ISE 307: Engineering Economic Analysis

Lecture 18: Rate of Return II — Incremental Analysis (Ch 7)

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Lecture 18

Why IRR Cannot Rank,
and the Incremental Method That Can

Lecture 18 Objectives

1. Explain why **ranking by IRR** fails for mutually exclusive alternatives
2. Build the **incremental cash flow** (larger − smaller investment)
3. Apply the rule: accept the larger iff $i^*_\Delta \geq$ MARR
4. Run the **champion–challenger** tournament for 3+ alternatives
5. Handle **unequal lives** (common horizon before incrementing)
6. Apply the same logic to two more ratio measures: **BC(i)** and **PI(i)**

The Ranking Paradox

Two mutually exclusive one-year projects, MARR $= 10\%$:

	$n=0$	$n=1$	IRR	$PW(10\%)$
A	$-2{,}000$	$+3{,}600$	80%	$+1{,}273$
B	$-8{,}000$	$+11{,}200$	40%	$+2{,}182$

IRR says A ($80\% > 40\%$); PW says **B** ($2{,}182 > 1{,}273$). **PW is right.**

IRR is a *percentage*; it ignores **scale**. You take profit to the bank in money, not in percent: 40% of 8,000 builds more wealth than 80% of 2,000.

The Incremental Method

The choice between A and B is really: "take A, **plus** invest the extra 6,000 in the increment B−A." So rate *that*:

	$n=0$	$n=1$
B − A	−6,000	+7,600

$i^*_\Delta = 7{,}600/6{,}000 - 1 = 26.7\% > 10\% = \text{MARR} \Rightarrow$ **choose B** ✓

Procedure (both alternatives already acceptable): 1. order by investment size; 2. form (larger − smaller); 3. if $i^*_\Delta \geq$ MARR $\rightarrow$ keep the **larger**; else keep the **smaller**.

Incremental Analysis Example

Two wheel loaders (10-yr lives; MARR = 15%):

	L1	**L2**
Investment (year 0)	24,000	36,000
Net annual (years 1–10)	6,600	9,000
Salvage (year 10)	2,400	6,000
IRR	24.8%	22.1%

1: both IRRs $> 15\%$ — both acceptable; L1 (smaller) is the champion.

2: increment L2−L1: $-12{,}000$; $+2{,}400$/yr; $+3{,}600$ at year 10 (simple).

3: $PW_\Delta(15\%) = +935$; $PW_\Delta(18\%) = -526 \Rightarrow i^* \approx 15\% + 3\% \, \frac{935}{1{,}461} \approx \boxed{16.9\%}$

Three or More Alternatives

1. Order by investment size: $D_1 < D_2 < D_3 < \ldots$ (may include do-nothing)
2. **Champion** = the smallest acceptable alternative
3. Challenge it with the next: rate the increment (challenger − champion)
4. $i^*_\Delta \geq$ MARR $\Rightarrow$ challenger becomes champion; else champion stays
5. Repeat until no challengers remain — the last champion wins

Every duel is pairwise and incremental. At no point does anyone compare raw IRRs.

Unequal Lives

- Incremental cash flows must be built over a **common analysis period** — the same fairness rule as Ch. 5
- With repeatability: replicate to the **LCM**, then subtract year by year
- Or compare **AE** over each life (Ch. 6 shortcut) when repetition is identical
- A stated **study period** overrides, as always

Nothing new to memorize: Ch. 5's horizon rules + Ch. 6's AE shortcut + today's increments compose.

Two More Ratio Measures: BC(i) and PI(i)

With B = PW of benefits, I = investment, C' = PW of operating costs (all at MARR):

BC ratio: $BC(i) = \dfrac{B}{I + C'}$ **Profitability index**: $PI(i) = \dfrac{B - C'}{I}$

accept iff ≥ 1 ($\equiv PW > 0$)

Projects A and B from the opening paradox ($C' = 0$, so $BC = PI$):

	I	B	PI	$PW(10\%)$
A	2,000	3,273	**1.64**	+1,273
B	8,000	10,182	1.27	+**2,182**

Ratios rank A; PW says B — the **same scale-blindness** as IRR, and the **same fix**: rate the increment, $\Delta B/\Delta I = 6{,}909/6{,}000 = 1.15 \geq 1 \Rightarrow$ **choose B**.

Practice Problem 1

A plant must pick one of two conveyor systems (8-yr lives; MARR = 12%):

	Investment (year 0)	Net annual (years 1–8)	Salvage (year 8)
C1	45,000	9,000	4,000
C2	70,000	13,500	7,000

Use **incremental IRR analysis** (bracket the increment with 10% and 12%) to decide.

Practice Problem 1: Solution

Steps 1–2: increment C2−C1: $-25{,}000$; $+4{,}500$/yr (years 1–8); $+3{,}000$ extra salvage at year 8. One sign change $\Rightarrow$ simple.

Step 3: $PW_\Delta(i) = -25{,}000 + 4{,}500\,(P/A,\, i,\, 8) + 3{,}000\,(P/F,\, i,\, 8)$:

$$PW_\Delta(10\%) = -25{,}000 + 4{,}500\,(5.3349) + 3{,}000\,(0.4665) = +407$$
$$PW_\Delta(12\%) = -25{,}000 + 4{,}500\,(4.9676) + 3{,}000\,(0.4039) = -1{,}434$$

$$i^*_\Delta \approx 10\% + 2\% \times \frac{407}{407 + 1{,}434} \approx \boxed{10.4\%}$$

$i^*_\Delta = 10.4\% < 12\% = \text{MARR} \Rightarrow$ the extra 25,000 does *not* earn its keep $\Rightarrow$ **choose C1**.

Quick Check

Mutually exclusive projects: $\text{IRR}_A = 30\%$, $\text{IRR}_B = 20\%$ (both simple, both $>$ MARR $= 12\%$; B is the larger investment). The incremental rate is $i^*_{B-A} = 15\%$. Which project should be chosen?

(A) A — it has the highest IRR

(B) B — the increment earns $15\% > 12\%$ MARR

(C) A — the increment earns less than IRR_A

(D) Neither — the IRRs disagree

Answer: (B). The extra capital in B earns 15%, above the 12% hurdle — take the larger project. Raw IRR ranking (A) is the classic error.

Summary

- IRR is a *percentage* — blind to scale; **never rank** mutually exclusive projects by raw IRR
- Rate the **increment** (larger − smaller): $i^*_\Delta \geq \text{MARR} \Rightarrow$ take the larger
- 3+ alternatives: champion–challenger, pairwise increments, smallest first
- Unequal lives: common horizon (LCM / AE / study period) *before* incrementing
- $\mathbf{BC}(i) = B/(I + C')$ and $\mathbf{PI}(i) = (B - C')/I$: accept iff ≥ 1; ratios share IRR's scale-blindness — increment before comparing
- Done correctly, incremental analysis always agrees with the PW ranking

Before Class

To do before our class meeting:

1. **Read Section 7.4** (IRR ranking flaws, incremental analysis, unequal lives)
2. Complete the **Lecture 18 pre-class quiz** on Blackboard before class
3. Warm-up: A costs 10,000 (IRR 25%); B costs 16,000 (IRR 19%); $i^*_{B-A} = 14\%$; MARR $= 12\%$ — which one?