per year. Second, the MARR is a *market* rate, in the sense of Section 2.2: it already reflects both earning power and inflation. Project cash flows are stated in actual money, so discounting them at the MARR accounts for inflation automatically; no separate adjustment is needed.

Payback period. The time required for the cumulative cash flows of a project to repay the initial investment, ignoring interest.

5.2 *The Conventional Payback Period*

The payback period asks the simplest question: how long until I get my money back? For a project with equal annual returns, the answer is a single division. For uneven flows, accumulate the cash flows year by year until the running total turns nonnegative.

Example 5.1 — Payback of an inspection robot

A phosphate plant buys an inspection robot to check its reactors. The robot costs 45,000 SAR and saves 12,000 SAR per year in labor and avoided downtime. Find the conventional payback period.

Solution.

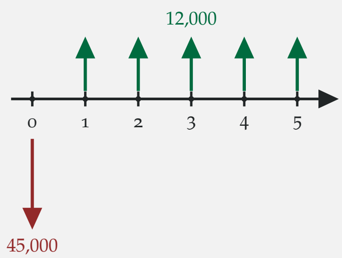

Figure 5.2. The inspection robot: one investment, five equal annual savings.

The savings are equal, so the payback is the investment divided by the annual saving:

$$\text{PP} = \frac{45{,}000}{12{,}000} = \boxed{3.75 \text{ years}}.$$

Payback is quick and intuitive, and many firms use it as a first filter. A common benchmark is to accept only projects that pay back within three years and to study those survivors more carefully. By that benchmark, the robot would not pass the first filter.

The payback period has two flaws, and you should be able to name both.

1. **It ignores the time value of money.** An amount received in year 4 counts the same as an amount received in year 1, which contradicts everything in Chapter 2.
2. **It ignores all cash flows after the payback point.** A project with large profits in its later years scores no better than one that ends the day after payback.

Insight. Payback measures *liquidity*, how fast the money returns, not *profitability*. A short payback tells a manager that the capital is exposed for a short time. It says nothing about how much value the project creates over its whole life. Payback is a screening tool; it is never a decision criterion on its own.

Discounted payback period. The time required for the cumulative *discounted* cash flows, at $i = \text{MARR}$, to repay the initial investment.

5.3 *The Discounted Payback Period*

The discounted payback period (DPP) repairs the first flaw. Instead of the raw cash flows, accumulate the *discounted* flows: each year's amount is multiplied by $(P/F, i, n)$ at the MARR before it enters the running total.

Example 5.2 — Discounted payback of the inspection robot

For the robot of Example 5.1 (invest 45,000 SAR, save 12,000 SAR per year for 5 years), find the discounted payback period at a MARR of 10%.

Solution.

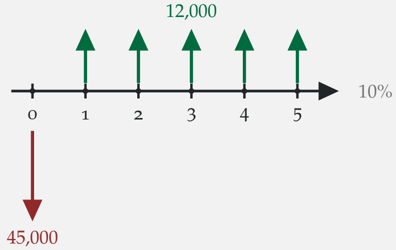

Figure 5.3. The same robot; the flows are now discounted at 10% before they are accumulated.

Each discounted flow is 12,000 $(P/F, 10\%, n)$. The ledger accumulates these amounts year by year:

n	0	1	2	3	4	5
Discounted flow	−45,000	10,909	9,917	9,016	8,196	7,451
Cumulative	−45,000	−34,091	−24,174	−15,158	−6,962	+**489**

Table 5.1. Discounted-payback ledger for the inspection robot at 10%. The cumulative row first turns nonnegative during year 5.

The cumulative row starts at −45,000 and climbs; it crosses zero during year 5. The shortfall at the end of year 4 is 6,962 SAR, and the year-5 discounted flow is 7,451 SAR, so

$$\text{DPP} = 4 + \frac{6{,}962}{7{,}451} = \boxed{4.93 \text{ years}},$$

compared with 3.75 years undiscounted. The discounted payback is always longer than the conventional payback, because the project must also earn interest on the capital while it is tied up.

Please note that the second flaw remains. Anything after the crossing point is still invisible, so even the discounted version is a screening tool, not a profitability measure.

5.4 *The Net Present Worth Criterion*

Net present worth. The sum of all project cash flows discounted to time zero at the MARR: $\text{PW}(i) = \sum_{n=0}^{N} A_n (1+i)^{-n}$.

The main criterion of this chapter discounts *every* cash flow of the project to time zero at the MARR and adds them up:

$$\text{PW}(i) = \sum_{n=0}^{N} A_n (1+i)^{-n} \qquad \text{evaluated at } i = \text{MARR}. \tag{5.1}$$

For a single project the decision rule is short:

- **accept** if $\text{PW} > 0$;
- **reject** if $\text{PW} < 0$;
- **indifferent** if $\text{PW} = 0$.

Every cash flow enters: the first cost, the revenues, the operating costs, and the salvage value. Nothing here is new mechanically. This is the equivalence calculation of Chapter 2, with (P/F) for single flows and (P/A) for series, evaluated at the MARR. What is new is the decision rule on top.

Example 5.3 — A haul-truck autonomy retrofit

A mine upgrades one haul truck to run autonomously. The kit costs 55,000 SAR, produces net savings of 16,000 SAR per year for 5 years, and is resold for 8,000 SAR at the end of year 5. The MARR is 12%. Should the mine accept the retrofit?

Solution.

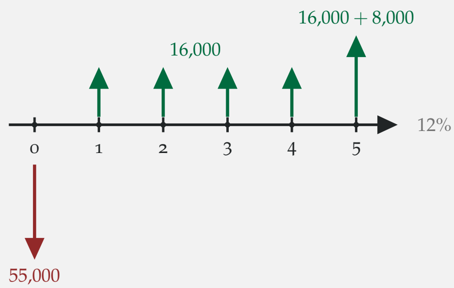

Figure 5.4. The retrofit: one arrow down at year 0, five annual savings up, with the resale value added to the last one.

Step 1: factor notation.

$$\text{PW}(12\%) = -55{,}000 + 16{,}000\,(P/A,\ 12\%,\ 5) + 8{,}000\,(P/F,\ 12\%,\ 5).$$

N	(P/A)	(P/F)
4	3.0373	0.6355
5	**3.6048**	**0.5674**
6	4.1114	0.5066

Table 5.2. Excerpt from the $i = 12\%$ interest factor table used for Example 5.3.

Step 2: read the table. On the 12% page, row $N = 5$ (Table 5.2) gives $(P/A) = 3.6048$ and $(P/F) = 0.5674$.

Step 3: substitute.

$$\begin{aligned} \text{PW}(12\%) &= -55{,}000 + 16{,}000\,(3.6048) + 8{,}000\,(0.5674) \\ &= -55{,}000 + 57{,}676.80 + 4{,}539.20 = \boxed{+7{,}216 \text{ SAR}}. \end{aligned}$$

The present worth is positive, so the mine should *accept* the retrofit.

It is worth pausing on what that number means. A positive present worth says the project does three jobs at once:

1. it **repays** the 55,000 SAR investment in full;
2. it **pays 12% interest** on every SAR while it is tied up in the retrofit;
3. it **leaves a surplus** of 7,216 SAR, measured in today's money.

Insight. PW $= 0$ does *not* mean "no profit." It means the project earns exactly the MARR: the returns repay the invested capital and pay interest at the hurdle rate, with no surplus and no shortfall. The surplus measured by a positive PW is profit *beyond* the hurdle rate, which is why the accept/reject boundary sits at zero.

5.5 *Net Future Worth*

Lecture 13. This section is covered by Lecture 13.

Reference. Park, 3rd ed., Sections 5.3.3–5.3.5 (pp. 206–213).

Net future worth. The project's cash flows carried forward to the end of the project: $\text{FW}(i) = \text{PW}(i)\,(F/P, i, N)$.

The net future worth (NFW) carries every cash flow forward to the end of the project instead of back to the start:

$$\text{FW}(i) = \sum_{n=0}^{N} A_n\,(1+i)^{N-n} = \text{PW}(i) \times (F/P,\, i,\, N). \tag{5.2}$$

It differs from the present worth by only one factor, $(F/P, i, N)$, and that factor is always positive. So the future worth has the same sign as the present worth, and it gives the same decision. Present worth and future worth are one equivalence statement, made at two different dates.

Example 5.4 — Future worth of the retrofit

Find the net future worth of the haul-truck retrofit of Example 5.3 at the end of year 5, and interpret it.

Solution.

The present worth is already known, so one factor moves it to year 5.

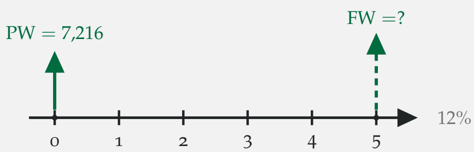

Figure 5.5. The single equivalent amount at year 0 carried forward to the end of the project.

$$\text{FW}(12\%) = 7{,}216\,(F/P,\ 12\%,\ 5) = 7{,}216\,(1.7623) = \boxed{12{,}717\text{ SAR}}.$$

The interpretation is concrete: if every cash flow were invested consistently at the MARR of 12%, the account would show a surplus of 12,717 SAR on the day the automation kit is resold.

The next example collects all four screening numbers for one project, in the ledger format used in the in-class activities. This is the full workflow: one table, four readings.

Example 5.5 — One project, four numbers

A workshop automation project costs 100,000 SAR at year 0 and returns 40,000 SAR at the end of each of years 1–4. The MARR is 10%. Find the conventional payback period, the discounted payback period, the net present worth, and the net future worth.

Solution.

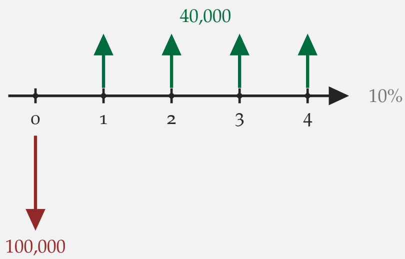

Figure 5.6. The automation project: one investment, four equal returns.

Step 1: build the ledger.

The first cumulative row uses the raw flows; the discounted rows multiply each flow by $(P/F, 10\%, n)$ first.

Year n	0	1	2	3	4
Cash flow A_n	−100,000	40,000	40,000	40,000	40,000
Cumulative (undiscounted)	−100,000	−60,000	−20,000	+20,000	+60,000
$(P/F, 10\%, n)$	1.0000	0.9091	0.8264	0.7513	0.6830
Discounted flow	−100,000	36,363.64	33,057.85	30,052.59	27,320.54
Cumulative (discounted)	−100,000	−63,636.36	−30,578.51	−525.92	+26,794.62

Table 5.3. The full screening ledger for Example 5.5: undiscounted and discounted cumulative rows at 10%.

Step 2: payback period.

The undiscounted cumulative row turns nonnegative during year 3. The shortfall at the end of year 2 is 20,000 SAR, against a year-3 flow of 40,000 SAR:

$$\text{PP} = 2 + \frac{20{,}000}{40{,}000} = \boxed{2.5 \text{ years}}.$$

Step 3: discounted payback period.

The discounted cumulative row turns nonnegative during year 4. The shortfall at the end of year 3 is 525.92 SAR, against a year-4 discounted flow of 27,320.54 SAR:

$$\text{DPP} = 3 + \frac{525.92}{27{,}320.54} = \boxed{3.02 \text{ years}}.$$

Step 4: net present worth.

The NPW is the last entry of the discounted cumulative row, because that row has already summed every discounted flow:

$$\text{PW}(10\%) = \boxed{+26{,}794.62 \text{ SAR}} \quad \Rightarrow \quad \textbf{accept}.$$

Step 5: net future worth.

One factor carries the NPW to year 4:

$$\begin{aligned}\text{FW}(10\%) &= 26{,}794.62\,(F/P,\ 10\%,\ 4)\\ &= 26{,}794.62\,(1.4641) = \boxed{39{,}230.00 \text{ SAR}}.\end{aligned}$$

Please note how the four numbers divide the work: the paybacks describe liquidity (when the capital comes back), and the two worths describe profitability (how much value is created, stated at two different dates).

Project balance. The year-by-year account of a project at the MARR: $PB_0 = A_0$ and $PB_n = PB_{n-1}(1+i) + A_n$.

5.6 *The Project Balance*

Net present worth and net future worth give a verdict, one number each. That verdict hides the journey. A manager asks three more questions. How much of our capital is at risk at the worst point? When do we climb out of the red? And if the project is cut short in year 2, where do we stand? A single present worth cannot answer any of these. The *project balance* can. It restates the same cash flows as a year-by-year account at the MARR:

$$PB_0 = A_0, \qquad PB_n = PB_{n-1}\,(1+i) + A_n \quad \text{(grow, then add)}. \tag{5.3}$$

Each year, two things happen in order: last year's balance grows at the MARR, and then the year's cash flow lands. Four readings come from one table:

- while $PB_n < 0$, the project still *owes* you money; the depth of the negative balance is the **risk exposure**;
- the first n with $PB_n \geq 0$ is the **discounted payback**;
- in the positive-PB years, the surplus compounds; this is the profit potential;
- the terminal balance PB_N equals the **net future worth**.

In practice. Read the project balance as a runway chart. Its deepest negative point is the financing that must actually be secured, and that can be more than the first cost when mid-life outflows arrive before the recovery; a positive-PW project can still sink a firm that cannot fund the years before that point.

Insight. You have seen this recursion before. The remaining balance of a loan grows by the interest each period and then drops by the payment. The project balance is the same bookkeeping with the roles reversed: there, you owed the bank; here, the project owes you, and each year's cash flow pays part of that debt back.

Example 5.6 — Project balance of the retrofit

Build the project balance table for the haul-truck retrofit of Example 5.3 (flows −55,000; then 16,000 for four years; 16,000 + 8,000 in year 5) at 12%. Read off the risk exposure, the discounted payback, and the terminal balance.

Solution.

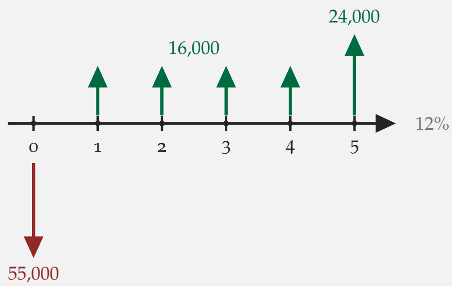

Figure 5.7. The retrofit flows again; the balance table below restates them as a running account.

Start at $PB_0 = -55{,}000$. Each later entry applies Equation (5.3); for example,

$$PB_1 = PB_0\,(1.12) + A_1 = -55{,}000\,(1.12) + 16{,}000 = -45{,}600 \text{ SAR}.$$

n	0	1	2	3	4	5
A_n	−55,000	16,000	16,000	16,000	16,000	24,000
PB_n	−55,000	−45,600	−35,072	−23,280.64	−10,074.32	+12,716.77

Table 5.4. Project balance of the retrofit at 12%. The balance turns nonnegative during year 5; the terminal entry equals the NFW.

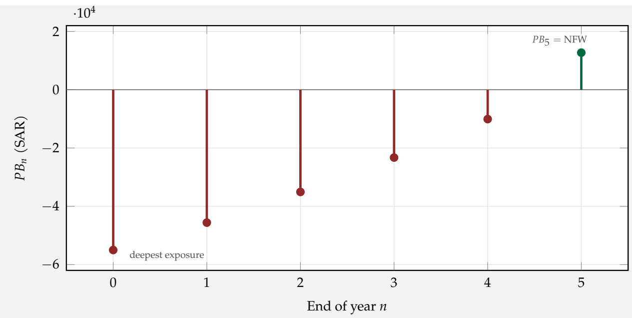

Figure 5.8. The project balance of the retrofit plotted year by year. The negative bars are the capital still at risk; the balance crosses zero during year 5 (the discounted payback) and ends at the net future worth.

Three readings (see Figure 5.8):

- *Risk exposure.* The deepest exposure is the full 55,000 SAR at year 0, and the project is still 10,074 SAR in the red at the end of year 4. If the project were cut short at any of these points, the balance shows the loss.
- *Discounted payback.* The balance first turns nonnegative during year 5.
- *Terminal balance.* $PB_5 = \boxed{+12{,}716.77 \text{ SAR}}$, which matches the net future worth of Example 5.4 up to rounding.

Please practice this recursion. A common exam question gives the table with one row blank and asks you to fill it in.

5.7 *The Capitalized Equivalent*

Capitalized equivalent. The present worth of a service that continues forever: $CE(i) = A/i$ for a perpetual uniform amount A, plus any initial cost.

Some projects have no end date. A haul-road bridge, a dam, a waqf endowment: the service runs indefinitely. That is a problem for the net present worth as used so far, because the sum in Equation (5.1) needs a finite N, and the factor tables stop at $N = 100$. But look at what discounting does to distant flows: they shrink. At 8%, 100 SAR in year 100 is worth less than 0.05 SAR today. So the present worth of perpetual service does not blow up; it converges to a finite number. All we need is that limiting value.

As $N \to \infty$, the series present-worth factor approaches a simple limit:

$$(P/A, i, N) = \frac{(1+i)^N - 1}{i\,(1+i)^N} \longrightarrow \frac{1}{i}, \qquad \text{so} \qquad CE(i) = \frac{A}{i}. \tag{5.4}$$

This quantity is the *capitalized equivalent* (CE). It is the perpetuity of Chapter 2, now used as a valuation tool for perpetual projects.

Insight. A quick check that A/i works. Deposit A/i in an account earning rate i. The interest each year is $i \times (A/i) = A$, which is exactly the required payment. You withdraw only the interest, the principal never shrinks, and the payments go on forever. One caution: A/i funds a fixed *actual* amount each year. If the service must keep today's buying power under inflation, the annual amount would have to grow, and a constant-money analysis would be needed instead.

Periodic renewal. A cost that returns every k years, forever. Convert it to an equal annual cost with $(A/F, i, k)$ before dividing by i.

The only wrinkle in practice is a cost that comes back every k years rather than every year, such as a major overhaul every 20 years. The fix is a two-step pattern. First, convert the every-k-years cost to an equal annual cost over one cycle with the sinking-fund factor $(A/F, i, k)$: a uniform deposit of $F\,(A/F, i, k)$ per year accumulates to exactly F at the end of each k-year cycle. Because every cycle looks the same, this annual cost runs forever. Second, capitalize all the annual costs together with A/i, and add any initial cost.

In practice. The renewal interval k is a corrosion or wear rate expressed in years. A coating that stretches a 20-year refurbishment cycle to 30 years changes $(A/F, i, k)$ and lowers the endowment; the chemistry drives the economics.

Example 5.7 — Endowing a safety-training center

A mining company endows a safety-training center *forever*: the building costs 200,000 SAR now, upkeep is 12,000 SAR per year, and a refurbishment costs 40,000 SAR every 20 years. The account earns 8%. What single amount funds the center forever?

Solution.

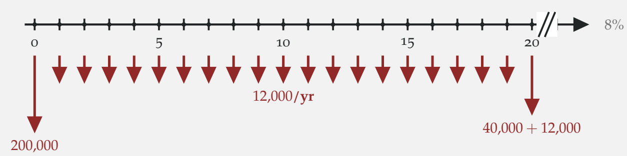

Figure 5.9. The endowment pattern: a build cost at year 0, upkeep every year, and a refurbishment every 20 years. The axis break indicates that the pattern repeats without end.

Step 1: convert the every-20-years cost to an equal annual cost.

Over one 20-year cycle, the equal annual amount that accumulates to

40,000 SAR is a sinking-fund deposit:

$$A_{\text{ref}} = 40{,}000\,(A/F,\, 8\%,\, 20) = 40{,}000\,(0.02185) = 874 \text{ SAR per year.}$$

Every 20-year block looks the same, so this 874 SAR per year runs forever alongside the regular upkeep.

Step 2: capitalize all annual costs and add the initial cost.

$$CE = 200{,}000 + \frac{12{,}000 + 874}{0.08} = 200{,}000 + 160{,}925 = \boxed{360{,}925 \text{ SAR}}.$$

A single deposit of about 360,925 SAR sustains the center forever: the 200,000 SAR builds it, and the remaining 160,925 SAR earns $0.08 \times 160{,}925 = 12{,}874$ SAR of interest each year, which covers the upkeep and the annual set-aside for each refurbishment.

5.8 *Comparing Mutually Exclusive Alternatives*

Until now we judged one project at a time: accept or reject, against the do-nothing baseline. Real decisions are usually choices between options. Crusher 1 or crusher 2; lease or buy; epoxy coating or standard coating.

Before any arithmetic, classify the situation. Three questions set up every comparison.

First, is do-nothing allowed? If yes, it competes too, with PW $= 0$, and every alternative must beat it. A *mandated* project, such as a required safety system or an environmental upgrade, excludes do-nothing; the only question is which option to pick.

Second, what kind of projects are these? Revenue projects differ in income as well as cost: different crushers process different tonnages, so the revenues differ, and we compare full cash flows and maximize the net present worth. *Service projects* deliver the same required output either way: the flotation circuit must run, the tank must be coated. The revenue is the same whichever option is picked, so it drops out of the comparison, and we minimize the present worth of cost.

Please pay attention to the sign here, because this is a common trap. For service projects the *least negative* number wins: a present worth of cost of $-32{,}000$ SAR beats $-44{,}000$ SAR, because it costs less to deliver the same service.

Lecture 14. This section is covered by Lecture 14.

Reference. Park, 3rd ed., Section 5.4 (pp. 213–224).

Mutually exclusive. Alternatives are mutually exclusive when choosing one excludes all the others. The question is not "is each good?" but "which is best?"

Do-nothing alternative. The baseline of investing nothing, with PW $= 0$. It competes unless the project is mandated.

In practice. For a venture, do-nothing is rarely a zero. The do-nothing baseline of PW $= 0$ assumes the capital otherwise earns the MARR; a founder should state what the cash would really do, because idle cash in a deposit account earns far less than the hurdle rate.

Revenue project. Alternatives differ in income as well as cost. Compare full cash flows and maximize NPW.

Service project. The same required output is delivered either way. Revenues are equal and drop out; minimize the PW of cost.

Analysis period. The common time span over which all alternatives are compared. A PW comparison is only fair over a common span of service.

Third, over what period? Present-worth comparisons are only fair over a **common time span**: all alternatives must deliver service over the same period. Suppose one machine lasts 3 years and another lasts 5. Comparing their present worths directly makes the 3-year machine look artificially cheap, because two years of service that the 5-year machine provides have been ignored, and someone has to cover those years. Three cases organize the rest of the chapter: the lives are *equal* (compare directly); the lives are *unequal* and no horizon is stated (replicate to the least common multiple of the lives); or the problem states a *study period* (use it for all alternatives).

5.8.1 *Equal lives: compare directly*

With equal lives, the rule is simply: maximize the present worth.

Example 5.8 — Choosing a crusher

A mineral-rock-aggregate plant must choose one crusher; both options last 5 years, and the MARR is 10%. These are revenue projects, because the two machines process different tonnages.

	Investment (year 0)	Net annual (years 1–5)	Salvage (year 5)
C1 (30 t/hr)	80,000	26,000	10,000
C2 (70 t/hr)	110,000	34,000	15,000

Which crusher should the plant buy?

Solution.

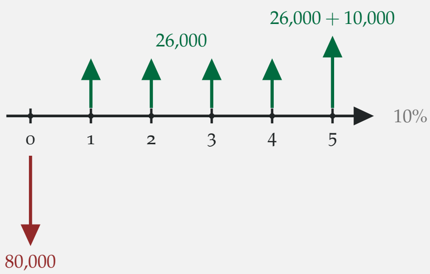

Figure 5.10. Crusher C1: investment, five net annual returns, and the salvage on top of the last return.

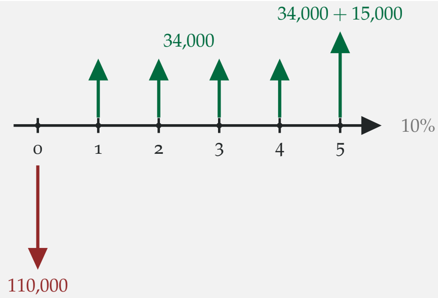

Figure 5.11. Crusher C2: larger investment, larger returns.

Write the general form first, then substitute each crusher's numbers:

$$\text{PW} = -P_0 + A\,(P/A,\,10\%,\,5) + S\,(P/F,\,10\%,\,5),$$

with $(P/A, 10\%, 5) = 3.7908$ and $(P/F, 10\%, 5) = 0.6209$.

$$\begin{aligned}\text{PW}_1 &= -80{,}000 + 26{,}000\,(3.7908) + 10{,}000\,(0.6209)\\ &= -80{,}000 + 98{,}560.80 + 6{,}209.00 = \boxed{+24{,}770 \text{ SAR}},\\ \text{PW}_2 &= -110{,}000 + 34{,}000\,(3.7908) + 15{,}000\,(0.6209)\\ &= -110{,}000 + 128{,}887.20 + 9{,}313.50 = \boxed{+28{,}201 \text{ SAR}}.\end{aligned}$$

Both are positive, so both beat do-nothing; the choice is between two good projects. Choose **C2**, despite the larger price tag: the extra 30,000 SAR of capital earns more than the 10% hurdle. Keep this idea of the extra capital earning its keep; it becomes the incremental method in a later chapter.

LCM replication. When lives differ and no horizon is stated, repeat each alternative's purchase cycle out to the least common multiple of the lives, so all alternatives cover the same span.

5.8.2 *Unequal lives: replicate to the least common multiple*

When the lives differ and no horizon is stated, replicate purchases out to the *least common multiple* (LCM) of the lives. Each repurchase is a single future amount, so it moves back to year 0 with (P/F), the same single-flow move from Chapter 2.

Example 5.9 — Coating a storage tank

A storage tank will be kept for ten years, and the supply manager must choose a coating. This is a service project: the tank gets coated either way. The MARR is 10%.

- **Epoxy coating**: 5,000 SAR, lasts 10 years.
- **Standard coating**: 3,000 SAR, lasts 5 years.

Which coating has the lower present worth of cost?

Solution.

The lives differ (5 and 10 years), so replicate to the LCM, which is 10 years. The standard coating is bought twice, now and again at year 5.

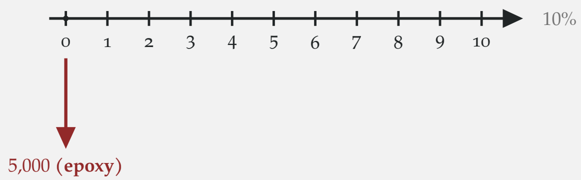

Figure 5.12. The epoxy option: one purchase covers the full ten years.

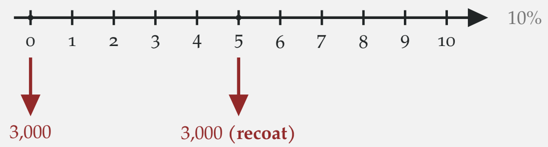

Figure 5.13. The standard option replicated to the 10-year LCM: the recoat at year 5 is a single future outflow.

The recoat is a single future amount, moved back with $(P/F, 10\%, 5) = 0.6209$:

$$\begin{aligned} \text{PW}_{\text{standard}} &= 3{,}000 + 3{,}000\,(P/F,\ 10\%,\ 5) \\ &= 3{,}000 + 3{,}000\,(0.6209) = \boxed{4{,}863 \text{ SAR}}, \\ \text{PW}_{\text{epoxy}} &= \boxed{5{,}000 \text{ SAR}}. \end{aligned}$$

The standard coating wins: it has the lower present worth of cost. Notice how discounting decided this. The second 3,000 SAR is paid five years from now, so it costs only about 1,863 SAR in today's money. Without discounting, two purchases of 3,000 would total 6,000 and epoxy would look better. The time value of money is what rescues the cheaper coating.

Insight. The LCM replication carries one hidden assumption: each future repurchase repeats the first one exactly, with the same price and the same performance. When that assumption is reasonable, the method is sound. Over long spans it is often unrealistic, which is one reason the textbook prefers an explicit study period when one exists. A later chapter introduces the annual-equivalence method, which reaches the same verdict without drawing the replication at all.

Study period. A horizon fixed by the firm or the problem statement. When stated, it is the analysis period for all alternatives, in place of the LCM.

Salvage (market) value. The amount an asset can be sold for at a given date. Under a study period, each asset is credited with its market value at the cutoff date, not at the end of its natural life.

5.8.3 *A stated study period overrides everything*

Sometimes the firm fixes the horizon: ''the mine closes in 4 years,'' ''the offtake contract runs 6 years.'' In that case the *study period*, not the LCM, is the analysis period for all alternatives. Two consequences follow. An asset that is still useful at the cutoff enters the analysis with an estimated salvage value at that date. And an asset whose life is shorter than the study period must be replaced within it, or the gap filled another way, for example by leasing.

Please remember the order of precedence: a stated study period overrides the LCM. Never replicate past a stated horizon.

Example 5.10 — Two forklifts and a closing site

A warehouse *must* run one forklift until the site closes in 4 years (MARR = 10%). ''Must'' excludes do-nothing, and the closure fixes a 4-year study period for both machines, even though their lives are 5 and 7 years. The salvage values are estimated *at the cutoff*:

	First cost	O&M per year	Life	Salvage at year 4
F1	40,000	5,000	5 yrs	12,000
F2	54,000	3,500	7 yrs	22,000

Which forklift should the warehouse buy?

Solution.

This is a service project over a stated 4-year study period, so compare the present worths of cost over exactly 4 years, with each machine's salvage credited at year 4.

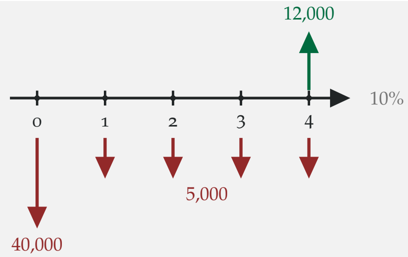

Figure 5.14. Forklift F1 over the 4-year study period: first cost, four years of O&M, and the market value credited at the cutoff.

Write the notation first, with $(P/A, 10\%, 4) = 3.1699$ and $(P/F, 10\%, 4) = 0.6830$; the salvage is subtracted because it is money coming back:

$$
\begin{aligned}
\text{PW}_{\text{cost},1} &= 40{,}000 + 5{,}000\,(P/A,\ 10\%,\ 4) - 12{,}000\,(P/F,\ 10\%,\ 4) \\
&= 40{,}000 + 15{,}849.50 - 8{,}196.00 = \boxed{47{,}653.50 \text{ SAR}}, \\
\text{PW}_{\text{cost},2} &= 54{,}000 + 3{,}500\,(P/A,\ 10\%,\ 4) - 22{,}000\,(P/F,\ 10\%,\ 4) \\
&= 54{,}000 + 11{,}094.65 - 15{,}026.00 = \boxed{50{,}068.65 \text{ SAR}}.
\end{aligned}
$$

Choose **F1**, the lower cost over the four years that matter. Notice what we did not do: we did not replicate to a common multiple of 5 and 7 years, because the stated horizon overrides it.

5.9 *Case Study from the Literature*

Published techno-economic studies apply exactly the criterion of this chapter: estimate the cash flows of a proposed system, discount them at a required rate, and read the sign of the present worth. The case study below reworks the published numbers of one such study with the tools of this chapter. Imam, Al-Turki, and Kumar (2020) evaluated a grid-connected rooftop photovoltaic (PV) system for a typical apartment building in Jeddah, Saudi Arabia, sized at 12.25 kW. Their inputs came from the IRENA renewable-cost database and were validated against quotations from the Saudi market, and their outputs are the same screening numbers this chapter has developed: a net present worth, a discounted payback period, and a per-unit energy cost.

Reference. Imam, Al-Turki, and Kumar (2020), *Sustainability*, 12(1):262. Open access; full reference at the end of the chapter.

Example 5.11 — Case study: rooftop solar in Jeddah

Imam, Al-Turki, and Kumar (2020) report the following inputs for the 12.25 kW rooftop system. All amounts are in US dollars, the currency used in the paper.

- Total capital cost: USD 10,529.46 at year 0.
- Operation and maintenance: 16 USD per kW per year, which is $16 \times 12.25 = 196$ USD per year.
- Inverter replacement: USD 1,346.79 at the end of year 15.
- Energy produced in the first year: 23,589 kWh.
- Residential electricity tariff: 0.048 USD per kWh for monthly consumption up to 6,000 kWh.
- Module life: 25 years.

Value the energy at the lower tariff tier, treat the first-year output as repeating for all 25 years, and screen the project at a MARR of 6%, then at 10%.

Solution.

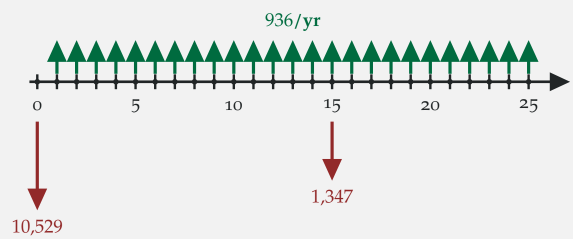

Figure 5.15. The Jeddah rooftop PV system, in USD: capital at year 0, net savings each year, and one inverter replacement at year 15.

Step 1: the net annual cash flow.

The energy is worth $23{,}589 \times 0.048 = 1{,}132.27$ USD per year, and the O&M takes 196 USD of it:

$$A = 1{,}132.27 - 196 = 936.27 \text{ USD per year.}$$

Step 2: present worth at 6%.

Three pieces: the capital, the 25-year savings series, and the single year-15 replacement, with $(P/A, 6\%, 25) = 12.7834$ and $(P/F, 6\%, 15) = 0.4173$:

$$\begin{aligned} \text{PW}(6\%) &= -10{,}529.46 + 936.27\,(12.7834) - 1{,}346.79\,(0.4173) \\ &= -10{,}529.46 + 11{,}968.69 - 562.02 = \boxed{+877 \text{ USD}} \;\Rightarrow\; \text{accept.} \end{aligned}$$

The system clears a 6% hurdle, but not by much: about 877 USD of surplus on a 10,529 USD investment, over 25 years.

Step 3: present worth at 10%.

With $(P/A, 10\%, 25) = 9.0770$ and $(P/F, 10\%, 15) = 0.2394$:

$$\begin{aligned} \text{PW}(10\%) &= -10{,}529.46 + 936.27\,(9.0770) - 1{,}346.79\,(0.2394) \\ &= -10{,}529.46 + 8{,}498.52 - 322.42 = \boxed{-2{,}353 \text{ USD}} \;\Rightarrow\; \text{reject.} \end{aligned}$$

The same cash flows fail a 10% hurdle. The rate at which the present worth crosses zero lies between the two, near 6.9% per year; that crossing rate is the subject of a later chapter. The conventional payback, for reference, is $10{,}529.46/936.27 = 11.2$ years, so this is a slow project by any screening standard.

What did the researchers conclude? Using a full simulation model with the tiered tariff, a 0.5% annual output degradation, and financing at Saudi rates, Imam, Al-Turki, and Kumar (2020) report a net present value of USD 4,378, a discounted payback period of 14.6 years, and a levelized energy cost of 0.0310 USD per kWh against a utility tariff of 0.048 USD per kWh. Their verdict matches the shape of ours: the system is economically viable, but the long payback period weakens its appeal to household investors. They also show that a time-of-use billing scheme would raise the net present value to USD 9,281 and cut the discounted payback to about 10.5 years, because the system's midday output would then be credited at peak rates.

Insight. The case study shows why the MARR must be stated before the verdict. The same Jeddah system is acceptable at 6% and rejected at 10%; neither answer is wrong, because the present worth prices the cash flows at whatever return the capital is required to earn. The researchers' reported NPV of USD 4,378 sits above our 6% figure mainly because their financing rate was lower still, near 2.5%. A published NPV is meaningful only next to its discount rate.

5.10 Chapter Summary

- The **MARR** is the hurdle rate: a project must beat what the firm's capital can earn elsewhere at similar risk. It is a market rate, so actual-money cash flows are discounted at it directly.
- **Payback** measures liquidity and has two flaws: it ignores the time value of money, and it ignores all flows after the payback point. The **discounted payback** repairs only the first flaw. Both are screening tools, not decision criteria.
- The **net present worth** at the MARR is the main criterion: for a single project, accept exactly when PW > 0. A positive PW means the project repays its capital, pays interest at the MARR, and leaves a surplus in today's money. PW $= 0$ means the project earns exactly the MARR.
- The **net future worth** is PW $\times$ $(F/P, i, N)$: same sign, same decision, stated at the end of the project.
- The **project balance** recursion is $PB_n = PB_{n-1}(1 + i) + A_n$ (grow, then add). One table shows the risk exposure, the discounted payback, and the terminal surplus, which equals the NFW.
- The **capitalized equivalent** values perpetual service: $CE = A/i$, plus any initial cost. A cost that returns every k years is first converted to an equal annual cost with $(A/F, i, k)$.
- For **mutually exclusive** alternatives, classify first: is do-nothing allowed? revenue or service project? equal or unequal lives? is a study period stated? With equal lives, maximize PW (for service projects, minimize the PW of cost; the least negative number wins).
- With **unequal lives** and no stated horizon, replicate to the LCM of the lives. A stated **study period** overrides the LCM, with salvage values estimated at the cutoff date. Never replicate past a stated horizon.

- All the arithmetic in this chapter is Chapter 2 equivalence: (P/A) for annual flows, (P/F) for single amounts, salvages, and repurchases. What is new is the decision rule and the framing of the choice.

5.11 *Exercises*

The problems move from screening one project with payback and present worth, through future worth, the project balance, and capitalized equivalent, to the comparison of mutually exclusive alternatives with equal and unequal lives. The longer in-class activities at the end run a single case from cash flow diagram to decision.

Screening a single project

Exercise 5.1. A flotation-cell control upgrade costs 30,000 SAR and saves 9,000 SAR per year for 5 years, with no salvage. The MARR is 10%. Find the conventional payback period and the net present worth, and state the decision.

Exercise 5.2. Repeat the analysis of Exercise 5.1 (invest 30,000 SAR, save 9,000 SAR per year for 5 years) at a MARR of 8%. Which of the two numbers changes, and why?

Exercise 5.3. A project's PW at the MARR is exactly 0. Which statement is correct?

(A) The project loses money and must be rejected.
(B) The project earns exactly the MARR on the invested capital.
(C) The project has no cash flows after payback.
(D) The payback period equals the project life.

Future worth and the project balance

Exercise 5.4. A conveyor upgrade costs 40,000 SAR and returns 15,000 SAR per year for 4 years. The MARR is 10%. Build the project balance table for $n = 0, \ldots, 4$, then read off the discounted payback and the terminal balance, and verify that the terminal balance equals the net future worth.

Exercise 5.5. A project's balance table ends with terminal balance $PB_N = +9{,}500$ SAR at the MARR. Which statement is correct?

(A) The project's NPW is 9,500.

(B) The project's NFW is 9,500, so its NPW is 9,500 $(P/F, i, N)$.

(C) The project never recovers its investment.

(D) The discounted payback is N years.

Capitalized equivalent

Exercise 5.6. What single deposit funds a payment of 25,000 SAR per year, forever, if the account earns 9%? Explain why the fund never runs out.

Mutually exclusive alternatives

Exercise 5.7. A crushing plant *must* install one of two dust-collection systems. Both last 5 years, and the MARR is 10%.

	First cost	O&M per year	Salvage
S1	12,000	3,000	2,000
S2	20,000	1,800	5,000

Classify the situation (do-nothing? revenue or service?), then compute the PW of cost for each system and state the decision.

Exercise 5.8. Two mutually exclusive projects have the cash flows below; the MARR is 10%.

n	A	B
0	−15,000	−13,000
1	6,000	X
2	5,000	5,500
3	7,000	X

What value of X makes the decision maker indifferent between A and B?

Exercise 5.9. Machine A lasts 3 years; machine B lasts 5 years. No study period is stated. Over what horizon should the PW comparison be made?

(A) 3 years — the shorter life.
(B) 5 years — the longer life.
(C) 15 years — the least common multiple.
(D) 8 years — the sum of the lives.

Exercise 5.10. Two machines have lives of 3 years and 6 years. With no stated study period, what is the common analysis horizon, and how many times is each machine purchased over it?

Project screening for a padel club (in-class activity)

A padel club is evaluating three possible investments. For each project, complete the screening ledger at a MARR of 10%, then compute the payback period, the discounted payback period, the net present worth, and the net future worth. The final exercise of the group makes the funding decision under a budget.

Exercise 5.11. Project A — covered courts. Two covered, climate-controlled padel courts let members play through the summer, when uncovered courts sit empty. The cash flows are: year 0, −280,000; then +95,000, +110,000, +120,000, +130,000 SAR at the ends of years 1–4. At a MARR of 10%, find the PP, the DPP, the NPW, and the NFW.

Exercise 5.12. Project B — booking app and smart lighting. A booking app cuts no-shows and phone-desk labor; motion-linked LED lighting lowers the electricity bill. The cash flows are: year 0, −60,000; then +22,000, +26,000, +28,000, +30,000 SAR at the ends of years 1–4. At a MARR of 10%, find the PP, the DPP, the NPW, and the NFW.

Exercise 5.13. Project C — pro shop and cafe corner. Selling rackets, grips, and drinks captures spending that members currently take off-site. The cash flows are: year 0, −120,000; then +30,000, +40,000, +55,000, +65,000 SAR at the ends of years 1–4. At a MARR of 10%, find the PP, the DPP, the NPW, and the NFW.

Exercise 5.14. The club's budget can fund the up-front investment of *at most two* of the three projects above, with a year-0 capital budget of 410,000 SAR. Using the results of Exercise 5.11–Exercise 5.13, which projects should the club fund if it maximizes total NPW, and what does the budget force it to give up?

A ten-year retrofit, start to finish (in-class activity)

Exercise 5.15. A mining contractor evaluates a 10-year automation retrofit for a haul-road conveyor. The kit costs 80,000 SAR today and generates net savings of 15,000 SAR per year in years 1–4. At the end of year 5, a scheduled mid-life overhaul costs 25,000 SAR, so the net flow that year is $15{,}000 - 25{,}000 = -10{,}000$ SAR. The overhaul boosts performance: net savings rise to 20,000 SAR per year in years 6–10, and the kit is resold for 10,000 SAR at the end of year 10. The MARR is 10%. (a) Compute the net present worth, state the decision, and explain what the number means to the contractor. (b) Build the project balance table for $n = 0, \ldots, 10$; state the year of discounted payback; explain why the balance moves backward between years 4 and 5; and verify that the terminal balance is consistent with the NPW. (c) After year 10, the mine owner wants the service kept running forever: upkeep of 6,000 SAR per year plus a 20,000 SAR overhaul every 10 years, at 10%. Find the capitalized equivalent of the perpetual service costs and interpret it.

Two decisions in a copper supply chain (in-class activity)

Najd Copper runs a concentrate supply chain: ore is crushed and milled, floated into copper concentrate, and hauled from the mill to a rail siding. The firm's MARR is 10% throughout the next two exercises.

Exercise 5.16. Debottlenecking the flotation circuit. An 8-year debottleneck costs 100,000 SAR today and raises net revenue by 24,000 SAR per year in years 1–8. A scheduled mid-life reline at the end of year 4 costs 18,000 SAR (net flow that year: $24{,}000 - 18{,}000 = 6{,}000$ SAR). The equipment is resold for 20,000 SAR at the end of year 8. (a) Compute the NPW at the MARR, state the decision, and explain what the number means. (b) A vendor offers an alternative: for the same 100,000 SAR upfront, a *perpetual* service yielding a net benefit of 12,000 SAR every year forever. Find the capitalized equivalent of the perpetual benefit and state which option creates more value today.

Exercise 5.17. Choosing the mill-to-siding haulage system. Concentrate *must* be moved from the mill to the rail siding regardless of the choice. The firm must install one of two systems (MARR = 10%):

Alternative	First cost	Annual O&M	Service life	Salvage
T — Truck fleet	60,000	22,000	5 yrs	8,000
C — Belt conveyor	140,000	9,000	10 yrs	20,000

(a) Classify the decision, draw the cash flow diagram, compute the PW of each alternative (replicating the shorter-lived system as needed), and state the choice. (b) Suppose instead the government requires the mine to close in exactly 6 years. Without recomputing, discuss how the analysis changes: the new horizon; how the first cost, O&M, and salvage of each alternative enter; and why the truck fleet's salvage credit at the new cutoff would be unusually high.

Two decisions in a phosphate supply chain (in-class activity)

Wadi Phosphate runs a rock supply chain: ore is mined, beneficiated into phosphate concentrate, and hauled from the pit to the processing plant. The firm's MARR is 8% throughout the next two exercises.

Exercise 5.18. Upgrading the beneficiation plant. A 6-year upgrade costs 90,000 SAR today and raises net revenue by 28,000 SAR per year in years 1–6. A scheduled mid-life overhaul at the end of year 3 costs 22,000 SAR (net flow that year: $28{,}000 - 22{,}000 = 6{,}000$ SAR). The equipment is resold for 12,000 SAR at the end of year 6. (a) Compute the NPW at the MARR, state the decision, and explain what the number means. (b) A vendor offers an alternative: for the same 90,000 SAR upfront, a perpetual service yielding a net benefit of 9,000 SAR every year forever. Find the capitalized equivalent of the perpetual benefit and state which option creates more value today.

Exercise 5.19. Choosing the pit-to-plant haulage system. Rock *must* be moved from the pit to the plant regardless of the choice. The firm must install exactly one of two systems (MARR = 8%):

Alternative	First cost	Annual O&M	Service life	Salvage
T — Truck haulage	50,000	26,000	4 yrs	6,000
P — Slurry pipeline	120,000	11,000	8 yrs	15,000

(a) Classify the decision, draw the cash flow diagram, compute the PW of each alternative (replicating the shorter-lived system as needed), and state the choice. (b) Suppose instead the pit will be exhausted in exactly 5 years. Without recomputing, discuss how the analysis changes.

Conflicting verdicts: payback vs. NPW (in-class activity)

Exercise 5.20. Two proposals compete for the same budget; the MARR is 10%. **Project A:** invest 40,000 SAR; returns 16,000 SAR per year in years 1–3, nothing after. **Project B:** invest 40,000 SAR; returns 9,000 SAR per year in years 1–8. (a) Compute the conventional payback of each; which does payback prefer? (b) Compute the PW of each at 10%; which does NPW prefer? (c) The verdicts conflict. Which criterion should govern an accept decision, and what information does payback still usefully convey?

Endowment design (in-class activity)

Exercise 5.21. An alumni group wants to endow a lab-maintenance fund: 18,000 SAR per year forever for consumables, plus a 60,000 SAR equipment refresh every 12 years. The account earns 9%. (a) Convert the 12-yearly refresh into its annual equivalent with $(A/F, 9\%, 12)$. (b) Compute the capitalized equivalent and state what happens to the fund balance over time if exactly this amount is deposited. (c) As a separate check of the project-balance recursion: a project has flows $-55{,}000$ at year 0 and $+16{,}000$ in each of years 1–2, at $i = 12\%$; compute PB_1 and PB_2.

The machine showdown: unequal lives (in-class activity)

Exercise 5.22. A packaging line needs one label printer; this is a service project at $i = 10\%$, with no salvage values. **P1:** costs 22,000 SAR, lasts 6 years, running cost 3,000 SAR per year. **P2:** costs 13,000 SAR, lasts 3 years, running cost 4,200 SAR per year. (a) *LCM route:* state the common horizon, draw P2's replicated cash flow, and compute the PW of cost of each printer over that horizon. (b) *Study-period route:* management imposes a 3-year study period; P1's market value at year 3 is estimated at 11,000 SAR. Recompute both PWs of cost over 3 years. (c) Did the verdict change? What economic assumption separates the two routes?

Exam-style problems

The following problems are written in the format of Major Exam 2: a scenario, lettered parts with point values, and full work expected. Classify the situation before computing, write the factor notation before substituting numbers, box each final answer with its units, and end with a decision sentence.

Exercise 5.23. Screening an ore-sorter at Najd Copper. An optical ore-sorter for the crushing circuit costs 150,000 SAR now, returns a net cash inflow of 38,000 SAR at the end of each year for 6 years, and is resold for 15,000 SAR at the end of year 6. The MARR is 10%. Company treasury policy requires every project to return the invested capital, *with interest at the MARR*, within 4 years, in case the pit plan changes. (a) (*8 pts*) Compute the net present worth at the MARR and state accept or reject on profitability alone. (b) (*10 pts*) Compute the conventional payback period and the discounted payback period. (c) (*6 pts*) State which of the two payback measures the treasury policy calls for, and give the final recommendation.

Exercise 5.24. Pit-to-mill haulage at Najd Copper. Ore *must* move from the pit to the mill regardless of the choice. The firm must install one of two systems (MARR = 10%):

Alternative	First cost	Annual O&M	Service life	Salvage
L — Loader fleet	70,000	24,000	3 yrs	7,000
V — Overland conveyor	150,000	8,000	6 yrs	18,000

(a) (*6 pts*) Classify the decision: is do-nothing admissible, is this a revenue or a service project, and what analysis horizon applies? (b) (*12 pts*) Compute the PW of cost of each alternative over that horizon, replicating as needed, and state the choice. (c) (*8 pts*) The mining license now ends in exactly 3 years. The loader fleet reaches the end of its natural life then (salvage 7,000 SAR), and the conveyor's market value at year 3 is estimated at 60,000 SAR. Recompute both PWs of cost over 3 years and state whether the verdict changes.

Exercise 5.25. Endowing a causeway at Wadi Lithium. A haul-road causeway across the sabkha must be kept in service indefinitely. Construction costs 500,000 SAR now, upkeep is 20,000 SAR per year, and a resurfacing costs 90,000 SAR every 15 years. The endowment account earns 8%. (a) (*6 pts*) Convert the 15-yearly resurfacing to an equal annual cost. (b) (*8 pts*) Compute the capitalized equivalent of the whole obligation. (c) (*6 pts*) Verify the answer by showing that the interest earned each year covers the annual costs, and state the deposit the firm should make.

Assessing outside recommendations

The two problems below present finished analyses produced by others. In each, the factor readings and the arithmetic are correct; the flaw is in the method. For each problem, (a) decide whether to accept the recommendation, (b) identify each error or omission precisely, and (c) produce the corrected numbers.

Exercise 5.26. A consultant ranks two blowers by payback. Two optional proposals for flotation-air blowers at Najd Copper's mill compete for the same budget; the MARR is 10% and do-nothing is allowed. **Proposal A:** invest 60,000 SAR; net savings of 24,000 SAR per year in years 1–3, after which the unit is worn out, with no salvage. **Proposal B:** invest 60,000 SAR; net savings of 14,000 SAR per year in years 1–8, no salvage. A consultant submits the memo below.

> *Proposal A recovers its 60,000 SAR in* $60{,}000/24{,}000 = 2.5$ *years; Proposal B needs* $60{,}000/14{,}000 = 4.3$ *years. Our client standard favors projects that recover capital within three years. A meets the standard, B does not. Recommendation: fund A; reject B.*

(a) (*4 pts*) Should the firm accept the recommendation? (b) (*8 pts*) Identify each error or omission precisely. (c) (*8 pts*) Compute the PW of each proposal at the MARR and state the correct funding decision.

Exercise 5.27. An AI compares two dewatering pumps. Wadi Phosphate must keep pit dewatering running; the MARR is 10%. **Pump P:** first cost 40,000 SAR, life 4 years, O&M 9,000 SAR per year, no salvage. **Pump Q:** first cost 64,000 SAR, life 8 years, O&M 5,000 SAR per year, salvage 8,000 SAR at year 8. An AI assistant produced the analysis below.

> $\mathrm{PW}_P = -40{,}000 - 9{,}000\,(P/A, 10\%, 4) = -40{,}000 - 9{,}000\,(3.1699) = -68{,}529$ *SAR.* $\mathrm{PW}_Q = -64{,}000 - 5{,}000\,(P/A, 10\%, 8) + 8{,}000\,(P/F, 10\%, 8) = -64{,}000 - 26{,}674.50 + 3{,}732.00 = -86{,}943$ *SAR. Pump P has the smaller cost,* $-68{,}529$ *against* $-86{,}943$*. Recommendation: install Pump P.*

(a) (*4 pts*) Should the firm accept the recommendation? (b) (*8 pts*) Identify the error precisely. (c) (*8 pts*) Produce the corrected comparison and state the correct choice.

References

Imam, A. A., Al-Turki, Y. A., & Kumar, R. S. (2020). Techno-economic feasibility assessment of grid-connected PV systems for residential buildings in Saudi Arabia—A case study.

Sustainability, 12(1), 262. https://www.mdpi.com/2071-1050/12/1/262
Park, C. S. (2012). *Fundamentals of Engineering Economics* (3rd ed.). Pearson.