A Formula Sheet

This sheet collects the formulas used throughout the book. Each factor is written in the ''find this, given that'' notation; look its value up in Appendix B. Chapter references use the part structure of the book.

Single-payment factors

$$F = P(1+i)^N = P\,(F/P, i, N) \qquad \text{compound amount}$$
$$P = F(1+i)^{-N} = F\,(P/F, i, N) \qquad \text{present worth}$$

The two are reciprocals: $(F/P, i, N)\,(P/F, i, N) = 1$.

Solving for the rate or the number of periods

$$N = \frac{\ln(F/P)}{\ln(1+i)}, \qquad i = \left(\frac{F}{P}\right)^{1/N} - 1, \qquad N_{\text{double}} \approx \frac{72}{100\,i} \text{ (Rule of 72).}$$

Uniform-series factors

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A\,(F/A,i,N) \qquad \text{series compound amount}$$

$$A = F\left[\frac{i}{(1+i)^N - 1}\right] = F\,(A/F,i,N) \qquad \text{sinking fund}$$

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = P\,(A/P,i,N) \qquad \text{capital recovery}$$

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A\,(P/A,i,N) \qquad \text{series present worth}$$

Reciprocal pairs on the same row: $(F/A)(A/F) = 1$ and $(P/A)(A/P) = 1$. The capital-recovery and sinking-fund factors are linked by $(A/P,i,N) = (A/F,i,N) + i$.

Perpetuity

$$P = \frac{A}{i} \qquad \text{(uniform payment } A \text{ forever).}$$

Deferred annuity

If the first of N equal payments arrives at period $d+1$,

$$P_0 = A\,(P/A,i,N)\,(P/F,i,d).$$

Gradient factors

$$P = G\left[\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right] = G\,(P/G,i,N) \qquad \text{linear gradient (first flow} = 0)$$

$$P = A_1\left[\frac{1-(1+g)^N(1+i)^{-N}}{i-g}\right] \qquad \text{geometric gradient, } i \neq g$$

$$P = \frac{A_1 N}{1+i} \qquad \text{geometric gradient, } i = g$$

Simple interest (for contrast)

$$F = P(1 + iN).$$

Money management (chapter 3)

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1, \qquad i_a = e^r - 1 \text{ (continuous)}$$

$$i = \left(1 + \frac{r}{M}\right)^C - 1, \quad C = M/K \qquad \text{effective rate per payment period}$$

$$B_t = A\,(P/A, i, N - t) \qquad \text{loan balance after payment } t$$

$$I_t = i\,B_{t-1}, \qquad PP_t = A - I_t \qquad \text{interest/principal split}$$

Add-on loan: $A = P(1 + rN_{\text{yrs}})/N$; the true rate solves $(P/A, i, N) = P/A$.

Inflation (chapter 4)

$$(1 + \bar{f})^N = \frac{\text{CPI}_N}{\text{CPI}_0}, \qquad A'_n = A_n\,(1 + \bar{f})^{-n}, \qquad 1 + i = (1 + i')(1 + \bar{f}).$$

Pairing rule: actual money ↔ market rate i; constant money ↔ inflation-free rate i'. Constant-value perpetuity: $P = A'/i'$.

Benefit–cost analysis (chapter 8)

$$BC(i) = \frac{B}{I + C'} \geq 1, \qquad PI(i) = \frac{B - C'}{I} \geq 1,$$
$$\frac{\Delta B}{\Delta I + \Delta C'} > 1 \text{ (incremental, ordered by size).}$$

Cost estimation (chapter 9)

$$C_2 = C_1 \left(\frac{Q_2}{Q_1}\right)^n, \qquad n = \frac{\ln(C_2/C_1)}{\ln(Q_2/Q_1)}, \qquad C_{\text{now}} = C_{\text{then}} \frac{I_{\text{now}}}{I_{\text{then}}}.$$

Contingency is set by estimate class (AACE Class 5 through Class 1), not by habit.

Project cash flows (chapter 11)

Tableau rows: $R - E - D = TI$; tax $= t\,TI$; $NI = TI(1-t)$; $ATCF = NI + D -$ capex $- \Delta WC +$ net salvage $+$ loan $-$ principal repayment. Working capital is injected early and recovered at the end; interest is deductible, principal is not.

Decision trees and information (chapter 13)

Roll back: expectation at chance nodes, maximum at decision nodes.

$$EVPI = E[\text{value with perfect information}] - E[\text{best act now}],$$

and EVSI uses posterior probabilities from Bayes revision.

Monte Carlo simulation (chapter 14)

Triangular mean $E[X] = (a+m+b)/3$; contingency $=$ chosen percentile $-$ base estimate; standard error of a probability $\sqrt{p(1-p)/n}$. Report P10/P50/P90 and the probability of loss, not the mean alone.

Real options (chapter 15)

$$q = \frac{(1+r)-d}{u-d}, \qquad C_0 = \frac{q\,C_u + (1-q)\,C_d}{1+r}, \qquad u = e^{\sigma\sqrt{\Delta t}},\ d = 1/u.$$

Sustainability economics (chapter 17)

Carbon enters as a cost line: annual carbon cost $=$ price $\times$ tCO_2e per year. Life-cycle cost $=$ PW of acquisition $+$ operation $+$ maintenance $+$ end-of-life (recycling revenue is a negative cost).

Multi-criteria decision analysis (chapter 18)

$$V_j = \sum_k w_k\,s_{jk}, \qquad \sum_k w_k = 1,$$

with scores s_{jk} scaled 0–100 per criterion before weighting. Report the flip weight of the ranking.

Automation economics (chapter 19)

Learning curve: $C_n = C_1\, n^{\log_2 \varphi}$, so $C_{2n} = \varphi\, C_n$. Wait-versus-adopt: adopt now unless $(AEC_1 - AEC_2)/i > AEC_D - AEC_1$.

Forecasting discipline (chapter 20)

Reference-class uplift: budget = base estimate $\times(1 + \text{uplift})$ at the chosen certainty percentile, with the uplift taken from the outside view (the distribution of outcomes in the project's class), not from the advocate's estimate.

Present-worth analysis (chapter 5)

$$PW(i) = \sum_{n=0}^{N} A_n\,(1+i)^{-n} \qquad \text{accept a single project iff } PW(\text{MARR}) > 0$$

$$FW(i) = PW(i)\,(F/P, i, N) \qquad \text{net future worth}$$

$$PB_0 = A_0, \qquad PB_n = PB_{n-1}(1+i) + A_n \qquad \text{project balance; } PB_N = FW$$

Payback period: the first year k whose cumulative cash flow is ≥ 0,

$$PP = (k-1) + \frac{\text{shortfall at year } k-1}{\text{cash flow in year } k}.$$

Discounted payback: the same rule on the discounted flows $A_n\,(P/F, i, n)$.

Capitalized equivalent (chapter 5)

$$CE = \frac{A}{i}, \qquad A_{\text{renewal}} = F\,(A/F, i, k) \text{ for a cost } F \text{ repeated every } k \text{ years.}$$

For mutually exclusive alternatives: equal lives, compare PW directly; unequal lives, replicate to the least common multiple of lives; a stated study period overrides replication (use market value at the end of the period).

Annual-equivalence analysis (chapter 6)

$$AE(i) = PW(i)\,(A/P, i, N) \qquad \text{same sign as } PW$$

$$CR(i) = (I - S)(A/P, i, N) + iS = I\,(A/P, i, N) - S\,(A/F, i, N) \qquad \text{capital recovery cost}$$

Unit cost = annual equivalent cost ÷ annual volume; break-even volume: set the two cost expressions equal and solve for Q. For unequal-life alternatives under repeatability, compare one-cycle AE values directly.

Rate-of-return analysis (chapter 7)

$$PW(i^*) = \sum_{n=0}^{N} A_n\,(1+i^*)^{-n} = 0 \qquad \text{definition of } i^*$$

$$i^* \approx i_L + (i_H - i_L)\,\frac{PW(i_L)}{PW(i_L) - PW(i_H)} \qquad \text{linear interpolation}$$

One sign change in the net cash flow ⇒ simple investment (unique i^*); more than one ⇒ nonsimple (up to as many roots; use PW at the MARR instead). Decision rule (simple investments): accept iff $i^* >$ MARR, which agrees with $PW(\text{MARR}) > 0$.

Incremental analysis (chapter 7)

Never rank alternatives by their individual rates. Order by investment size, form the increment $(B - A)$ = larger − smaller, and choose the larger alternative iff $i^*_{B-A} \geq$ MARR. For three or more alternatives, eliminate pairwise (champion–challenger). Note $PW_B(i) - PW_A(i) = PW_{B-A}(i)$: the two profiles cross at i^*_{B-A}.

Depreciation (chapter 10)

$$D_n = \frac{I - S}{N} \qquad \text{straight line (equal charges)}$$

$$D_n = \alpha\,B_{n-1}, \quad \alpha = \frac{\text{multiplier}}{N} \qquad \text{declining balance (DDB: } \alpha = 2/N)$$

$$D_n = \frac{u_n}{U}\,(I - S) \qquad \text{units of production}$$

$$D_n = I\,p_n \qquad \text{MACRS (percentages } p_n \text{ from the table)}$$

Book value: $B_n = B_{n-1} - D_n$, with $B_0 = I$ (cost basis, including freight, site preparation, and installation). Salvage floor: declining balance never depreciates below S. Switch to straight line when $(B_{n-1} - S)/(\text{remaining years})$ exceeds the DB charge. MACRS uses a zero salvage value, the half-year convention (the N-year class spans $N+1$ years), and a half charge in the disposal year.

Income taxes and after-tax cash flow (chapter 10)

$$
\begin{aligned}
TI &= R - E - D, \qquad \text{tax} = t\,TI, \qquad NI = TI\,(1-t) \\
ATCF &= NI + D = (R-E)(1-t) + tD
\end{aligned}
$$

The term tD is the depreciation tax shield. Disposal for salvage S against book value BV: gain (or loss) $= S - BV$; net proceeds $= S - t\,(S - BV)$ in both directions (a loss produces a tax credit).

Project uncertainty (chapter 12)

One-way sensitivity: recompute PW while one input swings (e.g. $\pm 10\%$, $\pm 20\%$); rank inputs by the swing in PW. Break-even: solve $PW(x) = 0$ for the input x; cushion $=$ (base $-$ break-even)/base. Scenario analysis: one PW per coherent scenario (pessimistic, most likely, optimistic). Expected present worth:

$$E[PW] = \sum_j p_j\, PW_j \qquad \text{(also report the probability and size of the losing outcomes).}$$

Replacement decisions (chapter 16)

Sunk costs are ignored; under the outsider (opportunity-cost) viewpoint the defender's investment is its market value today. Economic service life:

$$AEC(N) = (I - S_N)(A/P,i,N) + i\,S_N + \left[\textstyle\sum_{n=1}^{N} OC_n\,(P/F,i,n)\right](A/P,i,N),$$

minimized at N^*. Marginal (one-more-year) cost of keeping the defender:

$$MC = MV_{\text{now}}(1+i) - MV_{\text{in one year}} + OC_{\text{next year}};$$

keep the defender while $MC < AEC^*_{\text{challenger}}$. Unequal lives are compared by one-cycle AE under repeatability.