ISE 307: Engineering Economic Analysis

Lecture 15: Annual Equivalence I — The AE Criterion and Capital Recovery (Ch 6)

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Lecture 15

The Annual Equivalence Criterion
and the Capital Recovery Cost

Lecture 15 Objectives

1. Apply the **AE criterion**: $AE(i) = PW(i)\,(A/P,\, i,\, N)$; accept if > 0
2. Separate **capital (ownership)** costs from **operating** costs
3. Compute the **capital recovery cost**:
   $CR(i) = (I - S)(A/P,\, i,\, N) + iS$
4. Justify a project by comparing **annual savings** against $CR(i)$
5. Build a full **annual equivalent cost (AEC)**; annualize **perpetual** assets

The Annual Equivalence (AE) Criterion

$$AE(i) = PW(i) \times (A/P,\, i,\, N)$$

Accept a single project iff $AE > 0$. Since $(A/P) > 0$ always:
AE and PW agree in sign — fully consistent criteria.

Example: the haul-truck autonomy retrofit (from Lecture 12), with PW $= 7{,}216$ SAR at MARR $= 12\%$ and $n = 5$ years:

$$AE(12\%) = \text{PW}\,(A/P,\, i,\, n) = 7{,}216\,(0.2774) = \boxed{2{,}002 \text{ SAR/yr}}$$

AE shines when people think *per year*: budgets, rents, unit costs, and unequal-life comparisons.

Two Kinds of Cost

- **Capital (ownership) costs**: one-time purchase I at year 0, salvage S at year N
- Capital costs *must be annualized* before entering a per-year comparison
- **Operating costs**: recurring, such as: energy, labor, maintenance
- Operating costs are *already* on a per-year basis (when uniform)

The annualized capital cost has a name:
the **capital recovery cost**, $CR(i)$.

Capital Recovery Cost: $CR(i)$

$$CR(i) = (I - S)\,(A/P,\, i,\, N) + i\,S$$

Two parts: repay the *depreciating* portion $(I - S)$ in installments; pay *interest* on the capital parked in S.

Example: a site pickup truck. Buy at **78,000 SAR**, resale **30,000 SAR** after 4 years, $i = 8\%$:

$$\begin{aligned} CR(8\%) &= (I - S)\,(A/P,\, 8\%,\, 4) + i\,S \\ &= 48{,}000\,(0.3019) + 2{,}400 = \boxed{16{,}892 \text{ SAR/yr}} \end{aligned}$$

Vehicle and equipment leases are priced on exactly this structure.

Justifying a Project Example

A fabrication shop considers a pipe-threading machine ($MARR = 15\%$):

- cost **110,000 SAR**; salvage **10,000 SAR** after 5 years
- saves **33,000 SAR/yr** in outsourced work

1. Ownership per year:

$$CR(15\%) = (110{,}000 - 10{,}000)\,(A/P,\ 15\%,\ 5) + 0.15\,(10{,}000)$$

$$= 100{,}000\,(0.2983) + 1{,}500 = \boxed{31{,}332 \text{ SAR/yr}}$$

2. Verdict per year: $AE = 33{,}000 - 31{,}332 = \boxed{+1{,}668 \text{ SAR/yr}} \Rightarrow$ **accept**

The machine "earns its keep": savings exceed the yearly cost of owning it. Same verdict as PW, stated in per-year language.

Full AEC Example

A standby generator: cost **90,000**; salvage **9,000**; 5 years; operating **15,000/yr (yrs 1–2)** then **18,000/yr (yrs 3–5)**; MARR = 10%:

1. Capital: $CR = (81{,}000)(A/P, 10\%, 5) + 0.10\,(9{,}000) = 21{,}368 + 900 =$ 22,268 SAR/yr

2. Opr: $PW = 15{,}000\,(P/A, 10\%, 2) + 18{,}000\,(P/A, 10\%, 3)(P/F, 10\%, 2)$
$= 26{,}033 + 36{,}994 = 63{,}027 \Rightarrow \; AE = 63{,}027\,(A/P, 10\%, 5) = 16{,}627$ SAR/yr

3. Total: $AEC = 22{,}268 + 16{,}627 =$ $\boxed{38{,}895 \text{ SAR/yr}}$

The pattern: capital via $CR(i)$ + operating via PW $\rightarrow (A/P)$.

Perpetual Assets

Infinite life: capital's annual cost $= I \times i$ (interest forever); add annual O&M; spread every-k-yr repairs via $(A/F,\, i,\, k)$ or $(P/F,\, i,\, k)\,(A/P,\, i,\, k)$

Example: a mine-site water-treatment station. Build **3,000,000 SAR**; O&M **80,000/yr**; overhaul **1,000,000 every 25 years**; $i = 8\%$:

$$AE = \underbrace{3{,}000{,}000\,(0.08)}_{240{,}000} + \underbrace{80{,}000}_{\text{O\&M}} + \underbrace{1{,}000{,}000\,(A/F, 8\%, 25)}_{13{,}679} = \boxed{333{,}679 \text{ SAR/yr}}$$

The dual of Lecture 13's $CE = A/i$: there we *capitalized* an annual amount; here we *annualize* a capital amount.

Practice Problem 1

A CNC machine for the mine's maintenance shop costs **50,000 SAR**, has salvage **8,000 SAR** after **6 years**, at $i = 12\%$.

Find the **capital recovery cost**.

Practice Problem 1: Solution

Formula first, then substitute:

$$\begin{aligned} CR(12\%) &= (I - S)\,(A/P,\ 12\%,\ 6) + i\,S \\ &= (50{,}000 - 8{,}000)\,(0.2432) + 0.12\,(8{,}000) \\ &= 10{,}216 + 960 = \boxed{11{,}176 \text{ SAR/yr}} \end{aligned}$$

Owning the machine costs about **11,200 SAR/yr** before any operating expense — the number to beat if a vendor offers the same capability as a service.

Practice Problem 2

A tailings pump station will serve the site **indefinitely** (MARR $= 8\%$):

- construction **4,000,000 SAR**
- O&M **150,000 SAR/yr**
- major repair **1,500,000 SAR every 20 years**

Find the **annual equivalent cost** of the station.

Practice Problem 2: Solution

Three pieces, all per year:

1. Capital (forever): $4{,}000{,}000 \times 0.08 = 320{,}000$

2. O&M: 150,000 (already annual)

3. Repair spread over one 20-yr cycle: $1{,}500{,}000\,(A/F,\,8\%,\,20) = 1{,}500{,}000\,(0.02185) = 32{,}778$

$$AEC = 320{,}000 + 150{,}000 + 32{,}778 = \boxed{502{,}778 \text{ SAR/yr}}$$

Quick Check

A project has $PW(15\%) = -3{,}400$ SAR over 8 years. Without computing, what can you say about its $AE(15\%)$?

(A) Positive — AE and PW can disagree

(B) Zero — AE ignores the initial investment

(C) Negative — AE = PW $\times(A/P)$, and $(A/P) > 0$

(D) Cannot be determined without the cash flows

Answer: (C). $AE = -3{,}400\,(A/P,\, 15\%,\, 8) < 0$. AE and PW always carry the same sign — reject either way.

Summary

- $AE(i) = PW(i)\,(A/P,\, i,\, N)$ — always the same sign and decision as PW
- Capital vs. operating: one-time costs must be *annualized*; recurring costs already are
- $CR(i) = (I - S)(A/P,\, i,\, N) + iS$ — the yearly cost of ownership
- Justify a purchase: accept if annual savings $> CR(i)$
- Messy operating costs: PW first, then (A/P)
- Perpetual assets: $I \cdot i$ + O&M + (A/F)-spread repairs

Before Class

To do before our class meeting:

1. **Read Sections 6.1–6.2** (AE criterion, capital vs. operating costs, CR)
2. Complete the **Lecture 15 pre-class quiz** on Blackboard before class
3. Warm-up: a **60,000 SAR** machine, salvage **10,000**, 5 yrs, $i = 10\%$ — find CR