

ISE 307: Engineering Economic Analysis (Term 253)

Homework 1, Version C, Due Wednesday, 24 June 2026, 11:59 PM in Gradescope

Name: _____ Student ID: _____

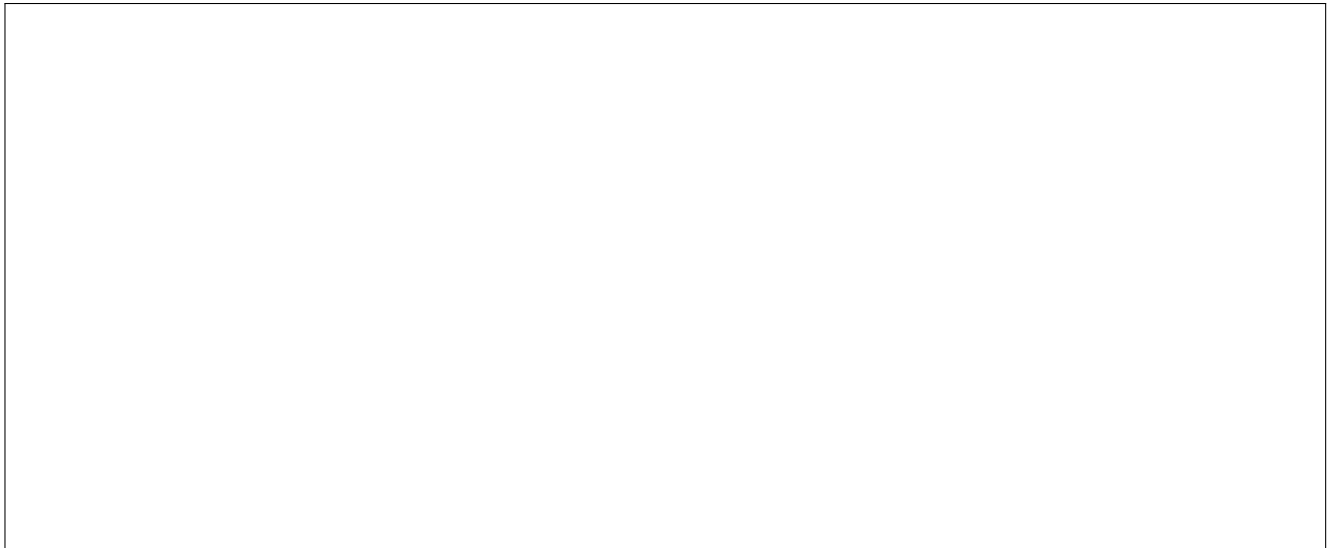
Section: _____ Date: _____

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- Show all work in the spaces provided; You may add extra space as needed. Partial credit might be awarded for correct setup.
 - The submitted work must be your own; you may use generative AI only to check your work, i.e., after you have formulated and computed your own answers.
 - Round rates to 4 decimals in percent; monetary answers to 2 decimals.
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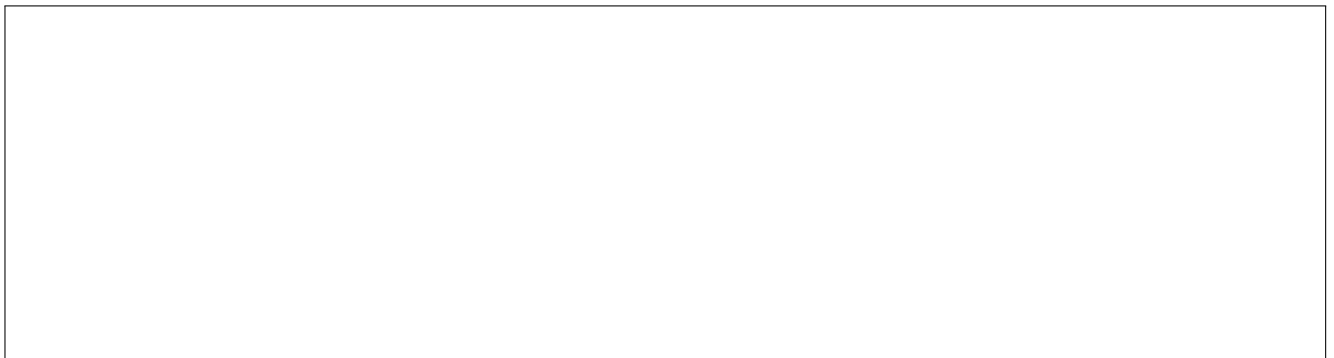
Problem 1 (20 pts)

A research foundation will fund an inspection-robotics lab one of two ways: (A) 250,000 SAR paid now, or (B) 32,000 SAR at the end of each year for 14 years.

(a) (6 pts) Draw a labeled cash flow diagram for each option.



(b) (6 pts) At $i = 7\%$, find the present worth of Option B. Which option is worth more today?



- (c) (8 pts) Find the interest rate at which the two options are equivalent. State the approximate rate and interpret it for the lab.

Problem 2 (20 pts)

A phosphate plant borrows **50,000 SAR at 10% compounded annually** to install a conveyor. The lender offers **Plan 1**: four equal end-of-year payments; **Plan 2**: one lump payment at the end of year 4.

- (a) (4 pts) Draw one labeled cash flow diagram per plan.

- (b) (4 pts) Find the Plan 1 annual payment.

- (c) (4 pts) Find the Plan 2 single payment.

(d) (5 pts) Report the total paid and the total interest paid under each plan.

(e) (3 pts) The totals differ, yet the lender is indifferent between the plans. Explain why in 2–3 sentences.

Problem 3 (20 pts)

A plant keeps an automation reserve earning **8% compounded annually**. It deposits 8,000 SAR now ($n=0$) and 6,000 SAR at the end of year 2, withdraws 3,000 SAR at the end of year 4, and deposits 5,000 SAR at the end of year 5.

(a) (4 pts) Draw the complete cash flow diagram of the account.

(b) (8 pts) Find the account balance at the end of year 8!

- (c) (8 pts) The plant will empty this balance with **four equal withdrawals at the ends of years 9–12**. Find the withdrawal amount. (The year-8 balance sits one period before the first withdrawal.)

Problem 4 (25 pts)

A lithium refinery projects its first five years of revenue two ways ($i = 10\%$ throughout):

- **Projection G (geometric):** 150,000 SAR in year 1, growing 12% per year through year 5.
- **Projection L (linear):** 150,000 SAR in year 1, increasing by 20,000 SAR each year through year 5.

- (a) (6 pts) Draw a labeled cash flow diagram for each projection.

- (b) (8 pts) Find the present worth of Projection G with the geometric-gradient formula.

- (c) (7 pts) Find the present worth of Projection L as a uniform series plus a linear gradient, using the tables.

- (d) (4 pts) Which projection is worth more today, and by how much?

MCQs (15 pts)

Circle the best answer for each multiple-choice question, and mark each statement True or False.

1. (3 pts) A riskier robotics venture and a safer one are otherwise comparable. Which principle explains why the riskier one must promise a higher expected return to attract capital?
 - (A) An earlier dollar is worth more than a later one.
 - (B) Only differences among alternatives matter.
 - (C) Marginal revenue must exceed marginal cost.
 - (D) Additional risk is not taken without additional expected return.
2. (3 pts) Replacing an aging haul truck “now, or if not now, when?” is an example of which type of strategic engineering economic decision?
 - (A) New product or expansion.
 - (B) Equipment replacement.
 - (C) Service and quality improvement.
 - (D) Cost reduction by make-or-buy.
3. (3 pts) Two cash-flow series are *economically equivalent* when they:
 - (A) have the same number of payments.
 - (B) have the same undiscounted total.
 - (C) have the same value at a common point in time for a given interest rate.
 - (D) both end at year 5.
4. (3 pts) **True / False:** Time and uncertainty are the defining aspects of any engineering economic decision.
5. (3 pts) **True / False:** Interest is the cost of money — a cost to the borrower and an earning to the lender.

ISE 307: Engineering Economic Analysis (Formula Sheet)

This formula sheet is compiled with the help of GenAI. It has been reviewed and corrected by the instructor, but may still miss some course content or contain mistakes. If you find any issues, please report it so we can correct it for everyone's use. *Current version: v1.1, last updated: 2026-06-23.*

Linear Interest

$$F = P(1 + iN)$$

Single Payment

$$F = P(1 + i)^N = P(F/P, i, N)$$

$$P = F(1 + i)^{-N} = F(P/F, i, N)$$

$$N = \frac{\ln(F/P)}{\ln(1+i)}, \quad i = \left(\frac{F}{P}\right)^{1/N} - 1$$

$$N_{\text{double}} \approx 72/i (\%) \quad (\text{Rule of 72})$$

Equal-Payment (Uniform) Series

$$F = A \left[\frac{(1+i)^N - 1}{i} \right] = A(F/A, i, N)$$

$$A = F \left[\frac{i}{(1+i)^N - 1} \right] = F(A/F, i, N)$$

$$P = A \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right] = A(P/A, i, N)$$

$$A = P \left[\frac{i(1+i)^N}{(1+i)^N - 1} \right] = P(A/P, i, N)$$

Linear Gradient Series

$$P = G \left[\frac{(1+i)^N - iN - 1}{i^2(1+i)^N} \right] = G(P/G, i, N)$$

$$A = G \left[\frac{(1+i)^N - iN - 1}{i[(1+i)^N - 1]} \right] = G(A/G, i, N)$$

Geometric Gradient Series

$$P = \begin{cases} A_1 \left[\frac{1 - (1+g)^N(1+i)^{-N}}{i - g} \right], & i \neq g \\ A_1 \left(\frac{N}{1+i} \right), & i = g \end{cases}$$

Useful Relationships

$$(P/F) = 1/(F/P) \quad \text{Reciprocals}$$

$$(A/F) = 1/(F/A)$$

$$(A/P) = 1/(P/A)$$

$$(A/P) = (A/F) + i \quad \text{Sinking/recovery}$$

$$(P/A) = (P/F) \cdot (F/A) \quad \text{Series link}$$

Nominal & Effective Rates

$$i = r/M$$

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1$$

$$i = \left(1 + \frac{r}{M}\right)^C - 1$$

$$i_a = e^r - 1, \quad F = P e^{rN}$$

Inflation

$$i = i' + f + i'f$$

$$i' = \frac{1+i}{1+f} - 1 = \frac{i-f}{1+f}$$

$$A'_n = A_n(1+f)^{-n}$$

$$\bar{f} = \left[\prod_{k=1}^N (1+f_k) \right]^{1/N} - 1$$

Perpetuity (Capitalized Cost)

$$P = A/i$$

Worth & Equivalent-Cost Methods

$$PW(i) = \sum_{n=0}^N A_n(1+i)^{-n}$$

$$FW(i) = PW(i)(F/P, i, N)$$

$$AE(i) = PW(i)(A/P, i, N)$$

$$CE(i) = A/i$$

Capital Recovery with Return

$$CR(i) = (I - S)(A/P, i, N) + iS$$

$$AEC = CR + AE \text{ of operating costs}$$

Rate of Return (IRR)

Find i^* (the IRR) that satisfies:

$$PW(i^*) = \sum_{n=0}^N A_n(1+i^*)^{-n} = 0$$

Accept if $i^* \geq \text{MARR}$

Benefit-Cost & Profitability

$$BC(i) = \frac{B}{I+C'}, \quad PI(i) = \frac{B-C'}{I}$$

Depreciation

$$D_n = \frac{I-S}{N} \quad \text{Straight-line}$$

$$D_n = (I - S) \frac{U_n}{U_{\text{total}}} \quad \text{Units of prod.}$$

$$D_n = \alpha B_{n-1} = \alpha I(1 - \alpha)^{n-1} \quad \text{Declining bal.}$$

$$D_n = I \times r_n \quad \text{MACRS } (r_n \text{ table})$$

$$B_n = I - \sum_{k=1}^n D_k \quad \text{Book value}$$

Gains, Losses & Taxes

$$\text{gain} = S - B_n$$

$$\text{gains tax} = (S - B_n)t$$

$$\text{net salvage} = S - (S - B_n)t$$

$$TI = \text{Rev} - \text{Exp} - D_n$$

$$\text{income tax} = TI \times t$$

$$ATCF = (\text{Rev} - \text{Exp})(1 - t) + t D_n$$

Replacement Analysis

$$AEC(N) = CR(N) + AEC \text{ of O\&M}(N)$$

Break-Even & Sensitivity

$$Q^* = \frac{\text{Fixed cost}}{\text{Price} - \text{Variable cost}}$$

Standardized Normal (Risk)

$$z = \frac{x - \mu}{\sigma}, \quad \Phi(-z) = 1 - \Phi(z)$$

Interest Factor Tables

$i = 1\%$

N	(F/P)	(P/F)	(F/A)	(A/F)	(P/A)	(A/P)	(A/G)	(P/G)
1	1.0100	0.9901	1.0000	1.0000	0.9901	1.0100	0.0000	0.0000
2	1.0201	0.9803	2.0100	0.4975	1.9704	0.5075	0.4975	0.9803
3	1.0303	0.9706	3.0301	0.3300	2.9410	0.3400	0.9934	2.9215
4	1.0406	0.9610	4.0604	0.2463	3.9020	0.2563	1.4876	5.8044
5	1.0510	0.9515	5.1010	0.1960	4.8534	0.2060	1.9801	9.6103
6	1.0615	0.9420	6.1520	0.1625	5.7955	0.1725	2.4710	14.3205
7	1.0721	0.9327	7.2135	0.1386	6.7282	0.1486	2.9602	19.9168
8	1.0829	0.9235	8.2857	0.1207	7.6517	0.1307	3.4478	26.3812
9	1.0937	0.9143	9.3685	0.1067	8.5660	0.1167	3.9337	33.6959
10	1.1046	0.9053	10.4622	0.0956	9.4713	0.1056	4.4179	41.8435
11	1.1157	0.8963	11.5668	0.0865	10.3676	0.0965	4.9005	50.8067
12	1.1268	0.8874	12.6825	0.0788	11.2551	0.0888	5.3815	60.5687
13	1.1381	0.8787	13.8093	0.0724	12.1337	0.0824	5.8607	71.1126
14	1.1495	0.8700	14.9474	0.0669	13.0037	0.0769	6.3384	82.4221
15	1.1610	0.8613	16.0969	0.0621	13.8651	0.0721	6.8143	94.4810
16	1.1726	0.8528	17.2579	0.0579	14.7179	0.0679	7.2886	107.2734
17	1.1843	0.8444	18.4304	0.0543	15.5623	0.0643	7.7613	120.7834
18	1.1961	0.8360	19.6147	0.0510	16.3983	0.0610	8.2323	134.9957
19	1.2081	0.8277	20.8109	0.0481	17.2260	0.0581	8.7017	149.8950
20	1.2202	0.8195	22.0190	0.0454	18.0456	0.0554	9.1694	165.4664
21	1.2324	0.8114	23.2392	0.0430	18.8570	0.0530	9.6354	181.6950
22	1.2447	0.8034	24.4716	0.0409	19.6604	0.0509	10.0998	198.5663
23	1.2572	0.7954	25.7163	0.0389	20.4558	0.0489	10.5626	216.0660
24	1.2697	0.7876	26.9735	0.0371	21.2434	0.0471	11.0237	234.1800
25	1.2824	0.7798	28.2432	0.0354	22.0232	0.0454	11.4831	252.8945
26	1.2953	0.7720	29.5256	0.0339	22.7952	0.0439	11.9409	272.1957
27	1.3082	0.7644	30.8209	0.0324	23.5596	0.0424	12.3971	292.0702
28	1.3213	0.7568	32.1291	0.0311	24.3164	0.0411	12.8516	312.5047
29	1.3345	0.7493	33.4504	0.0299	25.0658	0.0399	13.3044	333.4863
30	1.3478	0.7419	34.7849	0.0287	25.8077	0.0387	13.7557	355.0021

$i = 3\%$

N	(F/P)	(P/F)	(F/A)	(A/F)	(P/A)	(A/P)	(A/G)	(P/G)
1	1.0300	0.9709	1.0000	1.0000	0.9709	1.0300	0.0000	0.0000
2	1.0609	0.9426	2.0300	0.4926	1.9135	0.5226	0.4926	0.9426
3	1.0927	0.9151	3.0909	0.3235	2.8286	0.3535	0.9803	2.7729
4	1.1255	0.8885	4.1836	0.2390	3.7171	0.2690	1.4631	5.4383
5	1.1593	0.8626	5.3091	0.1884	4.5797	0.2184	1.9409	8.8888
6	1.1941	0.8375	6.4684	0.1546	5.4172	0.1846	2.4138	13.0762
7	1.2299	0.8131	7.6625	0.1305	6.2303	0.1605	2.8819	17.9547
8	1.2668	0.7894	8.8923	0.1125	7.0197	0.1425	3.3450	23.4806
9	1.3048	0.7664	10.1591	0.0984	7.7861	0.1284	3.8032	29.6119
10	1.3439	0.7441	11.4639	0.0872	8.5302	0.1172	4.2565	36.3088
11	1.3842	0.7224	12.8078	0.0781	9.2526	0.1081	4.7049	43.5330
12	1.4258	0.7014	14.1920	0.0705	9.9540	0.1005	5.1485	51.2482
13	1.4685	0.6810	15.6178	0.0640	10.6350	0.0940	5.5872	59.4196
14	1.5126	0.6611	17.0863	0.0585	11.2961	0.0885	6.0210	68.0141
15	1.5580	0.6419	18.5989	0.0538	11.9379	0.0838	6.4500	77.0002
16	1.6047	0.6232	20.1569	0.0496	12.5611	0.0796	6.8742	86.3477
17	1.6528	0.6050	21.7616	0.0460	13.1661	0.0760	7.2936	96.0280
18	1.7024	0.5874	23.4144	0.0427	13.7535	0.0727	7.7081	106.0137
19	1.7535	0.5703	25.1169	0.0398	14.3238	0.0698	8.1179	116.2788
20	1.8061	0.5537	26.8704	0.0372	14.8775	0.0672	8.5229	126.7987
21	1.8603	0.5375	28.6765	0.0349	15.4150	0.0649	8.9231	137.5496
22	1.9161	0.5219	30.5368	0.0327	15.9369	0.0627	9.3186	148.5094
23	1.9736	0.5067	32.4529	0.0308	16.4436	0.0608	9.7093	159.6566
24	2.0328	0.4919	34.4265	0.0290	16.9355	0.0590	10.0954	170.9711
25	2.0938	0.4776	36.4593	0.0274	17.4131	0.0574	10.4768	182.4336
26	2.1566	0.4637	38.5530	0.0259	17.8768	0.0559	10.8535	194.0260
27	2.2213	0.4502	40.7096	0.0246	18.3270	0.0546	11.2255	205.7309
28	2.2879	0.4371	42.9309	0.0233	18.7641	0.0533	11.5930	217.5320
29	2.3566	0.4243	45.2189	0.0221	19.1885	0.0521	11.9558	229.4137
30	2.4273	0.4120	47.5754	0.0210	19.6004	0.0510	12.3141	241.3613

$i = 2\%$

N	(F/P)	(P/F)	(F/A)	(A/F)	(P/A)	(A/P)	(A/G)	(P/G)
1	1.0200	0.9804	1.0000	1.0000	0.9804	1.0200	0.0000	0.0000
2	1.0404	0.9612	2.0200	0.4950	1.9416	0.5150	0.4950	0.9612
3	1.0612	0.9423	3.0604	0.3268	2.8839	0.3468	0.9868	2.8458
4	1.0824	0.9238	4.1216	0.2426	3.8077	0.2626	1.4752	5.6173
5	1.1041	0.9057	5.2040	0.1922	4.7135	0.2122	1.9604	9.2403
6	1.1262	0.8880	6.3081	0.1585	5.6014	0.1785	2.4423	13.6801
7	1.1487	0.8706	7.4343	0.1345	6.4720	0.1545	2.9208	18.9035
8	1.1717	0.8535	8.5830	0.1165	7.3255	0.1365	3.3961	24.8779
9	1.1951	0.8368	9.7546	0.1025	8.1622	0.1225	3.8681	31.5720
10	1.2190	0.8203	10.9497	0.0913	8.9826	0.1113	4.3367	38.9551
11	1.2434	0.8043	12.1687	0.0822	9.7868	0.1022	4.8021	46.9977
12	1.2682	0.7885	13.4121	0.0746	10.5753	0.0946	5.2642	55.6712
13	1.2936	0.7730	14.6803	0.0681	11.3484	0.0881	5.7231	64.9475
14	1.3195	0.7579	15.9739	0.0626	12.1062	0.0826	6.1786	74.7999
15	1.3459	0.7430	17.2934	0.0578	12.8493	0.0778	6.6309	85.2021
16	1.3728	0.7284	18.6393	0.0537	13.5777	0.0737	7.0799	96.1288
17	1.4002	0.7142	20.0121	0.0500	14.2919	0.0700	7.5256	107.5554
18	1.4282	0.7002	21.4123	0.0467	14.9920	0.0667	7.9681	119.4581
19	1.4568	0.6864	22.8406	0.0438	15.6785	0.0638	8.4073	131.8139
20	1.4859	0.6730	24.2974	0.0412	16.3514	0.0612	8.8433	144.6003
21	1.5157	0.6598	25.7833	0.0388	17.0112	0.0588	9.2760	157.7959
22	1.5460	0.6468	27.2990	0.0366	17.6580	0.0566	9.7055	171.3795
23	1.5769	0.6342	28.8450	0.0347	18.2922	0.0547	10.1317	185.3309
24	1.6084	0.6217	30.4219	0.0329	18.9139	0.0529	10.5547	199.6305
25	1.6406	0.6095	32.0303	0.0312	19.5235	0.0512	10.9745	214.2592
26	1.6734	0.5976	33.6709	0.0297	20.1210	0.0497	11.3910	229.1987
27	1.7069	0.5859	35.3443	0.0283	20.7069	0.0483	11.8043	244.4311
28	1.7410	0.5744	37.0512	0.0270	21.2813	0.0470	12.2145	259.9392
29	1.7758	0.5631	38.7922	0.0258	21.8444	0.0458	12.6214	275.7064
30	1.8114	0.5521	40.5681	0.0246	22.3965	0.0446	13.0251	291.7164

$i = 4\%$

N	(F/P)	(P/F)	(F/A)	(A/F)	(P/A)	(A/P)	(A/G)	(P/G)
1	1.0400	0.9615	1.0000	1.0000	0.9615	1.0400	0.0000	0.0000
2	1.0816	0.9246	2.0400	0.4902	1.8861	0.5302	0.4902	0.9246
3	1.1249	0.8890	3.1216	0.3203	2.7751	0.3603	0.9739	2.7025
4	1.1699	0.8548	4.2465	0.2355	3.6299	0.2755	1.4510	5.2670
5	1.2167	0.8219	5.4163	0.1846	4.4518	0.2246	1.9216	8.5547
6	1.2653	0.7903	6.6330	0.1508	5.2421	0.1908	2.3857	12.5062
7	1.3159	0.7599	7.8983	0.1266	6.0021	0.1666	2.8433	17.0657
8	1.3686	0.7307	9.2142	0.1085	6.7327	0.1485	3.2944	22.1806
9	1.4233	0.7026	10.5828	0.0945	7.4353	0.1345	3.7391	27.8013
10	1.4802	0.6756	12.0061	0.0833	8.1109	0.1233	4.1773	33.8814
11	1.5395	0.6496	13.4864	0.0741	8.7605	0.1141	4.6090	40.3772
12	1.6010	0.6246	15.0258	0.0666	9.3851	0.1066	5.0343	47.2477
13	1.6651	0.6006	16.6268	0.0601	9.9856	0.1001	5.4533	54.4546
14	1.7317	0.5775	18.2919	0.0547	10.5631	0.0947	5.8659	61.9618
15	1.8009	0.5553	20.0236	0.0499	11.1184	0.0899	6.2721	69.7355
16	1.8730	0.5339	21.8245	0.0458	11.6523	0.0858	6.6720	77.7441
17	1.9479	0.5134	23.6975	0.0422	12.1657	0.0822	7.0656	85.9581
18	2.0258	0.4936	25.6454	0.0390	12.6593	0.0790	7.4530	94.3498
19	2.1068	0.4746	27.6712	0.0361	13.1339	0.0761	7.8342	102.8933
20	2.1911	0.4564	29.7781	0.0336	13.5903	0.0736	8.2091	111.5647
21	2.2788	0.4388	31.9692	0.0313	14.0292	0.0713	8.5779	120.3414
22	2.3699	0.4220	34.2480	0.0292	14.4511	0.0692	8.9407	129.2024
23	2.4647	0.4057	36.6179	0.0273	14.8568	0.0673	9.2973	138.1284
24	2.5633	0.3901	39.0826	0.0256	15.2470	0.0656	9.6479	147.1012
25	2.6658	0.3751	41.6459	0.0240	15.6221	0.0640	9.9925	156.1040
26	2.7725	0.3607	44.3117	0.0226	15.9828	0.0626	10.3312	165.1212
27	2.8834	0.3468	47.0842	0.0212	16.3296	0.0612	10.6640	174.1385
28	2.9987	0.3335	49.9676	0.0200	16.6631	0.0600	10.9909	183.1424
29	3.1187	0.3207	52.9663	0.0189	16.9837	0.0589	11.3120	192.1206
30	3.2434	0.3083	56.0849	0.0178	17.2920	0.0578	11.6274	201.0618

$i = 5\%$									$i = 7\%$								
N	(F/P)	(P/F)	(F/A)	(A/F)	(P/A)	(A/P)	(A/G)	(P/G)	N	(F/P)	(P/F)	(F/A)	(A/F)	(P/A)	(A/P)	(A/G)	(P/G)
1	1.0500	0.9524	1.0000	1.0000	0.9524	1.0500	0.0000	0.0000	1	1.0700	0.9346	1.0000	1.0000	0.9346	1.0700	0.0000	0.0000
2	1.1025	0.9070	2.0500	0.4878	1.8594	0.5378	0.4878	0.9070	2	1.1449	0.8734	2.0700	0.4831	1.8080	0.5531	0.4831	0.8734
3	1.1576	0.8638	3.1525	0.3172	2.7232	0.3672	0.9675	2.6347	3	1.2250	0.8163	3.2149	0.3111	2.6243	0.3811	0.9549	2.5060
4	1.2155	0.8227	4.3101	0.2320	3.5460	0.2820	1.4391	5.1028	4	1.3108	0.7629	4.4399	0.2252	3.3872	0.2952	1.4155	4.7947
5	1.2763	0.7835	5.5256	0.1810	4.3295	0.2310	1.9025	8.2369	5	1.4026	0.7130	5.7507	0.1739	4.1002	0.2439	1.8650	7.6467
6	1.3401	0.7462	6.8019	0.1470	5.0757	0.1970	2.3579	11.9680	6	1.5007	0.6663	7.1533	0.1398	4.7665	0.2098	2.3032	10.9784
7	1.4071	0.7107	8.1420	0.1228	5.7864	0.1728	2.8052	16.2321	7	1.6058	0.6227	8.6540	0.1156	5.3893	0.1856	2.7304	14.7149
8	1.4775	0.6768	9.5491	0.1047	6.4632	0.1547	3.2445	20.9700	8	1.7182	0.5820	10.2598	0.0975	5.9713	0.1675	3.1465	18.7889
9	1.5513	0.6446	11.0266	0.0907	7.1078	0.1407	3.6758	26.1268	9	1.8385	0.5439	11.9780	0.0835	6.5152	0.1535	3.5517	23.1404
10	1.6289	0.6139	12.5779	0.0795	7.7217	0.1295	4.0991	31.6520	10	1.9672	0.5083	13.8164	0.0724	7.0236	0.1424	3.9461	27.7156
11	1.7103	0.5847	14.2068	0.0704	8.3064	0.1204	4.5144	37.4988	11	2.1049	0.4751	15.7836	0.0634	7.4987	0.1334	4.3296	32.4665
12	1.7959	0.5568	15.9171	0.0628	8.8633	0.1128	4.9219	43.6241	12	2.2522	0.4440	17.8885	0.0559	7.9427	0.1259	4.7025	37.3506
13	1.8856	0.5303	17.7130	0.0565	9.3936	0.1065	5.3215	49.9879	13	2.4098	0.4150	20.1406	0.0497	8.3577	0.1197	5.0648	42.3302
14	1.9799	0.5051	19.5986	0.0510	9.8986	0.1010	5.7133	56.5538	14	2.5785	0.3878	22.5505	0.0443	8.7455	0.1143	5.4167	47.3718
15	2.0789	0.4810	21.5786	0.0463	10.3797	0.0963	6.0973	63.2880	15	2.7590	0.3624	25.1290	0.0398	9.1079	0.1098	5.7583	52.4461
16	2.1829	0.4581	23.6575	0.0423	10.8378	0.0923	6.4736	70.1597	16	2.9522	0.3387	27.8881	0.0359	9.4466	0.1059	6.0897	57.5271
17	2.2920	0.4363	25.8404	0.0387	11.2741	0.0887	6.8423	77.1405	17	3.1588	0.3166	30.8402	0.0324	9.7632	0.1024	6.4110	62.5923
18	2.4066	0.4155	28.1324	0.0355	11.6896	0.0855	7.2034	84.2043	18	3.3799	0.2959	33.9990	0.0294	10.0591	0.0994	6.7225	67.6219
19	2.5270	0.3957	30.5390	0.0327	12.0853	0.0827	7.5569	91.3275	19	3.6165	0.2765	37.3790	0.0268	10.3356	0.0968	7.0242	72.5991
20	2.6533	0.3769	33.0660	0.0302	12.4622	0.0802	7.9030	98.4884	20	3.8697	0.2584	40.9955	0.0244	10.5940	0.0944	7.3163	77.5091
21	2.7860	0.3589	35.7193	0.0280	12.8212	0.0780	8.2416	105.6673	21	4.1406	0.2415	44.8652	0.0223	10.8355	0.0923	7.5990	82.3393
22	2.9253	0.3418	38.5052	0.0260	13.1630	0.0760	8.5730	112.8461	22	4.4304	0.2257	49.0057	0.0204	11.0612	0.0904	7.8725	87.0793
23	3.0715	0.3256	41.4305	0.0241	13.4886	0.0741	8.8971	120.0087	23	4.7405	0.2109	53.4361	0.0187	11.2722	0.0887	8.1369	91.7201
24	3.2251	0.3101	44.5020	0.0225	13.7986	0.0725	9.2140	127.1402	24	5.0724	0.1971	58.1767	0.0172	11.4693	0.0872	8.3923	96.2545
25	3.3864	0.2953	47.7271	0.0210	14.0939	0.0710	9.5238	134.2275	25	5.4274	0.1842	63.2490	0.0158	11.6536	0.0858	8.6391	100.6765
26	3.5557	0.2812	51.1135	0.0196	14.3752	0.0696	9.8266	141.2585	26	5.8074	0.1722	68.6765	0.0146	11.8258	0.0846	8.8773	104.9814
27	3.7335	0.2678	54.6691	0.0183	14.6430	0.0683	10.1224	148.2226	27	6.2139	0.1609	74.4838	0.0134	11.9867	0.0834	9.1072	109.1656
28	3.9201	0.2551	58.4026	0.0171	14.8981	0.0671	10.4114	155.1101	28	6.6488	0.1504	80.6977	0.0124	12.1371	0.0824	9.3289	113.2264
29	4.1161	0.2429	62.3227	0.0160	15.1411	0.0660	10.6936	161.9126	29	7.1143	0.1406	87.3465	0.0114	12.2777	0.0814	9.5427	117.1622
30	4.3219	0.2314	66.4388	0.0151	15.3725	0.0651	10.9691	168.6226	30	7.6123	0.1314	94.4608	0.0106	12.4090	0.0806	9.7487	120.9718

$i = 6\%$									$i = 8\%$								
N	(F/P)	(P/F)	(F/A)	(A/F)	(P/A)	(A/P)	(A/G)	(P/G)	N	(F/P)	(P/F)	(F/A)	(A/F)	(P/A)	(A/P)	(A/G)	(P/G)
1	1.0600	0.9434	1.0000	1.0000	0.9434	1.0600	0.0000	0.0000	1	1.0800	0.9259	1.0000	1.0000	0.9259	1.0800	0.0000	0.0000
2	1.1236	0.8900	2.0600	0.4854	1.8334	0.5454	0.4854	0.8900	2	1.1664	0.8573	2.0800	0.4808	1.7833	0.5608	0.4808	0.8573
3	1.1910	0.8396	3.1836	0.3141	2.6730	0.3741	0.9612	2.5692	3	1.2597	0.7938	3.2464	0.3080	2.5771	0.3880	0.9487	2.4450
4	1.2625	0.7921	4.3746	0.2286	3.4651	0.2886	1.4272	4.9455	4	1.3605	0.7350	4.5061	0.2219	3.3121	0.3019	1.4040	4.6501
5	1.3382	0.7473	5.6371	0.1774	4.2124	0.2374	1.8836	7.9345	5	1.4693	0.6806	5.8666	0.1705	3.9927	0.2505	1.8465	7.3724
6	1.4185	0.7050	6.9753	0.1434	4.9173	0.2034	2.3304	11.4594	6	1.5869	0.6302	7.3359	0.1363	4.6229	0.2163	2.2763	10.5233
7	1.5036	0.6651	8.3938	0.1191	5.5824	0.1791	2.7676	15.4497	7	1.7138	0.5835	8.9228	0.1121	5.2064	0.1921	2.6937	14.0242
8	1.5938	0.6274	9.8975	0.1010	6.2098	0.1610	3.1952	19.8416	8	1.8509	0.5403	10.6366	0.0940	5.7466	0.1740	3.0985	17.8061
9	1.6895	0.5919	11.4913	0.0870	6.8017	0.1470	3.6133	24.5768	9	1.9990	0.5002	12.4876	0.0801	6.2469	0.1601	3.4910	21.8081
10	1.7908	0.5584	13.1808	0.0759	7.3601	0.1359	4.0220	29.6023	10	2.1589	0.4632	14.4866	0.0690	6.7101	0.1490	3.8713	25.9768
11	1.8983	0.5268	14.9716	0.0668	7.8869	0.1268	4.4213	34.8702	11	2.3316	0.4289	16.6455	0.0601	7.1390	0.1401	4.2395	30.2657
12	2.0122	0.4970	16.8699	0.0593	8.3838	0.1193	4.8113	40.3369	12	2.5182	0.3971	18.9771	0.0527	7.5361	0.1327	4.5957	34.6339
13	2.1329	0.4688	18.8821	0.0530	8.8527	0.1130	5.1920	45.9629	13	2.7196	0.3677	21.4953	0.0465	7.9038	0.1265	4.9402	39.0463
14	2.2609	0.4423	21.0151	0.0476	9.2950	0.1076	5.5635	51.7128	14	2.9372	0.3405	24.2149	0.0413	8.2442	0.1213	5.2731	43.4723
15	2.3966	0.4173	23.2760	0.0430	9.7122	0.1030	5.9260	57.5546	15	3.1722	0.3152	27.1521	0.0368	8.5595	0.1168	5.5945	47.8857
16	2.5404	0.3936	25.6725	0.0390	10.1059	0.0990	6.2794	63.4592	16	3.4259	0.2919	30.3243	0.0330	8.8514	0.1130	5.9046	52.2640
17	2.6928	0.3714	28.2129	0.0354	10.4773	0.0954	6.6240	69.4011	17	3.7000	0.2703	33.7502	0.0296	9.1216	0.1096	6.2037	56.5883
18	2.8543	0.3503	30.9057	0.0324	10.8276	0.0924	6.9597	75.3569	18	3.9960	0.2502	37.4502	0.0267	9.3719	0.1067	6.4920	60.8426
19	3.0256	0.3305	33.7600	0.0296	11.1581	0.0896	7.2867	81.3062	19	4.3157	0.2317	41.4463	0.0241	9.6036	0.1041	6.7697	65.0134
20	3.2071	0.3118	36.7856	0.0272	11.4699	0.0872	7.6051	87.2304	20	4.6610	0.2145	45.7620	0.0219	9.8181	0.1019	7.0369	69.0898
21	3.3996	0.2942	39.9927	0.0250	11.7641	0.0850	7.9151	93.1136	21	5.0338	0.1987	50.4229	0.0198	10.0168	0.0998	7.2940	73.0629
22	3.6035	0.2775	43.3923	0.0230	12.0416	0.0830	8.2166	98.9412	22	5.4365	0.1839	55.4568	0.0180	10.2007	0.0980	7.5412	76.9257
23	3.8197	0.2618	46.9958	0.0213	12.3034	0.0813	8.5099	104.7007	23	5.8715	0.1703	60.8933	0.0164	10.3711	0.0964	7.7786	80.6726
24	4.0489	0.2470	50.8156	0.0197	12.5504	0.0797	8.7951	110.3812	24	6.3412	0.1577	66.7648	0.0150	10.5288	0.0950	8.0066	84.2997
25	4.2919	0.2330	54.8645	0.0182	12.7834	0.0782	9.0722	115.9732	25	6.8485	0.1460	73.1059	0.0137	10.6748	0.0937	8.2254	87.8041
26	4.5494	0.2198	59.1564	0.0169	13.0032	0.0769	9.3414	121.4684	26	7.3964	0.1352	79.9544	0.0125	10.8100	0.0925	8.4352	91.1842
27	4.8223	0.2074	63.7058	0.0157	13.2105	0.0757	9.6029	126.8600	27	7.9881	0.1252	87.3508	0.0114	10.9352	0.0914	8.6363	94.4390
28	5.1117	0.1956	68.5281	0.0146	13.4062	0.0746	9.8568	132.1420	28	8.6271	0.1159	95.3388	0.0105	11.0511	0.0905	8.8289	97.5687
29	5.4184	0.1846	73.6398	0.0136	13.5907	0.0736	10.1032	137.3096	29	9.3173	0.1073	103.9659	0.0096	11.1584	0.0896	9.0133	100.5738
30	5.7435	0.1741	79.0582	0.0126	13.7648	0.0726	10.3422	142.3588	30	10.0627	0.0994	113.2832	0.0088	11.2578	0.0888	9.1897	103.4558

$i = 9\%$									$i = 12\%$								
N	(F/P)	(P/F)	(F/A)	(A/F)	(P/A)	(A/P)	(A/G)	(P/G)	N	(F/P)	(P/F)	(F/A)	(A/F)	(P/A)	(A/P)	(A/G)	(P/G)
1	1.0900	0.9174	1.0000	1.0000	0.9174	1.0900	0.0000	0.0000	1	1.1200	0.8929	1.0000	1.0000	0.8929	1.1200	0.0000	0.0000
2	1.1881	0.8417	2.0900	0.4785	1.7591	0.5685	0.4785	0.8417	2	1.2544	0.7972	2.1200	0.4717	1.6901	0.5917	0.4717	0.7972
3	1.2950	0.7722	3.2781	0.3051	2.5313	0.3951	0.9426	2.3860	3	1.4049	0.7118	3.3744	0.2963	2.4018	0.4163	0.9246	2.2208
4	1.4116	0.7084	4.5731	0.2187	3.2397	0.3087	1.3925	4.5113	4	1.5735	0.6355	4.7793	0.2092	3.0373	0.3292	1.3589	4.1273
5	1.5386	0.6499	5.9847	0.1671	3.8897	0.2571	1.8282	7.1110	5	1.7623	0.5674	6.3528	0.1574	3.6048	0.2774	1.7746	6.3970
6	1.6771	0.5963	7.5233	0.1329	4.4859	0.2229	2.2498	10.0924	6	1.9738	0.5066	8.1152	0.1232	4.1114	0.2432	2.1720	8.9302
7	1.8280	0.5470	9.2004	0.1087	5.0330	0.1987	2.6574	13.3746	7	2.2107	0.4523	10.0890	0.0991	4.5638	0.2191	2.5515	11.6443
8	1.9926	0.5019	11.0285	0.0907	5.5348	0.1807	3.0512	16.8877	8	2.4760	0.4039	12.2997	0.0813	4.9676	0.2013	2.9131	14.4714
9	2.1719	0.4604	13.0210	0.0768	5.9952	0.1668	3.4312	20.5711	9	2.7731	0.3606	14.7757	0.0677	5.3282	0.1877	3.2574	17.3563
10	2.3674	0.4224	15.1929	0.0658	6.4177	0.1558	3.7978	24.3728	10	3.1058	0.3220	17.5487	0.0570	5.6502	0.1770	3.5847	20.2541
11	2.5804	0.3875	17.5603	0.0569	6.8052	0.1469	4.1510	28.2481	11	3.4785	0.2875	20.6546	0.0484	5.9377	0.1684	3.8953	23.1288
12	2.8127	0.3555	20.1407	0.0497	7.1607	0.1397	4.4910	32.1590	12	3.8960	0.2567	24.1331	0.0414	6.1944	0.1614	4.1897	25.9523
13	3.0658	0.3262	22.9534	0.0436	7.4869	0.1336	4.8182	36.0731	13	4.3635	0.2292	28.0291	0.0357	6.4235	0.1557	4.4683	28.7024
14	3.3417	0.2992	26.0192	0.0384	7.7862	0.1284	5.1326	39.9633	14	4.8871	0.2046	32.3926	0.0309	6.6282	0.1509	4.7317	31.3624
15	3.6425	0.2745	29.3609	0.0341	8.0607	0.1241	5.4346	43.8069	15	5.4736	0.1827	37.2797	0.0268	6.8109	0.1468	4.9803	33.9202
16	3.9703	0.2519	33.0034	0.0303	8.3126	0.1203	5.7245	47.5849	16	6.1304	0.1631	42.7533	0.0234	6.9740	0.1434	5.2147	36.3670
17	4.3276	0.2311	36.9737	0.0270	8.5436	0.1170	6.0024	51.2821	17	6.8660	0.1456	48.8837	0.0205	7.1196	0.1405	5.4353	38.6973
18	4.7171	0.2120	41.3013	0.0242	8.7556	0.1142	6.2687	54.8860	18	7.6900	0.1300	55.7497	0.0179	7.2497	0.1379	5.6427	40.9080
19	5.1417	0.1945	46.0185	0.0217	8.9501	0.1117	6.5236	58.3868	19	8.6128	0.1161	63.4397	0.0158	7.3658	0.1358	5.8375	42.9979
20	5.6044	0.1784	51.1601	0.0195	9.1285	0.1095	6.7674	61.7770	20	9.6463	0.1037	72.0524	0.0139	7.4694	0.1339	6.0202	44.9676
21	6.1088	0.1637	56.7645	0.0176	9.2922	0.1076	7.0006	65.0509	21	10.8038	0.0926	81.6987	0.0122	7.5620	0.1322	6.1913	46.8188
22	6.6586	0.1502	62.8733	0.0159	9.4424	0.1059	7.2232	68.2048	22	12.1003	0.0826	92.5026	0.0108	7.6446	0.1308	6.3514	48.5543
23	7.2579	0.1378	69.5319	0.0144	9.5802	0.1044	7.4357	71.2359	23	13.5523	0.0738	104.6029	0.0096	7.7184	0.1296	6.5010	50.1776
24	7.9111	0.1264	76.7898	0.0130	9.7066	0.1030	7.6384	74.1433	24	15.1786	0.0659	118.1552	0.0085	7.7843	0.1285	6.6406	51.6929
25	8.6231	0.1160	84.7009	0.0118	9.8226	0.1018	7.8316	76.9265	25	17.0001	0.0588	133.3339	0.0075	7.8431	0.1275	6.7708	53.1046
26	9.3992	0.1064	93.3240	0.0107	9.9290	0.1007	8.0156	79.5863	26	19.0401	0.0525	150.3339	0.0067	7.8957	0.1267	6.8921	54.4177
27	10.2451	0.0976	102.7231	0.0097	10.0266	0.0997	8.1906	82.1241	27	21.3249	0.0469	169.3740	0.0059	7.9426	0.1259	7.0049	55.6369
28	11.1671	0.0895	112.9682	0.0089	10.1161	0.0989	8.3571	84.5419	28	23.8839	0.0419	190.6989	0.0052	7.9844	0.1252	7.1098	56.7674
29	12.1722	0.0822	124.1354	0.0081	10.1983	0.0981	8.5154	86.8422	29	26.7499	0.0374	214.5828	0.0047	8.0218	0.1247	7.2071	57.8141
30	13.2677	0.0754	136.3075	0.0073	10.2737	0.0973	8.6657	89.0280	30	29.9599	0.0334	241.3327	0.0041	8.0552	0.1241	7.2974	58.7821

$i = 10\%$									$i = 15\%$								
N	(F/P)	(P/F)	(F/A)	(A/F)	(P/A)	(A/P)	(A/G)	(P/G)	N	(F/P)	(P/F)	(F/A)	(A/F)	(P/A)	(A/P)	(A/G)	(P/G)
1	1.1000	0.9091	1.0000	1.0000	0.9091	1.1000	0.0000	0.0000	1	1.1500	0.8696	1.0000	1.0000	0.8696	1.1500	0.0000	0.0000
2	1.2100	0.8264	2.1000	0.4762	1.7355	0.5762	0.4762	0.8264	2	1.3225	0.7561	2.1500	0.4651	1.6257	0.6151	0.4651	0.7561
3	1.3310	0.7513	3.3100	0.3021	2.4869	0.4021	0.9366	2.3291	3	1.5209	0.6575	3.4725	0.2880	2.2832	0.4380	0.9071	2.0712
4	1.4641	0.6830	4.6410	0.2155	3.1699	0.3155	1.3812	4.3781	4	1.7490	0.5718	4.9934	0.2003	2.8550	0.3503	1.3263	3.7864
5	1.6105	0.6209	6.1051	0.1638	3.7908	0.2638	1.8101	6.8618	5	2.0114	0.4972	6.7424	0.1483	3.3522	0.2983	1.7228	5.7751
6	1.7716	0.5645	7.7156	0.1296	4.3553	0.2296	2.2236	9.6842	6	2.3131	0.4323	8.7537	0.1142	3.7845	0.2642	2.0972	7.9368
7	1.9487	0.5132	9.4872	0.1054	4.8684	0.2054	2.6216	12.7631	7	2.6600	0.3759	11.0668	0.0904	4.1604	0.2404	2.4498	10.1924
8	2.1436	0.4665	11.4359	0.0874	5.3349	0.1874	3.0045	16.0287	8	3.0590	0.3269	13.7268	0.0729	4.4873	0.2229	2.7813	12.4807
9	2.3579	0.4241	13.5795	0.0736	5.7590	0.1736	3.3724	19.4215	9	3.5179	0.2843	16.7858	0.0596	4.7716	0.2096	3.0922	14.7548
10	2.5937	0.3855	15.9374	0.0627	6.1446	0.1627	3.7255	22.8913	10	4.0456	0.2472	20.3037	0.0493	5.0188	0.1993	3.3832	16.9795
11	2.8531	0.3505	18.5312	0.0540	6.4951	0.1540	4.0641	26.3963	11	4.6524	0.2149	24.3493	0.0411	5.2337	0.1911	3.6549	19.1289
12	3.1384	0.3186	21.3843	0.0468	6.8137	0.1468	4.3884	29.9012	12	5.3503	0.1869	29.0017	0.0345	5.4206	0.1845	3.9082	21.1849
13	3.4523	0.2897	24.5227	0.0408	7.1034	0.1408	4.6988	33.3772	13	6.1528	0.1625	34.3519	0.0291	5.5831	0.1791	4.1438	23.1352
14	3.7975	0.2633	27.9750	0.0357	7.3667	0.1357	4.9955	36.8005	14	7.0757	0.1413	40.5047	0.0247	5.7245	0.1747	4.3624	24.9725
15	4.1772	0.2394	31.7725	0.0315	7.6061	0.1315	5.2789	40.1520	15	8.1371	0.1229	47.5804	0.0210	5.8474	0.1710	4.5650	26.6930
16	4.5950	0.2176	35.9497	0.0278	7.8237	0.1278	5.5493	43.4164	16	9.3576	0.1069	55.7175	0.0179	5.9542	0.1679	4.7522	28.2960
17	5.0545	0.1978	40.5447	0.0247	8.0216	0.1247	5.8071	46.5819	17	10.7613	0.0929	65.0751	0.0154	6.0472	0.1654	4.9251	29.7828
18	5.5599	0.1799	45.5992	0.0219	8.2014	0.1219	6.0526	49.6395	18	12.3755	0.0808	75.8364	0.0132	6.1280	0.1632	5.0843	31.1565
19	6.1159	0.1635	51.1591	0.0195	8.3649	0.1195	6.2861	52.5827	19	14.2318	0.0703	88.2118	0.0113	6.1982	0.1613	5.2307	32.4213
20	6.7275	0.1486	57.2750	0.0175	8.5136	0.1175	6.5081	55.4069	20	16.3665	0.0611	102.4436	0.0098	6.2593	0.1598	5.3651	33.5822
21	7.4002	0.1351	64.0025	0.0156	8.6487	0.1156	6.7189	58.1095	21	18.8215	0.0531	118.8101	0.0084	6.3125	0.1584	5.4883	34.6448
22	8.1403	0.1228	71.4027	0.0140	8.7715	0.1140	6.9189	60.6893	22	21.6447	0.0462	137.6316	0.0073	6.3587	0.1573	5.6010	35.6150
23	8.9543	0.1117	79.5430	0.0126	8.8832	0.1126	7.1085	63.1462	23	24.8915	0.0402	159.2764	0.0063	6.3988	0.1563	5.7040	36.4988
24	9.8497	0.1015	88.4973	0.0113	8.9847	0.1113	7.2881	65.4813	24	28.6252	0.0349	184.1678	0.0054	6.4338	0.1554	5.7979	37.3023
25	10.8347	0.0923	98.3471	0.0102	9.0770	0.1102	7.4580	67.6964	25	32.9190	0.0304	212.7930	0.0047	6.4641	0.1547	5.8834	38.0314
26	11.9182	0.0839	109.1818	0.0092	9.1609	0.1092	7.6186	69.7940	26	37.8568	0.0264	245.7120	0.0041	6.4906	0.1541	5.9612	38.6918
27	13.1100	0.0763	121.0999	0.0083	9.2372	0.1083	7.7704	71.7773	27	43.5353	0.0230	283.5688	0.0035	6.5135	0.1535	6.0319	39.2890
28	14.4210	0.0693	134.2099	0.0075	9.3066	0.1075	7.9137	73.6495	28	50.0656	0.0200	327.1041	0.0031	6.5335	0.1531	6.0960	39.8283
29	15.8631	0.0630	148.6309	0.0067	9.3696	0.1067	8.0489	75.4146	29	57.5755	0.0174	377.1697	0.0027	6.5509	0.1527	6.1541	40.3146
30	17.4494	0.0573	164.4940	0.0061	9.4269	0.1061	8.1762	77.0766	30	66.2118	0.0151	434.7451	0.0023	6.5660	0.1523	6.2066	40.7526