ISE 307: Engineering Economic Analysis

Lecture 24: Replacement Decisions (Ch 12)

Term 253 (Summer)

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Lecture 24

Defender vs. Challenger, Economic Service Life,
and When to Replace

Lecture 24 Objectives

1. Frame the choice as **defender** vs. **challenger**; ignore **sunk costs**
2. Apply the **opportunity-cost (outsider) viewpoint**: the defender's investment is its *current market value*
3. Compare defender and challenger by **AE** over a common period
4. Compute the **economic service life** N^* that minimizes $AEC = CR(i) +$ operating AE
5. Decide **replace now vs. later** with the marginal one-more-year rule

Defender, Challenger, and Sunk Costs

- **Defender**: the asset we own and operate now
- **Challenger**: the best replacement available on the market

The company's ore-transfer truck:

	amount (SAR)	relevant?
purchase price, 3 years ago	85,000	**no** — sunk
engine repair, last year	6,000	**no** — sunk
book value on the company's ledger	34,000	**no** — accounting, not cash
market value today	**28,000**	**yes**

Only **future** cash flows count. What the defender *cost* is history; what it *would sell for today* is a real, forgone amount.

The Opportunity-Cost (Outsider) Viewpoint

Treat the defender's **current market value** as its **investment**. Keeping the truck = choosing *not* to collect 28,000 today — the same sacrifice as spending it.

- Ask: *what would an outsider pay to step into our position?* The outsider owns nothing — to get the defender's service they must "buy" it at market value
- Puts defender and challenger on **equal footing**: each option starts with its own investment at $n = 0$
- Both options are **service projects** (same haulage either way) $\Rightarrow$ compare by **AEC**; least cost wins

Worked Example: The Defender's AEC

Keep the truck 3 more years ($i = 10\%$): investment = **28,000** (market value now); operating costs **14,000 / 16,000 / 18,000**; salvage **10,000** at year 3:

1. **Capital:** $CR = (28{,}000 - 10{,}000)(A/P, 10\%, 3) + 0.10(10{,}000)$
   $= 18{,}000\,(0.4021) + 1{,}000 = 8{,}238$
2. **Operating:** $PW_{\text{op}} = 14{,}000(0.9091) + 16{,}000(0.8264) + 18{,}000(0.7513)$
   $= 12{,}727 + 13{,}222 + 13{,}523 = 39{,}473 \;\Rightarrow\; AE_{\text{op}} = 39{,}473\,(0.4021) = 15{,}873$
3. **Total:** $AEC_D = 8{,}238 + 15{,}873 = \boxed{\text{SAR } 24{,}111/\text{yr}}$

Worked Example: The Challenger's AEC

The challenger truck: price **72,000**; operating costs **8,000 / 11,000 / 15,000**; resale **33,000** after 3 years:

1. $CR = (72{,}000 - 33{,}000)(0.4021) + 0.10(33{,}000) = 15{,}682 + 3{,}300 = 18{,}982$
2. $PW_{\text{op}} = 8{,}000(0.9091) + 11{,}000(0.8264) + 15{,}000(0.7513) = 27{,}633$
   $\Rightarrow\ AE_{\text{op}} = 27{,}633\,(0.4021) = 11{,}112$
3. $AEC_C = 18{,}982 + 11{,}112 = \boxed{\text{SAR } 30{,}094/\text{yr}}$

$AEC_D = 24{,}111 < AEC_C = 30{,}094 \Rightarrow$ **keep the defender** (for now). Cheaper to run either option than to *own* the new one.

Economic Service Life: The Idea

How long should you keep an asset? Two opposing cost currents as the keep-period N grows:

- **Capital recovery** $CR(i) = (I - S_N)(A/P, i, N) + iS_N$ **falls** with N (price spread over more years)
- **Operating AE rises** with N (aging: maintenance, fuel, downtime)

$$AEC(N) = \underbrace{CR(i)}_{\text{falls}} + \underbrace{\left(\sum_{n=1}^{N} OC_n(P/F, i, n)\right)(A/P, i, N)}_{\text{rises}}$$

The **economic service life** N^* is the N that **minimizes** $AEC(N)$.

Economic Service Life: The Challenger Truck

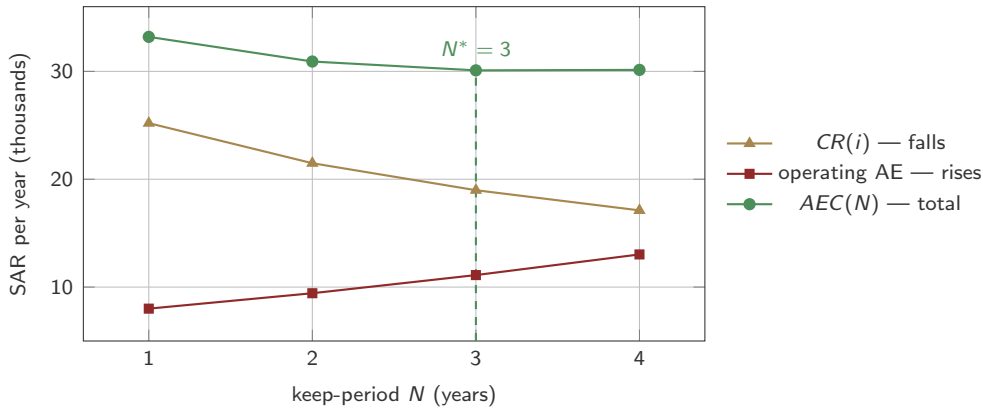
$I = 72{,}000$; $i = 10\%$; market value S_N and operating cost OC_n by year:

keep N	S_N	OC_N	$CR(10\%)$	op. AE	$AEC(N)$	
1	54,000	8,000	25,200	8,000	33,200	
2	42,000	11,000	21,486	9,429	30,914	
3	33,000	15,000	18,982	11,112	**30,094**	$\leftarrow N^*$
4	26,000	20,000	17,112	13,027	30,139	

N=1: $(72{,}000 - 54{,}000)(1.10) + 0.1(54{,}000) + 8{,}000 = 33{,}200$. N=2: $30{,}000(0.5762) + 4{,}200 = 21{,}486$; ops: $[8{,}000(0.9091) + 11{,}000(0.8264)](0.5762) = 9{,}429$.

$N^* = 3$ years; $AEC^* = 30{,}094$/yr = the challenger's **best possible annual cost** (its benchmark).

$AEC(N)$: Falling Capital, Rising Operations



The bottom is **flat** (30,094 vs. 30,139): being a year off N^* costs little — but the *benchmark value* AEC^* drives the replace decision.

Replace Now or Later? The One-More-Year Rule

What does it cost to keep the **defender one more year**? Three pieces (all future flows):

$$MC_{1\text{ yr}} = \underbrace{MV_{\text{now}}(1+i)}_{\text{forgone sale, with interest}} - \underbrace{MV_{\text{in 1 yr}}}_{\text{recovered at year-end}} + \underbrace{OC_{\text{next yr}}}_{\text{operations}}$$

The current truck: $MV_{\text{now}} = 28{,}000$; $MV_{\text{in 1 yr}} = 22{,}000$; $OC = 14{,}000$:

$$MC = 28{,}000\,(1.10) - 22{,}000 + 14{,}000 = 30{,}800 - 22{,}000 + 14{,}000 = \boxed{22{,}800}$$

$MC = 22{,}800 < AEC^{*}_{C} = 30{,}094 \Rightarrow$ **keep the defender this year**; re-run the check next year.

Year by Year: When Does the Verdict Flip?

Project the defender's marginal cost forward (MV path: 28,000 $\rightarrow$ 22,000 $\rightarrow$ 15,500 $\rightarrow$ 10,000):

keep year	$MV(1.10)$	$- MV_{\text{end}}$	$+ OC$	marginal cost
1	30,800	22,000	14,000	22,800
2	24,200	15,500	16,000	24,700
3	17,050	10,000	18,000	25,050

- Every year's marginal cost $< 30{,}094 \Rightarrow$ keep the truck to the end of its life, *then* buy the challenger
- Rule: **replace in the first year whose marginal cost exceeds** AEC_C^* — and recheck annually (prices and technology move)

Unequal Lives: AE under Repeatability

- Defender's remaining life (3 yrs) $\neq$ challenger's economic life (3 yrs here — but often they differ)
- Fix: compare **AEC over each option's own life**, under two assumptions:
  - the service is needed **indefinitely** (the hauling doesn't stop)
  - each asset is **replaced by an identical one** at the end of its life (*repeatability*)
- Then each AEC is a level, repeating annual cost — **directly comparable** (Lecture 16's one-cycle AE argument)

Defender over its *remaining* years vs. challenger at its *economic* life: compare AEC_D with AEC_C^* — no LCM needed.

Practice Problem

The company's tailings pump (the defender), at $i = 10\%$:

- market value now **13,000 SAR**; market value in one year **10,000 SAR**
- operating cost next year **7,500 SAR**
- the best challenger's minimum $AEC^* =$ **11,000 SAR/yr** (at its economic life)

(a) Compute the defender's marginal cost of one more year.

(b) Keep or replace now? One decision sentence.

(c) At what next-year operating cost would the verdict flip?

Practice Problem: Solution

(a) Three pieces:

$$MC = 13{,}000\,(1.10) - 10{,}000 + 7{,}500 = 14{,}300 - 10{,}000 + 7{,}500 = \boxed{11{,}800}$$

(b) $MC = 11{,}800 > AEC_C^* = 11{,}000 \Rightarrow$ **replace now**: one more year of the pump costs more than the challenger's steady annual cost.

(c) Flip when $MC = 11{,}000$:

$$OC^* = 11{,}000 - 14{,}300 + 10{,}000 = \boxed{6{,}700}$$

(A break-even solve — Lecture 23's move, applied to a replacement input. Note the sunk purchase price never appeared.)

Summary

- **Defender** = what we own; **challenger** = best on the market; **sunk costs never enter**
- **Outsider view**: the defender's investment is its **current market value**
- Both deliver the same service $\Rightarrow$ compare **AEC**; least cost wins
- **Economic service life** N^* minimizes $AEC(N) = CR(i)+$ operating AE (falling + rising $\Rightarrow$ interior minimum)
- **Replace** in the first year the defender's **one-more-year marginal cost** exceeds the challenger's AEC^*; with unequal lives, AE under **repeatability**

Before Class

To do before our class meeting:

1. **Read Sections 12.1–12.3** (defender vs. challenger, economic service life, when to replace)
2. Complete the **Lecture 24 pre-class quiz** on Blackboard before class
3. Warm-up: $MV_{\text{now}} = 9{,}000$; $MV_{\text{in 1 yr}} = 6{,}500$; $OC = 4{,}000$; $i = 10\%$ — marginal cost of one more year?

In class (ICA-24): "Replace the truck?" — build a challenger's economic-life table, run the defender's marginal check, then a break-even question: how far can the defender's operating cost rise? Ch 12 closes the final-exam material.