ISE 307: Engineering Economic Analysis

Lecture 14: Comparing Mutually Exclusive Alternatives (Ch 5)

Term 253 (Summer)

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Lecture 14

Mutually Exclusive Alternatives
and Analysis Periods

Announcements

1. Major 1 grade to be posted end of day today. Mean is 74.5, Median is 78. Std is 10.2.
2. Homework 3 is due Wednesday, July 22nd. Start early.
3. No class tomorrow, but there will be an online assignment.
4. Quiz 2 will be this Wednesday, MCQ (coverage chapter 5 all and 6 concepts)
5. Major 2 is Sunday, July 26th, 8-10pm. No changes to the schedule.

Lecture 14 Objectives

1. Set up choices correctly: **do-nothing**, **revenue** vs. **service** projects
2. Compare **equal-life** options by maximum PW (least cost for service)
3. Handle **unequal lives** with lowest-common-multiple replication
4. Apply a fixed **study period** when it is stated

Do-Nothing, Revenue, and Service Projects

- **Is "do nothing" allowed?** If yes, it competes too (with PW $= 0$).
- A **mandated** project (e.g. a required safety system) excludes **do-nothing**.
- **Revenue project**: the alternatives differ in *income* as well as cost $\rightarrow$ compare full cash flows; **maximize NPW**
- **Service project**: the same required output either way (flotation must run; the tank must be coated) $\rightarrow$ revenues are equal and can be dropped; **minimize the PW of cost**

For **mandated** service projects, the *least-negative* PW wins:
a cost of $-32{,}000$ beats a cost of $-44{,}000$.

The Common Analysis Period

- PW comparisons are only fair over a **common time span**: both alternatives must deliver service over the *same* period
- Comparing a 3-yr machine's PW directly against a 5-yr machine's PW ignores 2 years of service — the shorter option looks artificially cheap
- Three cases:
  - Lives **equal** (compare directly)
  - Lives **unequal, no horizon** (use the Least Common Multiple (LCM))
  - A **stated study period** (use it for all alternatives)
- **Mutually exclusive** means choosing one only (not any others).

Equal Lives Example

A mineral-rock-aggregate plant needs to choose a crusher machine; both options last for 5 years; MARR $= 10\%$:

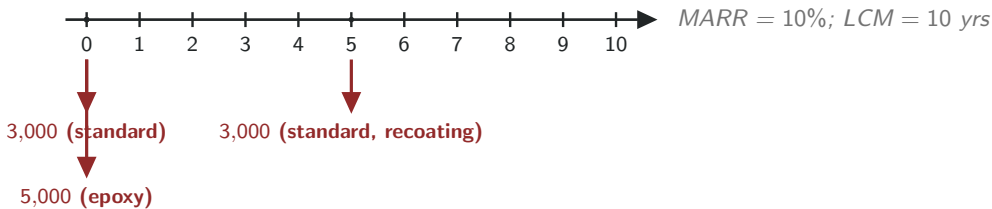
	Investment (year 0)	Net annual (year 1-5)	Salvage (year 5)
C1 (30 t/hr)	80,000	26,000	10,000
C2 (70 t/hr)	110,000	34,000	15,000

$$\begin{aligned} \mathrm{PW}_1 &= P_0 + A_n\,(P/A,\, i,\, n) + F_n\,(P/F,\, i,\, n) \\ &= -80{,}000 + 26{,}000\,(3.7908) + 10{,}000\,(0.6209) = \boxed{24{,}770 \text{ SAR}} \\ \mathrm{PW}_2 &= P_0 + A_n\,(P/A,\, i,\, n) + F_n\,(P/F,\, i,\, n) \\ &= -110{,}000 + 34{,}000\,(3.7908) + 15{,}000\,(0.6209) = \boxed{28{,}201 \text{ SAR}} \end{aligned}$$

Unequal Lives Example

Storage tank is planned to be kept for 10 years. The supply manager needs to choose between two coating options: MARR = 10% (a *service* project):

- **epoxy coating**: 5,000 SAR, lasts 10 yrs
- **standard coating**: 3,000 SAR, lasts 5 yrs



$$PW_{standard} = 3{,}000 + 3{,}000\,(P/F,\ 10\%,\ 5) = 3{,}000 + 3{,}000\,(0.6209) = \boxed{4{,}863 \text{ SAR}}$$

$PW_{epoxy} = \boxed{5{,}000 \text{ SAR}}$ **The standard coating wins** (least PW of cost)

When a Study Period Is Imposed

- Sometimes the firm fixes the horizon: *"the mine closes in 4 years"*, *"the offtake contract runs 6 years"*
- Then the **study period** — not the LCM — is the analysis period for *all*
- Assets still useful at the end considered as a **salvage value** at that date
- Shorter-lived assets: replace within the period, or fill the gap (e.g. **lease**)
- Textbook caution (Section 5.4.4): LCM assumes *identical* future repeats, often unrealistic; prefer an explicit study period when one exists

Order of precedence: **stated study period** > LCM replication.
Never replicate past a stated horizon.

Study Period Example

A warehouse **must** run one forklift until the site closes in **4 years** ($MARR = 10\%$, a service project). Estimated salvage *at the cutoff*:

	First cost	O&M/yr	Life	Salvage at yr 4
F1	40,000	5,000	5 yrs	12,000
F2	54,000	3,500	7 yrs	22,000

$$\begin{aligned} PW_{cost,1} &= 40{,}000 + 5{,}000\,(P/A,\ 10\%,\ 4) - 12{,}000\,(P/F,\ 10\%,\ 4) \\ &= 40{,}000 + 5{,}000\,(3.1699) - 12{,}000\,(0.6830) = \boxed{47{,}653 \text{ SAR}} \\ PW_{cost,2} &= 54{,}000 + 3{,}500\,(P/A,\ 10\%,\ 4) - 22{,}000\,(P/F,\ 10\%,\ 4) \\ &= 54{,}000 + 3{,}500\,(3.1699) - 22{,}000\,(0.6830) = \boxed{50{,}068 \text{ SAR}} \end{aligned}$$

Choose F1, all analysis period is four years.

Practice Problem 1

A crushing plant **must** install one of two dust-collection systems (5-yr lives; MARR $= 10\%$):

	First cost	O&M/yr	Salvage
S1	12,000	3,000	2,000
S2	20,000	1,800	5,000

Classify the situation (do-nothing? revenue or service?), then compute the PW of cost for each system, and state the decision.

Practice Problem 1: Solution

Classify: "must install" $\Rightarrow$ do-nothing excluded; same dust removal either way $\Rightarrow$ *service* project, minimize PW of cost. Equal lives $\Rightarrow$ compare directly:

$$\begin{aligned} \text{PW}_{\text{cost},1} &= 12{,}000 + 3{,}000\,(P/A,\ 10\%,\ 5) - 2{,}000\,(P/F,\ 10\%,\ 5) \\ &= 12{,}000 + 3{,}000\,(3.7908) - 2{,}000\,(0.6209) = \boxed{22{,}131 \text{ SAR}} \\ \text{PW}_{\text{cost},2} &= 20{,}000 + 1{,}800\,(3.7908) - 5{,}000\,(0.6209) = \boxed{23{,}719 \text{ SAR}} \end{aligned}$$

Same service either way $\Rightarrow$ **choose S1**, the lower PW of cost.

Practice Problem 2

Two mutually exclusive projects; MARR $= 10\%$:

n	A	B
0	$-15{,}000$	$-13{,}000$
1	6,000	X
2	5,000	5,500
3	7,000	X

What value of X makes the decision maker **indifferent** between A and B?

Practice Problem 2: Solution

Step 1 — PW of A at 10% (P/F: 0.9091, 0.8264, 0.7513):

$$\text{PW}_A = -15{,}000 + 6{,}000(0.9091) + 5{,}000(0.8264) + 7{,}000(0.7513) = -154$$

Step 2 — set $\text{PW}_B = \text{PW}_A$ and solve for X:

$$-13{,}000 + X(0.9091) + 5{,}500(0.8264) + X(0.7513) = -154$$

$$X\,(0.9091 + 0.7513) = -154 + 13{,}000 - 4{,}545 \;\Rightarrow\; X = \frac{8{,}301}{1.6604} = \boxed{\text{SAR } 4{,}999}$$

If $X > 4{,}999$, choose B; if $X < 4{,}999$, choose A.

Quick Check

Machine A lasts **3 years**; machine B lasts **5 years**. No study period is stated. Over what horizon should the PW comparison be made, and why?

(A) 3 years — the shorter life

(B) 5 years — the longer life

(C) 15 years — the lowest common multiple

(D) 8 years — the sum of the lives

Answer: (C). With no stated study period, replicate transactions to the lowest common multiple (LCM) (15 yrs) so both alternatives deliver service over the *same* span.

Summary

- Classify first: is **do-nothing** an option? revenue or **service**? equal or unequal **lives**? is a **study period** stated?
- Pick options based on PW (depends on service or revenue)
- Unequal lives: replicate to the **LCM** of the lives, then compare PW
- A stated **study period** overrides the LCM; include salvage at the cutoff
- Indifference problems: set $\text{PW}_A = \text{PW}_B$ and solve for the unknown
- All the arithmetic is **Ch 2 equivalence** — (P/A) for annual flows, (P/F) for salvages and repurchases

Before Class

To do before our class meeting:

1. **Read Section 5.4** (do-nothing, service vs. revenue, analysis periods)
2. Complete the **Lecture 14 pre-class quiz** on Blackboard before class
3. Warm-up: machines of **3-yr** and **6-yr** lives — what is the LCM horizon?