ISE 307: Engineering Economic Analysis

Lecture 22: Income Taxes (Ch 9)

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Lecture 22

From Profit to Cash:
Income Taxes in Project Analysis

Lecture 22 Objectives

1. Compute **taxable income**, the **tax**, and **net income**
2. Distinguish the **marginal** from the **average** tax rate
3. Build the **after-tax cash flow**: $\text{ATCF} = (R - E - D)(1 - t) + D$
4. Quantify the **depreciation tax shield**, $t \times D$
5. Handle **disposal**: gain $= S - BV$; net proceeds $= S - t\,(S - BV)$

How a Project's Profit Is Taxed

Taxable income = revenues − expenses − depreciation:

$$TI = R - E - D$$

$$\textbf{Income tax} = t \times TI = TI - \text{tax} = TI\,(1 - t) \qquad \textbf{Net income}$$

- R: gross revenues from products or services
- E: *cash* operating expenses — labor, material, rent, interest on debt
- D: the depreciation allowance — the **capital cost enters here**, never as a lump-sum expense
- t: the firm's tax rate (we use a single **marginal rate**)

Marginal vs. Average Tax Rate

Corporate tax schedules are often **progressive** (stair-step brackets). The textbook's U.S. example, used here only to show the idea:

bracket	rate	tax on $TI = 100{,}000$
first 50,000	15%	7,500
next 25,000	25%	6,250
above 75,000	34%	$0.34 \times 25{,}000 = 8{,}500$
		22,250

- **Marginal rate**: on the *last* unit earned = 34% — the rate that matters for a *new project*
- **Average rate**: total tax / total income = 22,250/100,000 = 22.25%

Example 1: Income Statement

The copper company, one operating year, marginal rate $t = 25\%$:

Revenues R	300,000 SAR
Cash operating expenses E	$-180{,}000$
Depreciation D	$-40{,}000$
Taxable income TI	**80,000**
Income tax (25%)	$-20{,}000$
Net income	**60,000 SAR**

Net income is the *accounting* answer. Is 60,000 the cash the company banked this year? **Not quite**. We will discuss in next slide.

After-Tax Cash Flow (ATCF)

Depreciation was deducted, but **no cash left the firm**. Add it back:

After-tax cash flow:

$$\text{ATCF} = \text{net income} + D = (R - E - D)(1 - t) + D$$

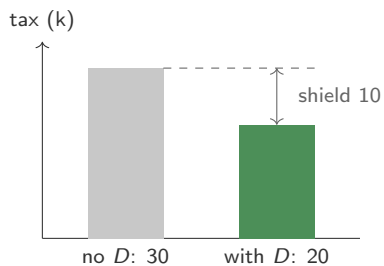
For our example, $\text{ATCF} = 60{,}000 + 40{,}000 = \boxed{100{,}000 \text{ SAR}}$

$$\begin{aligned} ATCF &= \underbrace{(R - E)(1 - t)}_{\text{after-tax operating profit}} + \underbrace{t\,D}_{\text{tax shield}} \\ &= (300{,}000 - 180{,}000)(1 - 0.25) + 0.25 \times 40{,}000 \\ &= 120{,}000 \times 0.75 + 10{,}000 = 90{,}000 + 10{,}000 = \boxed{100{,}000 \text{ SAR}} \end{aligned}$$

The Depreciation Tax Shield

Each SAR of depreciation cuts taxable income by 1 SAR, saving t in tax: **shield** $= t \times D$.

Tax bill, with and without D:



- Without depreciation: tax $= 0.25\,(120{,}000) = 30{,}000$
- With depreciation: tax $= 0.25 \times 80{,}000 = 20{,}000$
- Savings: $t\,D = 0.25 \times 40{,}000 = \mathbf{10{,}000}$
- Total D is fixed — so accelerated schedules (DDB, MACRS) deliver the shields **earlier**, and earlier money is worth more

Disposal: Gains and Losses

When an asset is **sold**, compare the selling price S with the **book value** BV at that moment:

- $S > BV$: **taxable gain** $= S - BV$ ("depreciation recapture"), taxed at t
- $S < BV$: **deductible loss** $= BV - S$ — offsets other income, so it *saves* $t\,(BV - S)$

One formula covers both cases: **net proceeds** $= S - t\,(S - BV)$

Intuition: the gain tax *settles the account*. If depreciation claimed more loss than reality delivered, the difference is taxed back; if it claimed less, the loss credit refunds it.

Example 2: Selling the Flotation Cell

After 3 SL years, the flotation cell's book value is $BV_3 = 23{,}000$ SAR (Lecture 21). $t = 25\%$. Two offers:

Offer A: $S = 30{,}000$ (above book value)

$$\text{gain} = 30{,}000 - 23{,}000 = 7{,}000; \quad \text{tax} = 0.25\,(7{,}000) = 1{,}750$$

$$\text{net proceeds} = 30{,}000 - 1{,}750 = \boxed{28{,}250 \text{ SAR}}$$

Offer B: $S = 18{,}000$ (below book value)

$$\text{loss} = 23{,}000 - 18{,}000 = 5{,}000; \quad \text{credit} = 0.25\,(5{,}000) = 1{,}250$$

$$\text{net proceeds} = 18{,}000 + 1{,}250 = \boxed{19{,}250 \text{ SAR}}$$

Example 3: A Full After-Tax Project (Setup)

The company considers a **concentrate bagging line**: cost basis **60,000 SAR**; SL over **4 years** to $S = 12{,}000$; extra revenue **50,000/yr**; cash expenses **20,000/yr**; $t = 25\%$; MARR $= 10\%$.

$$D = \frac{60{,}000 - 12{,}000}{4} = 12{,}000 \text{ SAR/yr}$$

years 1–4	$R - E - D$	tax (25%)	net income	**ATCF** $=$ NI $+D$
	$30{,}000 - 12{,}000 = 18{,}000$	4,500	13,500	**25,500**

Every operating year contributes **25,500 SAR** after tax. One flow is still missing: the **sale** at the end of year 4.

Example 3: Disposal and the After-Tax Decision

At the end of year 4 the line actually sells for **15,000 SAR**; $BV_4 = 12{,}000$:

$$\text{net proceeds} = 15{,}000 - 0.25\,(15{,}000 - 12{,}000) = 15{,}000 - 750 = 14{,}250$$

After-tax cash flow diagram: $-60{,}000$ now; $+25{,}500$ in years 1–4; $+14{,}250$ extra at year 4.

$$PW(10\%) = -60{,}000 + 25{,}500\,(P/A,\ 10\%,\ 4) + 14{,}250\,(P/F,\ 10\%,\ 4)$$

$$= -60{,}000 + 25{,}500\,(3.1699) + 14{,}250\,(0.6830) = \boxed{+30{,}565 \text{ SAR}}$$

$PW > 0$ at MARR $\Rightarrow$ **accept**. This is the full after-tax workflow: depreciate $\rightarrow$ tax $\rightarrow$ ATCF $\rightarrow$ net proceeds $\rightarrow$ PW.

Practice Problem 1

(a) A firm reports revenues of **180,000 SAR**, cash expenses of **100,000 SAR**, and depreciation of **25,000 SAR**; its marginal rate is **30%**. Run the pipeline: taxable income $\rightarrow$ tax $\rightarrow$ net income $\rightarrow$ **ATCF**.

(b) An asset with book value **14,000 SAR** is sold for **22,000 SAR**; gains are taxed at **25%**. Find the **net proceeds**.

Practice Problem 1: Solution

(a) The pipeline:

$$TI = 180{,}000 - 100{,}000 - 25{,}000 = 55{,}000$$

$$\text{tax} = 0.30\,(55{,}000) = 16{,}500; \qquad \text{NI} = 55{,}000 - 16{,}500 = 38{,}500$$

$$\text{ATCF} = 38{,}500 + 25{,}000 = \boxed{63{,}500 \text{ SAR}}$$

Check: $(R-E)(1-t)+tD = 80{,}000\,(0.70)+0.30\,(25{,}000) = 56{,}000+7{,}500 = 63{,}500$

(b) gain $= 22{,}000 - 14{,}000 = 8{,}000$; tax $= 0.25\,(8{,}000) = 2{,}000$:

$$\text{net proceeds} = 22{,}000 - 2{,}000 = \boxed{20{,}000 \text{ SAR}}$$

Quick Check

A firm's depreciation charge **doubles** this year; revenues, expenses, and the tax rate are unchanged. What happens?

(A) Net income falls and ATCF falls

(B) Net income falls and ATCF **rises**

(C) Net income rises and ATCF rises

(D) Neither changes — depreciation is non-cash

Answer: (B). Extra depreciation ΔD cuts net income by $\Delta D\,(1-t)$ but *adds* $t\,\Delta D$ of cash — the tax shield. Accounting profit and cash move in **opposite** directions.

Summary

- Pipeline: $TI = R - E - D$; tax $= t\,TI$; net income $= TI\,(1 - t)$
- **Marginal** rate (last unit; use for projects) vs. **average** rate (total tax / total income)
- **ATCF** $=$ NI $+D = (R - E - D)(1 - t) + D = (R - E)(1 - t) + t\,D$
- **Tax shield** $= t\,D$: same total under every method — accelerated schedules deliver it *earlier*
- **Disposal**: gain $= S - BV$; **net proceeds** $= S - t\,(S - BV)$ (works for gains and losses)

Before Class

To do before our class meeting:

1. **Read Section 9.4** (corporate taxes, net income, gains on disposal)
2. Complete the **Lecture 22 pre-class quiz** on Blackboard before class
3. Warm-up: $R = 140{,}000$, $E = 65{,}000$, $D = 30{,}000$, $t = 20\%$ — find the ATCF