ISE 307: Engineering Economic Analysis

Lecture 19: Major Exam II Review (Chapters 5–7)

Term 253 (Summer)

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Lecture 19

Objectives Recap, the Formula Sheet,
and Fully Worked Practice

Major Exam II: Format and Rules

- **75 minutes**, in class, covering **Chapters 5–7**
- **Part A: 8 MCQs** (3 pts each) + **Part B: 3 long problems with sub-parts** (76 pts)
- Long problems give **partial credit for shown work**
- Provided: the **Ch 5–7 formula sheet** and **factor tables**
- Non-programmable scientific calculator only

Lecture Summaries (Chapter 5, Lectures 12–14)

Lecture	**You should be able to . . .**
L12 (Ch 5)	state the MARR's role; compute payback and DPP; apply the NPW criterion
L13 (Ch 5)	compute NFW; build a project balance table; value perpetual projects with CE
L14 (Ch 5)	frame mutually exclusive choices (do-nothing, service vs revenue); compare over equal lives, the LCM, or a stated study period

Lecture Summaries (Chapters 6–7, Lectures 15–18)

Lecture	**You should be able to ...**
L15 (Ch 6)	apply the AE criterion; separate capital from operating costs; compute $CR(i)$ and a full AEC; annualize perpetual assets
L16 (Ch 6)	compute unit cost and minimum price; settle make-or-buy; find break-even usage; compare unequal lives by AE
L17 (Ch 7)	define IRR; find i^* (root, quadratic, interpolation); classify simple vs nonsimple; apply IRR vs MARR
L18 (Ch 7)	explain why IRR cannot rank; run incremental analysis and the champion–challenger tournament
L19 (today)	review materials

The Formula Sheet (Ch 5–7)

Screening and the Present-Worth Family

You want ...	**Formula**	**Trigger words**
payback	smallest n: $\sum A_k \geq 0$	"how fast is it recovered"
DPP	same, with $A_k(1+i)^{-k}$	"payback *considering interest*"
NPW	$\sum A_n(1+i)^{-n}$ at MARR	"should we accept" (single project)
NFW	$PW \times (F/P, i, N)$	"worth *at the end*"
project balance	$PB_n = PB_{n-1}(1+i) + A_n$	"balance at year n," table with a blank
CE	A/i (+ initial cost)	"forever," "perpetual service"

DPP fraction = remaining deficit ÷ that year's discounted flow.
Terminal PB_N = NFW.

Mutually Exclusive Choices (Ch 5)

Situation	Rule	Trigger
revenue projects	maximize NPW	incomes differ
service projects	minimize PW of cost	"must install," same output
unequal lives	replicate to the **LCM**	lives differ, no horizon stated
stated study period	use it for *all*; salvage at cutoff	"plant closes in N years"
indifference value	set $PW_A = PW_B$, solve X	"what X makes them equal"

Least-negative wins for service projects: $-32{,}000$ beats $-44{,}000$.

The Annual-Equivalence Family (Ch 6)

You want ...	**Formula**	**Trigger**
AE	$PW \times (A/P, i, N)$	"per year"
ownership cost	$CR = (I - S)(A/P) + iS$	"annual cost of owning"
full AEC	CR + operating (PW $\rightarrow A/P$)	mixed cost problems
perpetual asset	$I \cdot i$ + O&M + $C(A/F, i, k)$	infinite life, periodic repair
unit cost / min price	$AEC \div$ volume	"minimum price," "per unit"
break-even usage	$AEC_1(h) = AEC_2(h)$	"hours to justify..."
unequal lives	one-cycle AE each	lives differ, identical repetition

Rate of Return (Ch 7)

Situation	**Method / rule**	**Trigger**
single future flow	$i^* = (F/P)^{1/N} - 1$	two amounts + time
two periods	quadratic in $x = \frac{1}{1+i}$	3-row cash flow table
general series	bracket + interpolate	"find the IRR"
classification	count sign changes	"simple or nonsimple?"
decision (simple)	IRR $\gtrless$ MARR	accept/reject wording
nonsimple	abandon IRR; use PW/AE	multiple sign changes
mutually exclusive	incremental i^*_Δ vs. MARR	"which one," 2+ alternatives

At i^*: NPW $= 0$ (by definition). No sign change $\Rightarrow$ no ROR exists.
Never rank by raw IRR.

Practice Problems
(Major 2 Problem Set A)

Problem 1(a)

A dewatering project has the following cash flows at $i = 10\%$:

n	0	1	2	3	4	5	6
A_n	$-6{,}000$	0	0	6,000	1,500	1,500	300

Find the **discounted payback period**.

Try it: build the cumulative discounted cash flow year by year.

Problem 1(a): Solution

Discount each flow, keep a running total:

n	0	1	2	3	4	5	6
$A_n(1.10)^{-n}$	$-6{,}000$	0	0	4,508	1,025	931	169
cumulative	$-6{,}000$	$-6{,}000$	$-6{,}000$	$-1{,}492$	-468	$+464$	$+633$

First nonnegative during year 5. Fraction $= 468/931 = 0.50$:

$$\boxed{\text{DPP} = 4.5 \text{ years}} \qquad (\text{and NPW} = +633 > 0\text{: acceptable})$$

Problem 1(b)

A screening plant asset at $i = 8\%$ per year: initial cost **25,000**; annual operating cost **4,500**; major maintenance **18,000** at end of year 3; salvage **3,500**; life **6 years**. Find the **net future worth**.

Try it: collapse to PW with (P/A) and (P/F), then carry forward with (F/P).

Problem 1(b): Solution

PW of each piece at 8%:

$$\begin{aligned}PW &= -25{,}000 - 4{,}500\,(P/A, 8\%, 6) - 18{,}000\,(P/F, 8\%, 3)\\ &\qquad + 3{,}500\,(P/F, 8\%, 6)\\ &= -25{,}000 - 4{,}500(4.6229) - 18{,}000(0.7938) + 3{,}500(0.6302)\\ &= -25{,}000 - 20{,}803 - 14{,}288 + 2{,}206 = -57{,}885\end{aligned}$$

$$\text{NFW} = PW\,(F/P, 8\%, 6) = -57{,}885\,(1.5869) = \boxed{-91{,}860}$$

A cost-only asset: NFW is negative by construction — the exam asks for its *magnitude and sign*, not a decision.

Problem 1(c)

A project ($-50{,}000$; then $18{,}000$/yr for 4 years) has this partial **project balance** table at MARR $= 10\%$:

n	0	1	2	3	4
PB_n	$-50{,}000$	$-37{,}000$	**?**	$-6{,}970$	$+10{,}333$

Fill in PB_2, and state the **DPP** and the **NFW**.

Try it: grow, then add.

Problem 1(c): Solution

Grow last year's balance at 10%, then add the year's flow:

$$
\begin{aligned}
PB_2 &= PB_1\,(1.10) + A_2 = -37{,}000\,(1.10) + 18{,}000 \\
&= -40{,}700 + 18{,}000 = \boxed{-22{,}700}
\end{aligned}
$$

DPP: first nonnegative PB is at $n = 4 \Rightarrow$ payback *during year 4*.
NFW $=$ terminal balance $PB_4 = \boxed{+10{,}333}$ — no extra computation needed.

Problem 2(a)

At $i = 10\%$, find the **capitalized equivalent cost** of: **12,000** at year 0; **4,000 per year** in years 1–5; and **1,500 per year thereafter, indefinitely**.

Try it: finite piece by (P/A)*; the perpetual tail by* A/i*, then discount the tail to year 0.*

Problem 2(a): Solution

Three pieces, all valued at year 0:

$$
\begin{aligned}
CE &= 12{,}000 + 4{,}000\,(P/A, 10\%, 5) + \frac{1{,}500}{0.10}\,(P/F, 10\%, 5) \\
&= 12{,}000 + 4{,}000(3.7908) + 15{,}000(0.6209) \\
&= 12{,}000 + 15{,}163 + 9{,}314 = \boxed{\text{SAR } 36{,}477}
\end{aligned}
$$

The perpetuity A/i sits **one period before** its first payment (year 5 here, first payment year 6) — so it still needs $(P/F, 10\%, 5)$.

Problem 2(b)

Two mutually exclusive projects, MARR $= 12\%$:

n	A	B
0	$-9{,}000$	$-8{,}000$
1	3,500	X
2	2,800	3,000
3	4,600	X

What value of X makes the decision maker **indifferent** between A and B?

Problem 2(b): Solution

$(P/F, 12\%)$: 0.8929, 0.7972, 0.7118. First, PW_A:

$$PW_A = -9{,}000 + 3{,}500(0.8929) + 2{,}800(0.7972) + 4{,}600(0.7118) = -369$$

Set $PW_B = PW_A$:

$$-8{,}000 + X(0.8929) + 3{,}000(0.7972) + X(0.7118) = -369$$

$$X\,(1.6047) = -369 + 8{,}000 - 2{,}392 = 5{,}239 \;\Rightarrow\; X = \boxed{\text{SAR } 3{,}265}$$

Problem 2(c)

A tank must stay lined for **6 years** ($i = 10\%$, a service project). **Liner A**: 4,000 SAR, lasts 6 years. **Liner B**: 2,500 SAR, lasts 3 years. Which liner, and by how much (in PW of cost)?

Try it: common horizon = 6 yrs; B is bought twice.

Problem 2(c): Solution

Replicate B over the 6-yr horizon (repurchase at year 3):

$$PW_B = 2{,}500 + 2{,}500\,(P/F, 10\%, 3) = 2{,}500 + 2{,}500(0.7513) = \boxed{4{,}378}$$

$$PW_A = \boxed{4{,}000}$$

Choose A — cheaper by 378 SAR in PW of cost over the common horizon. (The “cheap” liner loses once its repurchase is priced in.)

Problem 3(a): The Ownership Chain

Problem 3 works one asset from ownership cost to break-even.

A crew-cab truck is purchased for **24,000 SAR** and will be resold after **3 years** for **14,640 SAR**; $i = 6\%$. Find the annual **capital recovery cost**.

Problem 3(a): Solution

Formula, then substitute:

$$\begin{aligned} CR(6\%) &= (I - S)(A/P, 6\%, 3) + iS \\ &= (24{,}000 - 14{,}640)(0.3741) + 0.06(14{,}640) \\ &= 9{,}360\,(0.3741) + 878 = 3{,}502 + 878 = \boxed{\text{SAR } 4{,}380/\text{yr}} \end{aligned}$$

Owning the truck costs about 4,380/yr — the depreciation installments plus interest on the parked salvage.

Problem 3(b)

A CAD workstation cluster costs **120,000 SAR**, salvage **12,000** after **5 years**. Operating costs: **22,000/yr** for years 1–2, then **27,000/yr** for years 3–5. MARR $= 10\%$. Compute the **annual equivalent cost**.

Try it: CR for the capital; PW $\rightarrow (A/P)$ *for the stepped operating costs.*

Problem 3(b): Solution

1. **Capital:** $CR = (108{,}000)(A/P, 10\%, 5) + 0.10(12{,}000) = 28{,}490 + 1{,}200 = 29{,}690$
2. **Operating:** $PW_{\text{op}} = 22{,}000\,(P/A, 10\%, 2) + 27{,}000\,(P/A, 10\%, 3)(P/F, 10\%, 2)$
   $= 38{,}181 + 55{,}494 = 93{,}675 \;\Rightarrow\; AE_{\text{op}} = 93{,}675\,(0.2638) = 24{,}712$
3. **Total:** $AEC = 29{,}690 + 24{,}712 = \boxed{\text{SAR } 54{,}402/\text{yr}}$

(Decision use: buy only if the same capability costs more than $\approx$54,400/yr to rent or outsource.)

Problem 3(c)

A consumer-products plant will make **7.5 million units/yr** for **10 years**: investment **5,000,000**; operating **1,750,000/yr**; salvage **1,200,000**; MARR $= 25\%$. Find the **minimum unit price**.

Try it: CR + operating = AEC; then divide by volume.

Problem 3(c): Solution

1. $CR = (3{,}800{,}000)(A/P, 25\%, 10) + 0.25(1{,}200{,}000)$
   $= 3{,}800{,}000(0.2801) + 300{,}000 = 1{,}064{,}380 + 300{,}000 = 1{,}364{,}380$
2. $AEC = 1{,}364{,}380 + 1{,}750{,}000 = 3{,}114{,}380$ SAR/yr
3. minimum price $= \dfrac{3{,}114{,}380}{7{,}500{,}000} = \boxed{0.415 \text{ SAR/unit}}$

At 0.415 per unit the plant earns exactly 25%; pricing below it destroys value.

Problem 3(d)

Two slurry pumps ($i = 6\%$, 10-yr lives, no salvage). **P1**: 6,000 SAR, energy **0.48/hr**; **P2**: 4,800 SAR, energy **0.56/hr**. Annual maintenance = 15% of first cost. Find the **minimum operating hours per year** that justify buying the more expensive P1.

Try it: write $AE_1(h)$ and $AE_2(h)$, set equal, solve for h.

Problem 3(d): Solution

$(A/P, 6\%, 10) = 0.1359$. Fixed + variable for each pump:

$$AE_1(h) = 6{,}000(0.1359) + 900 + 0.48h = 1{,}715 + 0.48h$$
$$AE_2(h) = 4{,}800(0.1359) + 720 + 0.56h = 1{,}372 + 0.56h$$

Set equal:

$$1{,}715 + 0.48h = 1{,}372 + 0.56h \;\Rightarrow\; 0.08h = 343 \;\Rightarrow\; \boxed{h^* \approx 4{,}288 \text{ hrs/yr}}$$

Above 4,288 hrs/yr the efficient P1 wins; below it, the cheaper P2.

Problem 4(a)

Classify each cash flow series (simple / nonsimple / no ROR), and state what that implies about i^*:

	$n = 0$	1	2	3
D1	$-5{,}000$	$+2{,}000$	$+2{,}000$	$+2{,}000$
D2	$-3{,}000$	$+9{,}000$	$-7{,}000$	—
D3	$+2{,}000$	$+1{,}000$	$+500$	—

Try it: scan the signs left to right and count the changes.

Problem 4(a): Solution

- **D1**: $-, +, +, + \Rightarrow$ **one** sign change $\Rightarrow$ **simple**: a unique i^* exists and equals the IRR — the IRR rule applies
- **D2**: $-, +, - \Rightarrow$ **two** changes $\Rightarrow$ **nonsimple**: possibly multiple i^* — abandon IRR, decide by PW at the MARR
- **D3**: all positive $\Rightarrow$ **no** sign change $\Rightarrow$ **no rate of return exists** (nothing is ever invested)

Problem 4(b)

Find the IRR of this investment:

n	0	1	2
Cash flow	$-9{,}000$	$+4{,}000$	$+7{,}000$

Try it: quadratic in $x = 1/(1+i)$*; keep the positive root.*

Problem 4(b): Solution

With $x = \frac{1}{1+i}$: $-9{,}000 + 4{,}000x + 7{,}000x^2 = 0$:

$$x = \frac{-4{,}000 + \sqrt{4{,}000^2 + 4(7{,}000)(9{,}000)}}{2(7{,}000)} = \frac{-4{,}000 + 16{,}371}{14{,}000} = 0.8836$$

$$i^* = \frac{1}{0.8836} - 1 = \boxed{13.2\%}$$

One sign change $\Rightarrow$ simple $\Rightarrow$ this 13.2% *is* the IRR. (Accept iff MARR $<$ 13.2%.)

Problem 4(c)

A project costs **10,000** now and pays a single **25,000** at the end of year **5**. Three companies are interested, with MARRs of **15%**, **22%**, and **25%**. Find the **IRR** and state **which companies** should accept.

Try it: single flow $\Rightarrow$ take a root; then compare against each MARR.

Problem 4(c): Solution

One amount in, one out:

$$(1 + i^*)^5 = \frac{25{,}000}{10{,}000} = 2.5 \;\Rightarrow\; i^* = 2.5^{1/5} - 1 = \boxed{20.11\%}$$

Accept iff MARR $< 20.11\%$: the **15% firm accepts**; the 22% and 25% firms **reject**. Same project, different hurdles, different (correct) answers.

Problem 4(d)

Two mutually exclusive dryers (8-yr lives, no salvage; MARR $= 14\%$):

	Investment	Net annual	IRR
P	16,000	4,000	18.6%
Q	24,000	5,800	17.5%

Use **incremental analysis** (bracket with 14% and 16%) to choose.

Problem 4(d): Solution

Both IRRs $> 14\%$: both acceptable. Increment Q$-$P: $-8{,}000$; $+1{,}800$/yr (simple):

$$PW_\Delta(14\%) = -8{,}000 + 1{,}800\,(P/A, 14\%, 8) = -8{,}000 + 8{,}350 = +350$$
$$PW_\Delta(16\%) = -8{,}000 + 1{,}800\,(4.3436) = -182$$

$$i^*_\Delta \approx 14\% + 2\%\,\frac{350}{350+182} \approx 15.3\% \Rightarrow \boxed{\textbf{choose Q}}$$

Raw-IRR ranking would (wrongly) pick P. The extra 8,000 earns 15.3% — above the hurdle, so the larger project wins.

Problem 5: Recommend a Decision

A firm is offered a **10-year** project at MARR $= 12\%$. Its cash flows, listed separately (not netted):

- initial investment **40,000** at year 0
- revenue **13,000** per year, years 1–10
- operating cost **3,000** per year, years 1–10
- overhaul **5,000** at the end of years 4 and 8
- added compliance cost **2,500** per year, years 6–10

Should the firm undertake this project? Doing nothing — leaving the money to earn the MARR — is an acceptable choice.

Try it: put every separate flow on one timeline, reduce it to a single worth figure, then compare against doing nothing.

Problem 5: Solution

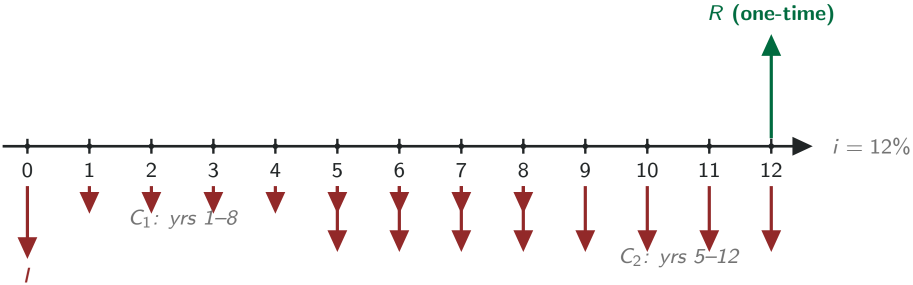
No method is prescribed — take present worth at 12% and compare to the do-nothing value of 0:

$$\begin{aligned} PW &= -40{,}000 + (13{,}000 - 3{,}000)\,(P/A, 12\%, 10) \\ &\quad - 5{,}000\big[(P/F, 12\%, 4) + (P/F, 12\%, 8)\big] \\ &\quad - 2{,}500\,(P/A, 12\%, 5)(P/F, 12\%, 5) \\ &= -40{,}000 + 10{,}000(5.6502) - 5{,}000(1.0394) - 2{,}500(2.0454) \\ &= -40{,}000 + 56{,}502 - 5{,}197 - 5{,}114 = \boxed{\text{SAR } +6{,}191} \end{aligned}$$

$PW > 0$, so the project beats doing nothing — **undertake it**. AE or FW would give the same accept: one criterion in different units.

Problem 6: A Mandated Project

A regulation **requires** the firm to install a treatment system (MARR $= 12\%$); declining is *not* allowed. Its cash flows:



A colleague computes the **IRR**, finds it positive, and says “the numbers work.” **(a)** Is IRR vs. MARR the right basis here? **(b)** State the rule that governs a mandated choice, and the quantity to minimize.

Problem 6: Solution

Worth at the MARR (a required project, so a cost):

$$\begin{aligned} PW(12\%) &= -30{,}000 - 4{,}000\,(P/A, 12\%, 8) \\ &\quad - 6{,}000\,(P/A, 12\%, 8)(P/F, 12\%, 4) + 220{,}000\,(P/F, 12\%, 12) \\ &= -30{,}000 - 19{,}870 - 18{,}943 + 56{,}469 = \boxed{-12{,}344} \end{aligned}$$

The IRR trap. One sign change $\Rightarrow$ a single $i^* \approx 9.4\%$. It is *positive* — which tempts the eye — but *below* the 12% MARR, so the IRR rule would **reject**, not accept.

(a) No. The project is mandated, so "reject" is off the menu; and $i^* <$ MARR anyway. **(b)** Compare the required alternatives on **PW (or AE) of cost** and pick the **least negative**. IRR cannot rank service projects.

Multiple Choice Questions (Major 2 Problem Set A)

MCQ 1

A 5-year project with an initial outlay and uniform annual inflows has a **DPP strictly before the end of year 5** at $i = 8\%$ ($=$ MARR). What can you conclude?

(A) The project is acceptable — $PW(8\%) > 0$
(B) The project is unacceptable
(C) Nothing — DPP ignores the time value of money
(D) The IRR is below 8%

MCQ 1: Solution

Answer: (A). DPP $< N$ means the *discounted* flows recover the investment before the project ends — the remaining years' inflows are pure surplus, so $PW(8\%) > 0$ and IRR $> 8\%$.

Caution: this works because DPP *does* discount. The same claim for the plain payback period would be (C)-style wrong.

MCQ 2

Service projects A and B have $PW_A = -32{,}000$ and $PW_B = -44{,}000$ at the MARR. Which statement is correct?

(A) Reject both — their PWs are negative
(B) Select A — it has the lower incurred cost
(C) Select B — more is invested in it
(D) Neither can be evaluated without revenues

MCQ 2: Solution

Answer: (B). Service projects deliver a *required* output — negative PW is expected. The rule is least PW of cost: $-32{,}000$ beats $-44{,}000$.

(A) wrongly applies the single-project accept/reject rule; the service *must* be delivered, so "reject both" isn't on the menu.

MCQ 3

When evaluating a project, the verdict of AE analysis compared with PW analysis (same MARR, same horizon) is:

(A) Sometimes different, because AE spreads costs over time

(B) Always the same — $AE = PW \times (A/P)$ and $(A/P) > 0$

(C) The same only for uniform cash flows

(D) The same only when salvage is zero

MCQ 3: Solution

Answer: (B). *AE* is *PW* multiplied by a *positive* factor, so the sign — and the decision — can never differ. This holds for any cash flow shape and any salvage.

The same consistency holds for NFW ($PW \times (F/P)$). All three are one criterion in three units.

MCQ 4

For a **simple** investment with MARR $>$ IRR, which set of statements is fully correct?

(A) Accept; $PW(\text{MARR}) > 0$; $PW(\text{IRR}) = 0$
(B) Reject; $PW(\text{MARR}) < 0$; $PW(\text{IRR}) = 0$
(C) Reject; $PW(\text{MARR}) < 0$; $PW(\text{IRR}) < 0$
(D) Accept; $PW(\text{MARR}) = 0$; $PW(\text{IRR}) > 0$

MCQ 4: Solution

Answer: (B). MARR $>$ IRR $\Rightarrow$ reject. On the falling profile, discounting at a rate above i^* lands in the negative region, so $PW(\text{MARR}) < 0$. And $PW(i^*) = 0$ *by definition*.

Two exam staples in one: the rejection rule, and "NPW at the IRR is zero."

MCQ 5

Which statement about **nonsimple** investments is correct?

(A) Every nonsimple investment has multiple rates of return

(B) A nonsimple investment cannot be evaluated at all

(C) Multiple i^* are *possible*; when in doubt, decide by PW at the MARR

(D) A series with no sign change has exactly one rate of return

MCQ 5: Solution

Answer: (C). Multiple sign changes make multiple roots *possible*, not guaranteed. The project is still evaluated — by PW or AE at the MARR. (D) is wrong: no sign change $\Rightarrow$ *no* ROR at all.

The true/false section loves the words "all" and "always" here. Possibility, not certainty.

Before the Exam

1. Re-solve every part on these slides **without looking** at the solution
2. For each, write 4 steps: **setup/classification, factor notation, substitution, boxed answer**
3. Drill the three sign-sensitive habits: least-negative wins (service), the deferred-perpetuity shift, and counting sign changes before trusting an IRR
4. Practice **under time**: 75 minutes is tight if you re-derive instead of using the sheet
5. Try also Major 2 Problem Set B in Blackboard — **HW4 is due and Quiz 4 (Ch. 7) runs before class**

Bring your questions to class! We'll work these problems together and answer any questions.