4 *Inflation and Purchasing Power*

After this chapter you will be able to:

1. compute a yearly inflation rate from a price index, $f_n = (\mathrm{CPI}_n - \mathrm{CPI}_{n-1})/\mathrm{CPI}_{n-1}$
2. compute the average inflation rate $\bar{f}$ over N years as a geometric average, not an arithmetic one
3. distinguish the general inflation rate from a specific rate for one commodity, such as steel or fuel
4. convert between actual money and constant money with $(1+\bar{f})^{\pm n}$
5. derive the real rate from $1+i = (1+i')(1+\bar{f})$, which gives $i' = (i-\bar{f})/(1+\bar{f})$
6. choose the matching rate in every equivalence calculation: the market rate i for actual money, the real rate i' for constant money

So far, money has had one job: to earn interest over time. That earning power, developed in Chapter 2 and priced precisely in chapter 3, is the first reason money has time value. There is a second force, and this chapter brings it in: prices rise. A given amount of money today does not buy what the same amount bought ten years ago, and that loss of buying power is inflation. The chapter first measures inflation with a price index, computing year-by-year rates and the average rate over several years. It then separates the two ways any future amount of money can be stated, as actual money (the cash that changes hands) or as constant money (the same flow in today's buying power), and connects the three rates that govern them: the market rate i, the inflation-free rate i', and the inflation rate $\bar{f}$. One rule, the pairing rule, then keeps every equivalence calculation clean: actual money is discounted at the market rate, constant money at the real rate, and the two routes always agree. The chapter applies the rule to contract and salary comparisons and to perpetuities that must keep their buying power, and closes with a remark that matters later: depreciation allowances are fixed in actual SAR, so inflation quietly erodes the tax shield of chapter 10. None of this is a side topic. For a project that runs ten or twenty years, ignoring inflation understates future costs, overstates fixed future income, and can change which projects look worthwhile.

Where subtracting inflation goes wrong.

Q1. A price index reads 100 today and 121 two years from now. What is the average inflation rate per year?

Q2. A contract pays 50,000 SAR in *actual* money at the end of year 4. With inflation at 5% per year, what is that payment worth in today's buying power?

Q3. The market rate is 10% and inflation is 6%. Is the real (inflation-free) rate 4%?

Answers.

Q1. $(1+\bar{f})^2 = 121/100$, so $\bar{f} = \sqrt{1.21} - 1 = 10\%$ per year. The average is geometric: the yearly growth factors multiply.

Q2. Deflate it: $50{,}000\,(1.05)^{-4} = 41{,}135$ SAR. The payer still hands over 50,000 SAR, but it buys only what 41,135 SAR buys today.

Q3. *No.* $i' = (i - \bar{f})/(1+\bar{f}) = 0.04/1.06 = 3.77\%$. The naive subtraction ignores that the inflation compensation itself must grow with prices; the division by $1+\bar{f}$ corrects for it.

Lecture 10. This section is covered by Lecture 10.

Reference. Park, 3rd ed., Section 4.1 (pp. 159–166).

Inflation. A loss in the purchasing power of money over time: the same amount of money buys less of an item as years pass.

Deflation. The opposite: prices fall and a fixed amount of money gains purchasing power. Far less common than inflation.

Consumer price index (CPI). An index tracking the cost of a fixed basket of goods and services to the average consumer, relative to a base period set at 100.

4.1 *Measuring Inflation: Price Indexes*

Before inflation can enter an equivalence calculation, it must be measured. A *price index* tracks the cost of a fixed basket of goods over time; the consumer price index (CPI) is the best-known example. Two readings matter. First, the index takes the *buyer's* perspective: it records what the basket costs a consumer, not the price set by a seller. Second, the index is a *level*, while the inflation rate is its *change*. One index value tells you how expensive the basket is at one date; you need two index values to get one inflation rate. The rate in year n is the relative change from the year before:

$$f_n = \frac{\mathrm{CPI}_n - \mathrm{CPI}_{n-1}}{\mathrm{CPI}_{n-1}}. \tag{4.1}$$

As a quick illustration, suppose an index reads 100, then 106, then 116.6, then 114.3 over three years. The yearly rates are $f_1 = (106 - 100)/100 = +6\%$, $f_2 = (116.6 - 106)/106 = +10\%$, and $f_3 = (114.3 - 116.6)/116.6 = -2\%$. The third year is a year of deflation: the level fell, so the rate is negative.

> **Insight.** Keep the level and the change separate. An index of 114.3 does *not* mean "inflation is 14.3%"; it means the basket costs 14.3% more than it did in the base year, which may be many years back. A common exam trap asks for this year's inflation rate and offers the index level as a distractor.

4.2 *The Average Inflation Rate*

To summarize several years in one number, we need the single rate that, applied every year, gives the same total growth. Because each year's inflation compounds on the last, the yearly *factors multiply*:

$$(1+\bar{f})^N \;=\; (1+f_1)(1+f_2)\cdots(1+f_N) \;=\; \frac{\text{CPI}_N}{\text{CPI}_0}. \tag{4.2}$$

This is a geometric average, not an arithmetic one, and the two differ.

One distinction matters for engineers. The *general* inflation rate is computed from the CPI, and the market interest rate responds to it. A *specific* inflation rate tracks a single item, a reagent, steel, diesel, and can run far from the CPI in either direction. Park (2012) tabulates a year in which gasoline prices rose 24% while the CPI rose 2.5%; an estimate escalated with the wrong index misses by an order of magnitude in such a year.

Average inflation rate. The single rate $\bar{f}$ that, applied for N years, produces the same total price growth as the actual yearly rates: a geometric average.

General vs. specific inflation. The general rate $\bar{f}$ is computed from the CPI for the whole basket; a specific rate f_j tracks one item or industry, such as steel or fuel, and can differ sharply from the CPI.

In practice. Cost estimators escalate equipment and construction costs with specific indexes (plant-cost and commodity indexes), not with the CPI. Choosing the index is an engineering judgment about what the estimate is made of: a steel-intensive structure follows the steel index, a labor-intensive installation follows wages, and only the project's general revenue follows anything like the CPI.

Example 4.1 — Reagent prices: yearly rates and the average

A flotation reagent cost 1,000 SAR per tonne three years ago. The price then moved to 1,060, then 1,166, then 1,142.68 SAR per tonne. Find the inflation rate of the reagent in each year and the average annual rate over the three years.

Solution.

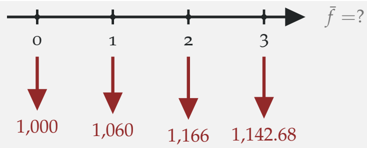

Figure 4.1. The price paid per tonne of reagent in each of the four years. The average inflation rate is the single growth rate that connects the first arrow to the last.

***Step 1: year-by-year rates from Equation* (4.1).**

$$f_1 = \frac{1{,}060 - 1{,}000}{1{,}000} = +6\%,$$
$$f_2 = \frac{1{,}166 - 1{,}060}{1{,}060} = +10\%,$$
$$f_3 = \frac{1{,}142.68 - 1{,}166}{1{,}166} = -2\%.$$

Step 2: the average is geometric.

The three growth factors multiply:

$$(1+\bar{f}\,)^3 = (1.06)(1.10)(0.98) = 1.14268$$
$$\Rightarrow \quad \bar{f} = (1.14268)^{1/3} - 1 = \boxed{4.55\% \text{ per year}}.$$

The arithmetic mean $(6 + 10 - 2)/3 = 4.67\%$ is *not* the answer: rates compound through time, so their factors multiply, and the correct average is the geometric one. Please also check the direct route: the price ratio $1{,}142.68/1{,}000 = 1.14268$ gives the same $\bar{f}$ without computing the yearly rates at all.

Reference. Park, 3rd ed., Section 4.2 (pp. 166–172).

Actual (current) money. A_n: the cash that actually changes hands in year n, the number written on the contract or the cheque.

Constant (real) money. A'_n: the same flow expressed in the buying power of a reference year, in this book year 0, today.

4.3 *Actual and Constant Money*

Any future amount of money can be stated two ways. *Actual* money is the cash that changes hands in year n. *Constant* money expresses that same flow in today's buying power. Converting between them is compounding at the inflation rate:

$$A'_n = A_n\,(1+\bar{f}\,)^{-n} \qquad \text{(deflate)}, \qquad A_n = A'_n\,(1+\bar{f}\,)^{n} \qquad \text{(inflate)}. \quad (4.3)$$

Constant money is always measured against a reference year; in this book the reference is today, year 0. If a problem chose a different base year, the conversions would work the same way against that base.

Example 4.2 — An offtake payment in two currencies of time

An offtake contract pays 60,000 SAR in *actual* money at the end of year 3. Average inflation is 4% per year. (a) State the payment in today's buying power. (b) Check the conversion by inflating your answer back.

Solution.

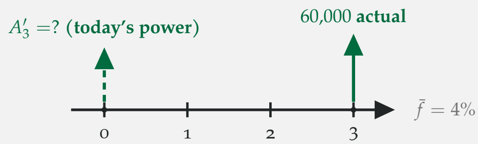

Figure 4.2. The year-3 cheque restated in year-0 buying power. The move uses the inflation rate, not an interest rate: no money moves through time, only its unit changes.

(*a*) *Deflate.*

$$A_3' = 60{,}000\,(1.04)^{-3} = \boxed{53{,}340 \text{ SAR}} \quad \text{in today's buying power.}$$

The buyer still hands over the full 60,000 SAR in year 3, but it buys what 53,340 SAR buys today.

(*b*) *Inflate back.*

$$A_3 = 53{,}340\,(1.04)^3 = \boxed{60{,}000 \text{ SAR}}.$$

Deflating and inflating are exact opposites; applying both returns the original figure. Please note what this example did *not* do: it did not discount anything. Stating a flow in constant money changes its unit, not its date.

Example 4.3 — What idle cash loses, and what invested cash keeps

A plant keeps a 60,000 SAR emergency reserve. Inflation averages 4% per year. (a) If the reserve sits idle for 5 years, what is it worth in today's buying power? (b) If instead it is deposited at a market rate of 6%, what is the balance after 5 years, and what is that balance worth in today's buying power?

Solution.

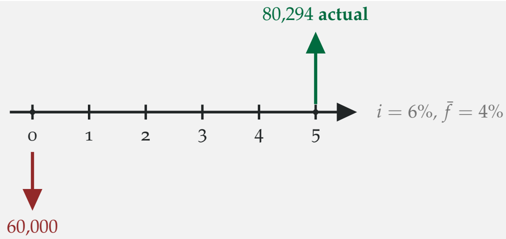

Figure 4.3. The invested reserve: the actual balance grows at the market rate, but its buying power grows only at the real rate.

(*a*) *Idle cash.* The drawer still holds 60,000 SAR, but its buying power falls at the inflation rate:

$$60{,}000\,(1.04)^{-5} = \boxed{49{,}316 \text{ SAR}} \quad \text{of today's power.}$$

Doing nothing quietly costs about 10,684 SAR of buying power over the five years.

(*b*) *Invested cash.* The actual balance compounds at the market rate, and the buying power is that balance deflated:

$$F = 60{,}000\,(1.06)^5 = 80{,}294 \text{ SAR}, \qquad F' = 80{,}294\,(1.04)^{-5} = \boxed{65{,}995 \text{ SAR}}.$$

The balance *looks* like it grew 6% per year, but its buying power grew from 60,000 to 65,995, about 1.9% per year. That smaller rate is the real rate, defined next. Idle cash is the extreme case with a market rate of zero, so its real rate is negative.

In practice. A venture's runway is quoted in months of cash, but the cash is actual money while the burn is a basket of salaries, rent, and cloud services whose prices rise. At 6% inflation, an 18-month runway funds about 17.3 months of today's spending; a founder who plans to the actual number, not the constant one, runs out early.

Insight. Money has time value for *two* separate reasons: earning power, which interest captures, and buying power, which inflation erodes. Chapter 2 priced the first force. This chapter separates the two, and the market rate of the next section is where they are combined into one number.

Reference. Park, 3rd ed., Section 4.3.1 (pp. 172–173).

Market interest rate. i: the rate banks quote and the rate used throughout Chapters 2 and 3. It carries both earning power and inflation compensation.

Inflation-free (real) rate. i': the growth rate of true buying power, with inflation stripped out.

4.4 *The Market Rate, the Real Rate, and the Inflation Identity*

Three rates govern this chapter, bound by one identity:

$$1 + i \;=\; (1 + i')(1 + \bar{f}\,), \qquad \text{so} \qquad i' \;=\; \frac{i - \bar{f}}{1 + \bar{f}}. \tag{4.4}$$

Read the identity in words. One SAR grows by the market rate in actual terms. Part of that growth only keeps pace with rising prices; the factor $(1 + \bar{f}\,)$ accounts for it. What remains, the factor $(1 + i')$, is the true gain in what the money can buy. The market rate is therefore not pure earning: some of it is compensation for inflation, and only the leftover, the real rate, increases buying power.

Two quick computations show why the naive subtraction $i - \bar{f}$ is not good enough. With $i = 9\%$ and $\bar{f} = 4\%$,

$$i' = \frac{0.09 - 0.04}{1.04} = 4.81\%, \quad \text{not } 5\%;$$

with $i = 12\%$ and $\bar{f} = 7\%$,

$$i' = \frac{0.12 - 0.07}{1.07} = 4.67\%, \quad \text{not } 5\%.$$

The gap looks small in one year, but discounted over twenty or thirty years it moves a present worth noticeably. Note also that $i > i'$ whenever $\bar{f} > 0$; only deflation can push the real rate to or above the market rate.

Insight. Equation (4.4) explains a remark made in chapter 5: the MARR is a *market* rate. Because project cash flows are usually estimated in actual money, discounting them at the market MARR handles inflation automatically; both sides of the equivalence carry inflation once. The pairing rule below is the general statement of that bookkeeping.

4.5 *The Pairing Rule*

Pairing rule. Discount actual money at the market rate i; discount constant money at the real rate i'. Never mix the pairs.

One rule keeps every inflation problem clean:

Cash flows stated in ...	Discount at ...	Why
actual money	market rate i	i already carries inflation
constant money	real rate i'	inflation already stripped out

Both routes give the same present worth, and Equation (4.4) guarantees the agreement. A one-line check: a payment of 104 SAR in actual money at year 1, with $i = 9\%$ and $\bar{f} = 4\%$ (so $i' = 4.81\%$). Route one, actual at market: $104/1.09 = 95.41$. Route two, convert to constant ($104/1.04 = 100$) then discount at the real rate: $100/1.0481 = 95.41$. Identical.

Now break the pairing and watch the damage. Constant money at the market rate gives $100/1.09 = 91.74$, too low, because inflation was removed twice. Actual money at the real rate gives $104/1.0481 = 99.23$, too high, because inflation was never removed at all. The two wrong routes land on opposite sides of the right answer.

Insight. The procedure for any inflation problem: (1) label each cash flow as actual or constant; (2) pick one route, all-actual or all-constant; (3) convert any mismatched flow with Equation (4.3); (4) discount with the matching rate. Almost every error in this chapter is a skipped step (1).

Example 4.4 — Two offtake contracts, one pairing rule

A plant compares two 3-year supply contracts. Contract A pays 100,000 SAR per year in *constant* money (each payment keeps today's buying power). Contract B pays 105,000 SAR per year in *actual* money. The market rate is $i = 8\%$ and inflation is $\bar{f} = 5\%$. Which contract is worth more today to the supplier receiving the payments?

Solution.

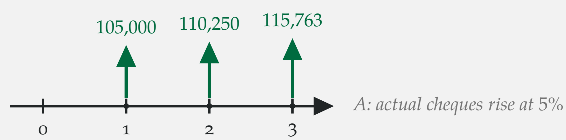

Figure 4.4. Contract A in actual money: constant buying power means the cheques climb with inflation.

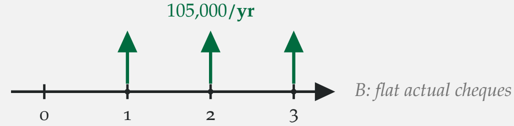

Figure 4.5. Contract B in actual money: the cheques stay flat.

Step 1: the real rate.

$$i' = \frac{0.08 - 0.05}{1.05} = 2.857\%.$$

Step 2: pair each contract with its rate.

Contract A is stated in constant money, so it is discounted at the real rate; Contract B is actual money at the market rate.

N	$(P/A, 8\%, N)$
2	1.7833
3	**2.5771**
4	3.3121

Table 4.1. Excerpt from the $i = 8\%$ factor table used for Contract B in Example 4.4. Contract A's rate, 2.857%, is not a table rate, so its factor is computed directly.

$$P_A = 100{,}000\,(P/A,\,2.857\%,\,3) = 100{,}000\,(2.8364) = \boxed{283{,}640\text{ SAR}},$$
$$P_B = 105{,}000\,(P/A,\,8\%,\,3) = 105{,}000\,(2.5771) = \boxed{270{,}596\text{ SAR}}.$$

Step 3: explain the gap.

Contract A wins by about 13,000 SAR even though its stated number looks smaller. The reason sits in the diagrams: A's actual cheques start at 105,000 SAR and climb to 115,763 SAR, while B's stay flat at 105,000 SAR. The two contracts are equal in year 1 and diverge after. Decision sentence: take Contract A; its inflation-protected payments are worth about 13,000 SAR more today.

Figure 4.6 puts both contracts in both currencies on one axis; look at year 1, where all four numbers meet, and then at how far apart the pairs have travelled by year 3.

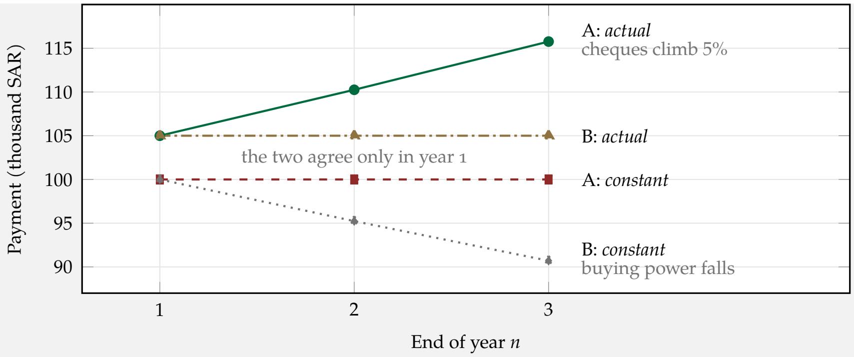

Figure 4.6. The two offtake contracts, each drawn in both currencies of time. Contract A is flat in constant riyals and climbing in actual riyals; Contract B is the mirror image, flat in actual riyals and falling in constant ones. The two agree in year 1 and part company after, which is the whole 13,000 SAR gap in present worth.

Reference. Park, 3rd ed., Sections 4.3.2–4.3.4 (pp. 173–181).

4.6 *Equivalence Calculations under Inflation*

Constant-value perpetuity. A payment of fixed *buying power* every year forever. Its actual cheques grow with inflation, and its present worth is $P = A'/i'$.

With the pairing rule, every equivalence tool of Chapters 2 and 3 carries over to an inflationary world. Single amounts, series, and gradients all work at either rate, provided the flows and the rate are matched. One tool deserves its own treatment because endowments and long-term funds depend on it.

A *fixed* perpetuity pays the same actual amount forever, and Chapter 2 priced it at $P = A/i$ with the market rate. A *constant-value* perpetuity pays the same *buying power* forever, so its actual cheques rise with inflation. By the pairing rule it is valued at the real rate:

$$P = \frac{A'}{i'}. \tag{4.5}$$

Because $i' < i$ whenever inflation is positive, the constant-value promise always needs the larger fund.

Example 4.5 — Endowing mineral-research training forever

A national fund wants to pay 3,000,000 SAR per year *of today's buying power*, forever, to train mineral-research engineers. The market rate is 8% and inflation averages 3%. What deposit is required today, and what mistake would the naive A/i route make?

Solution.

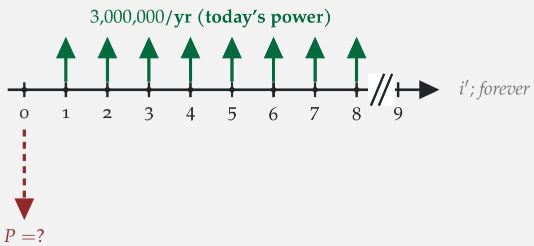

Figure 4.7. A perpetual withdrawal of constant buying power. The actual cheques grow at 3% per year without end, which is why the real rate prices them.

Step 1: the real rate.

$$i' = \frac{0.08 - 0.03}{1.03} = 4.854\%.$$

Step 2: the constant-value perpetuity.

$$P = \frac{A'}{i'} = \frac{3{,}000{,}000}{0.04854} = \boxed{61.8 \text{ million SAR}}.$$

Step 3: the classic mistake.

Dividing by the market rate gives $3{,}000{,}000/0.08 = 37.5$ million SAR, which looks like a saving of 24 million. It is not. That smaller fund pays a *fixed* 3 million actual SAR forever, and at 3% inflation the real value of the payment falls every year; after 24 years it buys only about half of what it promised. The larger deposit is exactly what lets the fund pay out *and* grow enough each year to cover rising prices. Decision sentence: deposit 61.8 million SAR; the promise is stated in buying power, so the real rate must price it.

In practice. Long-lived assets raise the same issue in reverse. A sinking fund that must *replace a machine* in 15 years should target the machine's future *actual* price, today's price inflated at the equipment-specific escalation rate, not today's price itself. Funding the constant amount and ignoring escalation leaves the fund short by exactly the inflation factor.

4.7 *Inflation, Depreciation, and the Chapters Ahead*

Two remarks connect this chapter to the rest of the book.

First, the default in the project chapters is *actual-money analysis at the market rate*. The MARR of chapter 5 is a market rate, and project cash flows are ordinarily estimated in actual money, so the pairing is satisfied without further work. Whenever a problem hands you flows "in today's money," that is the signal to switch rails: either inflate the flows or use the real rate.

Second, one important cash flow refuses to grow with prices. Depreciation allowances, introduced in chapter 10, are fixed in *actual* SAR by the tax rules at the moment the asset is bought. The tax saving they generate, the depreciation tax shield, is therefore a flat actual-money series, and inflation erodes its buying power year after year, exactly like Contract B in Example 4.4.

Example 4.6 — Inflation shrinks a depreciation tax shield

A machine will generate a depreciation allowance of 100,000 SAR per year for 5 years, saving $0.25 \times 100{,}000 = 25{,}000$ SAR of tax each year, fixed in actual money. The market rate is 10% and inflation is 4%. (a) Find the present worth of the shield. (b) Find what the shield would be worth if the tax rules indexed it to inflation, and read the difference as the cost of inflation to the firm.

Solution.

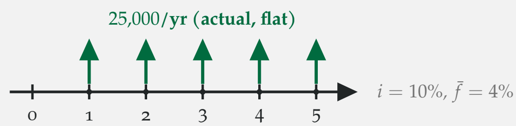

Figure 4.8. The tax shield: a flat actual-money series. An indexed shield would climb with inflation like Contract A of Example 4.4.

(*a*) *The shield as written.* Flat actual flows pair with the market rate:

$$\mathrm{PW} = 25{,}000\,(P/A,\,10\%,\,5) = 25{,}000\,(3.7908) = \boxed{94{,}770\text{ SAR}}.$$

(*b*) *An indexed shield.* If the 25,000 SAR kept today's buying power, it would be a constant-money series, priced at the real rate $i' = (0.10 - 0.04)/1.04 = 5.769\%$:

$$\mathrm{PW} = 25{,}000\,(P/A,\,5.769\%,\,5) = 25{,}000\,(4.2389) = \boxed{105{,}973\text{ SAR}}.$$

Inflation costs the firm the difference, about $105{,}973 - 94{,}770 = 11{,}203$ SAR of present worth, roughly 12% of the shield. The year-5 allowance alone is worth only $25{,}000\,(1.04)^{-5} = 20{,}548$ SAR of today's power. Please carry this into chapter 10: the higher inflation runs, the more valuable *early* depreciation becomes, because it cashes the fixed allowance before prices erode it.

4.8 Case Study from the Literature

Every number in this chapter so far came from a made-up index. The tools, however, run directly on the official one. The General Authority for Statistics (GASTAT) publishes the Saudi consumer price index monthly and annually, with the base period 2018 set at 100, together with section indexes for housing, food, transport, and other groups. Its report *Annual Consumer Price Index 2024* states that the general index averaged 111.3 in 2024 against 109.5 in 2023, an average annual inflation rate of 1.7%, and that the sections moved very differently: housing, water, electricity, gas, and other fuels rose 8.8% (driven by a 10.6% rise in housing rents), food and beverages rose 0.8%, while clothing and footwear fell 3.4% and transport fell 2.4%. All figures below are read from that report; the index is unitless, and the contract amounts we deflate with it are in SAR.

Reference. GASTAT (2025), *Annual Consumer Price Index 2024*. Open data; full reference at the end of the chapter.

Example 4.7 — Case study: reading the Saudi CPI like an engineer

Using the GASTAT figures above (general index: 100 in 2018, 109.5 in 2023, 111.3 in 2024; food and beverages index 124.4 in 2024), (a) verify the reported 2024 general inflation rate from the two index levels; (b) find the average general inflation rate over 2018–2024; (c) find the average specific inflation rate of food over the same period and compare; (d) a site-services contract signed in 2018 still pays a flat 200,000 SAR per year in actual money; state the 2024 payment in 2018 buying power.

Solution.

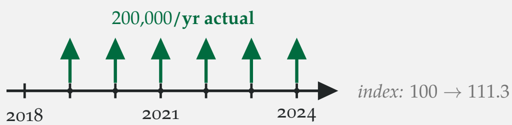

Figure 4.9. The flat 200,000 SAR contract signed in 2018. Each cheque is the same actual amount; the index measures how much buying power each one has lost.

Step (a): one year, two index levels.

By Equation (4.1),

$$f_{2024} = \frac{111.3 - 109.5}{109.5} = \boxed{1.6\%},$$

which matches GASTAT's reported 1.7% up to the rounding of the published index levels (GASTAT computes from unrounded indexes).

Step (b): the six-year average.

The base 2018 = 100, so by Equation (4.2),

$$\bar{f} = \left(\frac{111.3}{100}\right)^{1/6} - 1 = \boxed{1.80\% \text{ per year}}.$$

Step (c): a specific rate.

Food and beverages reached 124.4 on the same base:

$$\bar{f}_{\text{food}} = \left(\frac{124.4}{100}\right)^{1/6} - 1 = \boxed{3.71\% \text{ per year}},$$

about twice the general rate. Within one economy and one period, the specific index a cost estimate needs can sit far from the CPI, in either direction: over 2023–2024 alone, housing rose 8.8% while transport *fell* 2.4%. An estimator escalating a transport-heavy cost with the general 1.7% would overstate it; one escalating rents with it would badly understate them.

Step (d): deflate the contract.

The 2024 payment of 200,000 SAR actual, stated in 2018 buying power, is

$$A' = 200{,}000 \times \frac{100}{111.3} = \boxed{179{,}695 \text{ SAR}}.$$

The contractor still receives 200,000 SAR, but it buys what 179,695 SAR bought when the contract was signed, a real pay cut of about 10% with no renegotiation and no breach. Decision sentence: index long service contracts to the CPI, or reprice them on a fixed schedule; a flat actual price is a slow concession to the counterparty.

Insight. The case study shows the measurement chain end to end: a published index (a level), converted to rates (changes), averaged geometrically, split into general and specific components, and finally used to deflate a real contract. Nothing beyond Equations (4.1)–(4.3) was needed; the only judgment call was *which* index matched the cash flow, and that call belongs to the engineer who knows what the money is spent on.

4.9 Chapter Summary

- A **price index** is a level; the **inflation rate** is its year-to-year change: $f_n = (\mathrm{CPI}_n - \mathrm{CPI}_{n-1})/\mathrm{CPI}_{n-1}$. The index takes the buyer's perspective.

- The **average inflation rate** is geometric: $(1 + \bar{f}\,)^N = (1 + f_1) \cdots (1 + f_N) = \mathrm{CPI}_N/\mathrm{CPI}_0$. Never average inflation rates arithmetically.

- Distinguish the **general** rate (CPI, the whole basket) from **specific** rates (steel, fuel, rents), which can differ sharply; escalate each estimate with the index that matches it.

- **Actual money** is the cash that changes hands; **constant money** is the same flow in today's buying power. Convert with $(1 + \bar{f}\,)^{\pm n}$. Idle cash loses buying power at the rate $\bar{f}$.

- The three rates obey $1 + i = (1 + i')(1 + \bar{f}\,)$, so $i' = (i - \bar{f})/(1 + \bar{f})$, not $i - \bar{f}$.

- The **pairing rule**: discount actual money at the market rate i and constant money at the real rate i'. Both routes agree; mixing them double-counts or ignores inflation.

- A **constant-value perpetuity** pays fixed buying power forever: $P = A'/i'$. Dividing by the market rate funds a promise that decays at the inflation rate.

- **Depreciation** allowances are fixed in actual SAR, so inflation erodes the tax shield of chapter 10; early depreciation is worth more when inflation is high.

4.10 Exercises

The problems start with measuring inflation from published indexes and converting between actual and constant money, then move to equivalence calculations where each flow must be paired with the matching market or real rate.

Measuring inflation

Exercise 4.1. A published price index for a refinery's inputs reads 150.0 now and 178.65 three years from now. Find the average annual inflation rate of the inputs over the three years.

Exercise 4.2. A manufacturer's lithium purchase price changed by $+6\%$, then $+9\%$, then -3% over three years. (a) Find the average annual inflation rate of the lithium price. (b) Separately, with a market rate of $i = 10\%$ and general inflation of $\bar{f} = 4\%$, find the inflation-free rate i'.

Equivalence under inflation

Exercise 4.3. Two 4-year supply contracts are offered to a manufacturer, who will make the payments. Contract A pays 50,000 SAR per year in *constant* money (each payment keeps today's buying power). Contract B pays 53,000 SAR per year in *actual* money. With $i = 10\%$ and $\bar{f} = 4\%$, find the present worth of each contract's payments and state which is cheaper for the manufacturer today.

Exercise 4.4. A company wants to endow a training fund paying 80,000 SAR per year *of today's purchasing power*, forever. With $i = 10\%$ and $\bar{f} = 4\%$, how much must be deposited now?

Exercise 4.5. A payment of 90,000 SAR will be made in *actual* money at the end of year 5. With average inflation of 4% per year, which expression gives its value in *today's buying power*?

(A) $90{,}000\,(F/P, 4\%, 5)$

(B) $90{,}000\,(P/F, 4\%, 4)$

(C) $90{,}000$

(D) $90{,}000\,(P/F, 4\%, 5)$

Exam-style problems

The following problems are written in the format of the Major Exam: a scenario, lettered parts with point values, and full work expected. Label each cash flow as actual or constant before computing, pair it with the matching rate, box each final answer with its units, and end with a decision sentence.

Exercise 4.6. Two salary packages at a processing plant. An engineer must choose between two 4-year compensation packages. **Package X** pays a flat 240,000 SAR per year in *actual* money. **Package Y** pays 215,000 SAR per year in *constant* money: the cheque is indexed and rises with inflation so that each year's payment keeps today's buying power. The market rate is $i = 9\%$ and inflation averages $\bar{f} = 5\%$. (a) (*4 pts*) Find the inflation-free rate i'. (b) (*10 pts*) Find the present worth of each package using the pairing rule. (c) (*6 pts*) Tabulate Package Y's actual cheques for years 1–4 and state in which year they overtake Package X's. (d) (*4 pts*) State the decision and the single mistake that would have reversed it.

Exercise 4.7. A mining input index, start to finish. The price index of a mine's consumables reads 100, 103, 109.18, and 112.46 over three consecutive years. (a) (*6 pts*) Find the inflation rate of the consumables in each of the three years. (b) (*6 pts*) Find the average annual inflation rate over the three years. (c) (*6 pts*) A spare-parts contract pays 120,000 SAR in actual money at the end of year 8. Using the average rate from (b), rounded to 4%, state that payment in today's buying power. (d) (*6 pts*) The mine wants to endow a safety-training program paying 50,000 SAR per year of today's buying power, forever, with a market rate of 9% and general inflation of 4%. Find the required deposit.

Assessing outside recommendations

The two problems below present finished analyses produced by others. In each, the arithmetic shown is correct; the flaw is in the method. For each problem, (a) decide whether to accept the recommendation, (b) identify each error or omission precisely, and (c) produce the corrected numbers.

Exercise 4.8. A consultant rejects a rehabilitation project. A tailings-line rehabilitation costs 180,000 SAR now and saves "40,000 SAR per year, in today's money" for 6 years, an engineering estimate made in constant money. The firm's market MARR is 10% and inflation averages 4%. A consultant reports:

> $\text{PW} = -180{,}000 + 40{,}000\,(P/A, 10\%, 6) = -180{,}000 + 40{,}000\,(4.3553) = -5{,}788$ *SAR. The present worth is negative. Recommendation: reject the rehabilitation.*

(a) (*4 pts*) Should the firm accept the recommendation? (b) (*8 pts*) Identify the error precisely. (c) (*8 pts*) Produce the corrected present worth and decision.

Exercise 4.9. A bank memo on "zero real return." A firm parks cash in a deposit account. Its bank writes:

> *Our account pays 6% compounded monthly, and inflation is running at 6% per year, so your real return is exactly zero. For a benchmark, remember the simple rule: real rate = market rate − inflation, so at 9% market and 4% inflation you would earn a real 5%.*

(a) (*4 pts*) Should the firm accept the memo's two claims? (b) (*8 pts*) Identify each error precisely. (c) (*8 pts*) Produce the corrected numbers.

References

General Authority for Statistics (GASTAT) (2025). *Annual Consumer Price Index 2024* (base period 2018 = 100). Riyadh, Saudi Arabia. `https://www.stats.gov.sa/en/w/cpi-1`

Park, C. S. (2012). *Fundamentals of Engineering Economics* (3rd ed.). Pearson.