ISE 307: Engineering Economic Analysis

Lecture 17: Rate of Return I — Finding and Using i^* (Ch 7)

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Lecture 17

The Internal Rate of Return:
Definition, Computation, and Decision Rule

Lecture 17 Objectives

1. Define the **IRR**: the rate i^* at which $PW(i^*) = 0$
2. Compute i^* by **direct solution** (single flow; two-period quadratic)
3. Compute i^* by **trial-and-error** with linear interpolation
4. Classify investments: **simple** vs. **nonsimple** (sign changes)
5. Apply the **IRR decision rule** against the MARR — and know when *not* to

What Is the Rate of Return?

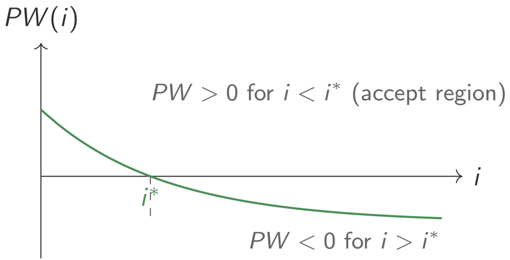
The **internal rate of return** i^* is the rate that makes the project break even:

$$PW(i^*) = \sum_{n=0}^{N} \frac{A_n}{(1+i^*)^n} = 0$$

- Interpretation: the **interest earned on the unrecovered balance** — like a loan in reverse (Lecture 9)
- At $i = i^*$: NPW $= 0$ and NFW $= 0$ — repay capital $+$ earn exactly i^*
- Revisit Lec 11, Problem 1(c): one amount in, one amount out, solve i

The NPW Profile

For an *investment* (money out first, returns later), $PW(i)$ **falls** as i rises:



Discounting harder hurts far-away *returns* more than the up-front *cost* $\Rightarrow$ the curve falls; it crosses zero once at i^* (for a *simple* investment).

Direct Solution Example 1

A drill-bit patent license costs **8,000 SAR** today and pays a single **20,000 SAR** royalty at the end of year **5**. Find i^*:

$$PW(i^*) = -8{,}000 + \frac{20{,}000}{(1+i^*)^5} = 0$$

$$(1+i^*)^5 = \frac{20{,}000}{8{,}000} = 2.5$$

$$i^* = 2.5^{1/5} - 1 = \boxed{20.11\%}$$

Same procedure as Lecture 11: two amounts + time $\Rightarrow$ take a root. No table.

Direct Solution Example 2

A used screening deck: buy for **10,000 SAR**, net **6,000** in year 1, then **7,200** in year 2 (then scrapped at ≈ 0 value). Let $x = \frac{1}{1+i}$:

$$-10{,}000 + 6{,}000\,x + 7{,}200\,x^2 = 0$$

Quadratic formula (keep the positive root):

$$x = \frac{-6{,}000 + \sqrt{6{,}000^2 + 4\,(7{,}200)(10{,}000)}}{2\,(7{,}200)} = \frac{-6{,}000 + 18{,}000}{14{,}400} = 0.8333$$

$$1 + i^* = \frac{1}{0.8333} = 1.20 \;\Rightarrow\; \boxed{i^* = 20\%}$$

Trial-and-Error Example

A conveyor drive replacement: **75,000 SAR** now, saves **24,400 SAR/yr** for **5 years**. $PW(i) = -75{,}000 + 24{,}400\,(P/A,\, i,\, 5)$:

try i	15%	18%	20%
$(P/A,\, i,\, 5)$	3.3522	3.1272	2.9906
$PW(i)$	$+6{,}793$	$+1{,}303$	$-2{,}029$

Sign change between 18% and 20% $\Rightarrow$ interpolate:

$$i^* \approx 18\% + 2\% \times \frac{1{,}303}{1{,}303 - (-2{,}029)} = 18\% + 2\%\,(0.391) = \boxed{18.8\%}$$

Simple vs. Nonsimple Investments

Scan the cash flow signs left to right; count the changes:

	$n = 0$	1	2	3	sign changes
A (simple)	$-1{,}000$	$+500$	$+800$	$+900$	1
B (nonsimple)	$-1{,}000$	$+3{,}900$	$-5{,}030$	$+2{,}145$	3
C (no ROR)	$+1{,}200$	$+500$	$+300$	$+100$	0

- **Simple**: exactly **one** sign change $\Rightarrow$ a **unique** i^*, and $i^* = \text{IRR}$
- **Nonsimple**: multiple sign changes $\Rightarrow$ *possibly* multiple i^* (not always)
- **No sign change** $\Rightarrow$ no rate of return exists

The IRR Decision Rule

For a **simple** investment with unique i^*:

- IRR $>$ MARR: **accept**
- IRR $=$ MARR: indifferent
- IRR $<$ MARR: **reject**

- Same decision as PW: MARR $< i^* \Leftrightarrow PW(\text{MARR}) > 0$
- With $i^* = 18.8\%$: accept at MARR $= 12\%$; reject at MARR $= 20\%$
- One project, different firms: a firm with a higher MARR can rightly *reject* what another firm might accept

Nonsimple Investments: When i^* Misleads

A strip-mining project (in thousands): $-1{,}000$ now, $+2{,}300$ in year 1, $-1{,}320$ reclamation in year 2. Two sign changes. Set $PW = 0$ with $x = \frac{1}{1+i}$:

$$-1{,}000 + 2{,}300\,x - 1{,}320\,x^2 = 0 \;\Rightarrow\; x = \tfrac{10}{11} \text{ or } \tfrac{10}{12} \;\Rightarrow\; i^* = \boxed{10\% \textbf{ and } 20\%}$$

Which one is "the" return? Neither. With multiple roots, i^* loses its meaning as an earned rate.

Rule for nonsimple investments: abandon IRR. Decide by PW (or AE) at the MARR. Here: $PW(15\%) = +1.89 > 0 \Rightarrow$ **accept**.

Practice Problem 1

A packaging machine (MARR $= 15\%$):

- costs **25,000 SAR**; salvage **5,000 SAR** after 6 years
- saves **7,465 SAR/yr** in labor; costs **2,000 SAR/yr** to operate

(a) Classify the investment (simple or nonsimple).

(b) Find i^* by trial-and-error, bracketing with 10% and 15%.

(c) State the accept/reject decision.

Practice Problem 1: Solution

(a) Net flow: $-25{,}000$; then $7{,}465 - 2{,}000 = +5{,}465$/yr; $+5{,}000$ salvage. **One** sign change $\Rightarrow$ **simple** $\Rightarrow$ unique i^*.

(b) $PW(i) = -25{,}000 + 5{,}465\,(P/A,\ i,\ 6) + 5{,}000\,(P/F,\ i,\ 6)$:

$$PW(10\%) = -25{,}000 + 5{,}465\,(4.3553) + 5{,}000\,(0.5645) = +1{,}624$$
$$PW(15\%) = -25{,}000 + 5{,}465\,(3.7845) + 5{,}000\,(0.4323) = -2{,}156$$

$$i^* \approx 10\% + 5\% \times \frac{1{,}624}{1{,}624 + 2{,}156} = 10\% + 5\%\,(0.430) \approx \boxed{12.1\%}$$

(c) $i^* \approx 12\% < \text{MARR} = 15\% \Rightarrow$ **reject**. (Check: $PW(15\%) < 0$ — same verdict.)

Quick Check

A project's cash flows are $+4{,}000$, $+2{,}500$, $+1{,}000$ in years 0–2 (no negative flow anywhere). Its rate of return is:

(A) 0%

(B) Infinite — money for nothing

(C) Nonexistent — no sign change, so no rate of return is defined

(D) Equal to the MARR

Answer: (C). With no sign change there is no invested balance to earn a return — $PW(i) > 0$ for every i, so no root exists.

Summary

- i^* solves $PW(i^*) = 0$: the interest earned on the *unrecovered balance*
- Direct solution: single flow $\Rightarrow$ take a root; two periods $\Rightarrow$ quadratic in $x = \frac{1}{1+i}$
- General: bracket a sign change of $PW(i)$, then interpolate
- Count sign changes: **one** $\Rightarrow$ simple, unique i^*; **several** $\Rightarrow$ nonsimple, maybe multiple; **none** $\Rightarrow$ no ROR
- Simple: accept iff IRR $>$ MARR (agrees with PW). Nonsimple: **abandon IRR**; use PW/AE at the MARR

Before Class

To do before our class meeting:

1. **Read Sections 7.1–7.3** (ROR definition, computing i^*, IRR criterion)
2. Complete the **Lecture 17 pre-class quiz** on Blackboard before class
3. Warm-up: invest **5,000**; receive **12,000** in year 6 — find i^*