ISE 307: Engineering Economic Analysis

Lecture 23: Handling Project Uncertainty (Ch 11)

Term 253 (Summer)

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Lecture 23

Sensitivity, Break-Even, and Scenario Analysis

Lecture 23 Objectives

1. Explain why decisions built on **point estimates** can mislead
2. Run a **one-way sensitivity analysis** and read a **sensitivity graph**
3. Rank inputs by impact — which estimate **dominates** the PW
4. Find **break-even values** by solving $PW = 0$ for one input, and the **break-even quantity** Q^*
5. Build a **scenario table** and compute the **expected PW** over states
6. Measure the spread with **Var[PW]** and σ, and read loss probabilities from the **standardized normal**

Why Point Estimates Mislead

- Our habit so far: one "best estimate" per input $\Rightarrow$ **one** PW $\Rightarrow$ accept/reject
- Two problems with that habit:
  1. no guarantee the estimates will *match the actual values*
  2. no measure of *risk*: how likely is the project to lose money?
- **What-if analysis**: if demand is 10% lower, what happens to PW?

A single PW hides how *fragile* that PW is. Chapter 11 measures that with sensitivity analysis.

The Base Case: A Mobile Crushing Unit

A mining company plans a mobile ore-crushing unit, with most-likely estimates:

investment	55,000 SAR	annual revenue	42,000 SAR
annual operating cost	26,000 SAR	salvage (yr 5)	5,000 SAR
life	5 years	MARR	10%

$$PW = -55{,}000 + (42{,}000 - 26{,}000)(P/A, 10\%, 5) + 5{,}000\,(P/F, 10\%, 5)$$

$$= -55{,}000 + 16{,}000\,(3.7908) + 5{,}000\,(0.6209) = \boxed{+8{,}757}$$

Base-case decision: **accept**.

Today's question: *how much of a mis-estimate does that decision is still correct?*

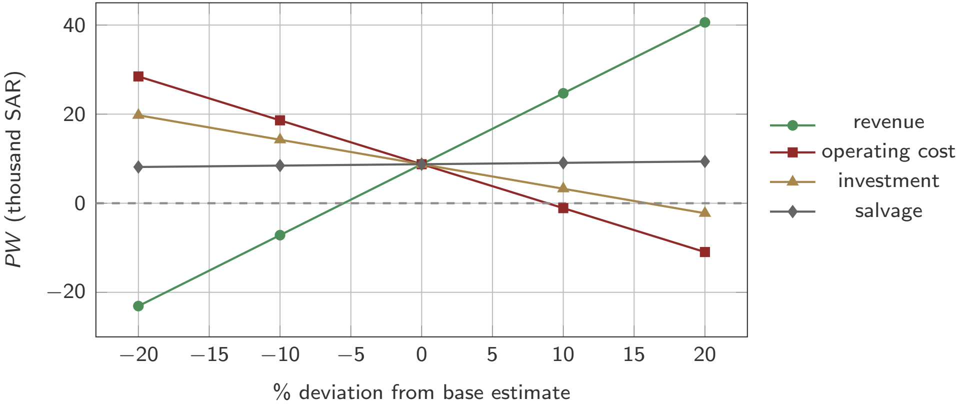
One-Way Sensitivity Analysis

Vary **one input at a time** by $\pm 10\%$ and $\pm 20\%$; recompute PW; hold everything else at base:

PW (SAR)	−20%	−10%	base	+10%	+20%
annual revenue	−23,085	−7,164	8,757	24,678	40,600
operating cost	28,469	18,613	8,757	−1,099	−10,955
investment	19,757	14,257	8,757	3,257	−2,243
salvage	8,136	8,447	8,757	9,068	9,378

A 10% revenue slip already turns the PW **negative** (−7,164); a 20% salvage error moves it by only ± 621.

The Sensitivity Graph



All lines cross at the base case. **The steeper the line, the more dangerous the input.** Where a line crosses $PW = 0$ is that input's *break-even* point.

Which Input Dominates?

Rank inputs by **total PW changes** across (tornado-style, widest bar on top):

rank	input	PW range (SAR)	changes/swing
1	annual revenue	$-23{,}085 \rightarrow 40{,}600$	**63,685**
2	operating cost	$-10{,}955 \rightarrow 28{,}469$	39,424
3	investment	$-2{,}243 \rightarrow 19{,}757$	22,000
4	salvage	$8{,}136 \rightarrow 9{,}378$	1,242

Estimation effort and management attention go to the **top of the ranking**: firm up the revenue forecast; don't waste a week refining the salvage guess.

Break-Even Analysis: Solve $PW = 0$

How far can the annual net A fall before it is unacceptable? Set $PW(A) = 0$:

$$-55{,}000 + A^*(3.7908) + 5{,}000\,(0.6209) = 0$$

$$A^* = \frac{55{,}000 - 3{,}104}{3.7908} = \boxed{13{,}690 \text{ SAR/yr}} \quad (\text{base: } 16{,}000)$$

- **Cushion**: $(16{,}000 - 13{,}690)/16{,}000 = \mathbf{14.4\%}$ — the net can slip that far
- Revenue terms: $R^* = 13{,}690 + 26{,}000 = 39{,}690$ — only **5.5%** below base
- Check: $PW(13{,}690) = -55{,}000 + 51{,}896 + 3{,}104 \approx 0$ ✓

Break-Even Quantity: When the Input Is a Volume

Often the uncertain input is **how much we sell**. With a price p and variable cost v per unit, each unit contributes $p - v$ toward the **fixed cost** FC per year:

Break-even quantity: $Q^* = \dfrac{FC}{p - v}$ units per year — below Q^* the year runs at a loss.

The company also crushes for outside clients: charge $p = 12$ SAR/tonne, variable cost $v = 8$ SAR/tonne, fixed cost $FC = 96{,}000$ SAR/yr:

$$Q^* = \frac{96{,}000}{12 - 8} = \boxed{24{,}000 \text{ tonnes/yr}}$$

To respect the time value of money, put the **capital recovery cost** $CR(i)$ into FC — then Q^* is the volume at which $AE = 0$, the annual twin of the $PW = 0$ solve.

Scenario Analysis

One-way analysis moves inputs *alone*; in a bad year they move *together*. Bundle them into coherent stories:

	pessimistic	most likely	optimistic
annual revenue	36,000	42,000	46,000
operating cost	28,000	26,000	24,000
salvage	3,000	5,000	7,000
annual net	8,000	16,000	22,000
$PW(10\%)$	−**22,811**	+**8,757**	+**32,744**

The spread (−22,811 to +32,744) *is* the project's risk picture — one number per coherent story.

Attaching Probabilities: Expected PW

The planning team's read: $P(\text{pess}) = 0.25$, $P(\text{likely}) = 0.50$, $P(\text{opt}) = 0.25$:

$$E[PW] = \sum_j p_j \, PW_j = 0.25(-22{,}811) + 0.50(8{,}757) + 0.25(32{,}744)$$

$$= -5{,}703 + 4{,}379 + 8{,}186 = \boxed{+6{,}862}$$

- $E[PW] > 0$: **accept on average** ...
- ... but there is a **25% chance of losing** $\approx$ 22,800 (the pessimistic state)

Report **both**: the center ($E[PW]$) and the risk (probability and size of the losing state). A risk-averse owner may demand more than a positive average.

Measuring the Spread: Variance and Standard Deviation

$$\mathrm{Var}[PW] = \sum_j p_j \left(PW_j - E[PW]\right)^2$$
$$\sigma = \sqrt{\mathrm{Var}[PW]} \text{ (same money units as } PW\text{.)}$$

The crushing unit ($E[PW] = 6{,}862$):

scenario	p_j	PW_j	$PW_j - E[PW]$	$p_j(PW_j - E[PW])^2$
pessimistic	0.25	−22,811	−29,673	220.1×10^6
most likely	0.50	8,757	1,895	1.8×10^6
optimistic	0.25	32,744	25,882	167.5×10^6
			$\mathrm{Var}[PW]$	389.4×10^6

$$\sigma = \sqrt{389.4 \times 10^6} = \boxed{19{,}733 \text{ SAR}}$$

Report both: $E[PW] = 6{,}862 \pm 19{,}733$ (stdev $\approx$ 3 times mean = is risky.

From Spread to Probability: The Standardized Normal

Model PW as **normal** with mean $\mu = E[PW]$ and standard deviation σ. Then:

$$
\begin{aligned}
z &= \frac{x-\mu}{\sigma} \\
P(PW \le x) &= \Phi(z) \\
\Phi(-z) &= 1 - \Phi(z) \quad (\Phi : \text{standard normal table})
\end{aligned}
$$

Chance the crushing unit loses money ($x = 0$; $\mu = 6{,}862$, $\sigma = 19{,}733$):

$$z = \frac{0 - 6{,}862}{19{,}733} = -0.35 \quad \Rightarrow \quad P(PW < 0) = \Phi(-0.35) = 1 - \Phi(0.35) = 1 - 0.6368 = \boxed{0.36}$$

The discrete table said 25% (the pessimistic state); the normal model, using only μ and σ, says 36%. Both are estimates of the same risk — state which model you used.

Practice Problem

The company studies a satellite ore-testing lab (MARR $= 10\%$):

- investment **60,000 SAR**; salvage **6,000 SAR** after **5 years**
- most-likely annual net cash flow: **18,000 SAR/yr**

(a) Compute the base-case PW.

(b) Find the break-even annual net A^* and the % cushion.

(c) If the net comes in 15% below the estimate, is the lab still acceptable?

Practice Problem: Solution

(a) Base case:

$$PW = -60{,}000 + 18{,}000\,(3.7908) + 6{,}000\,(0.6209) = \boxed{+11{,}960}$$

(b) Break-even: $A^*(3.7908) = 60{,}000 - 3{,}725$:

$$A^* = \frac{56{,}275}{3.7908} = \boxed{14{,}845} \qquad \text{cushion} = \frac{18{,}000 - 14{,}845}{18{,}000} = \mathbf{17.5\%}$$

(c) At -15%: $A = 15{,}300 > A^* \Rightarrow$ still acceptable:

$$PW = -60{,}000 + 15{,}300\,(3.7908) + 3{,}725 = \boxed{+1{,}725 > 0}\ \checkmark$$

Quick Check

From the crushing unit's sensitivity analysis, which statement is **INCORRECT**?

(A) The PW responds more to revenue than to any other input

(B) The salvage line is nearly flat, so refining that estimate adds little

(C) The project stays acceptable if revenue falls 10%

(D) The operating-cost line slopes downward: higher cost, lower PW

Answer: (C). At -10% revenue, $PW = -7{,}164 < 0$, the revenue cushion is only 5.5%.

Summary

- Point estimates give **no risk measure** — ask "what if" instead
- **Sensitivity**: vary one input at a time; the **steepest line** on the graph is the most dangerous input; rank inputs by PW swing
- **Break-even**: set $PW = 0$, solve for the input, report the **% cushion**; for volumes, $Q^* = FC/(p - v)$
- **Scenario**: categorize: pessimistic / most likely / optimistic, each with PW
- With probabilities: $E[PW] = \sum p_j PW_j$; $\mathrm{Var}[PW]$, $\sigma = \sqrt{\mathrm{Var}[PW]}$, report the **mean and the stdev**
- Normal model: $z = (x - \mu)/\sigma$; $P(PW \leq x) = \Phi(z)$; $\Phi(-z) = 1 - \Phi(z)$

Before Class

To do before our class meeting:

1. **Read Sections 11.1–11.2** (sensitivity, break-even, scenario) + Section 11.3 (expected value, variance, risk probabilities)
2. Complete the **Lecture 23 pre-class quiz** on Blackboard before class