ISE 307: Engineering Economic Analysis

Lecture 13: Future Worth, Project Balance, and Capitalized Equivalent

Term 253 (Summer)

Instructor: Mansur M. Arief
Industrial and Systems Engineering, KFUPM

Office: 22-219 — Email: mansur.arief@kfupm.edu.sa

Lecture 13

Net Future Worth, the Project Balance,
and the Capitalized Equivalent

Lecture 13 Objectives

1. Compute the **net future worth (NFW)** and relate it to NPW
2. Build a **project balance** table (exposure, DPP, terminal surplus)
3. Value perpetual projects with the **capitalized equivalent (CE)**
4. Handle CE with **periodic** (every-k-years) costs

Three Screens So Far: Strengths and Limits

Tool	Strength	Limitation
PP	quick; measures liquidity	ignores TVM; ignores cash flows after payback
DPP	adds the time value of money	still ignores cash flows after payback
NPW	considers MARR; full profitability criterion	only *single* number at time 0

NPW says **how much** value you expect to get relative only to the initial investment. We will see what you **end** with (NFW), how the project **changes** year by year (project balance), or how to price service that **never ends** (CE)

Net Future Worth (NFW)

$$\text{NFW} = \text{FW}(i) = \sum_{n=0}^{N} A_n\,(1+i)^{N-n} = \text{PW}(i) \times (F/P,\, i,\, N)$$

Since $(F/P) > 0$ always: **same sign, same decision** as NPW.

The haul-truck autonomy retrofit from Lec 12 (NPW $= 7{,}216$ SAR at 12%, $N = 5$):

$$\text{FW}(12\%) = 7{,}216\,(F/P,\, 12\%,\, 5) = 7{,}216\,(1.7623) = \boxed{\text{SAR } 12{,}717}$$

Interpretation: if every cash flow were invested consistently at MARR $= 12\%$, the account shows a **12,717 SAR surplus** on the day the kit is resold.

Midway Metrics (Project Balance)

NPW and NFW give a **decision**, but they hide the **process**. We may also ask:

- "How much of our capital is tied up and at risk at the **worst point**?"
- "**How and when** do we climb out of the negative balance?"
- "If the project is cut short midway in year 2, **where do we stand**?"

The **project balance**restates the same cash flows as a year-by-year account at the interest rate = MARR

The Project Balance

A bank statement for the project

$$PB_0 = A_0$$
$$PB_n = PB_{n-1}\,(1+i) + A_n \quad (\textbf{grow} \text{ then } \textbf{add})$$

- $PB_n < 0$: the project still *owes* you money (**risk exposure**)
- First n with $PB_n \geq 0$: the **discounted payback**
- Positive-PB years: surplus compounding — the **profit potential**
- PB_N (terminal balance) = **NFW**

Project Balance: The Truck Autonomy Retrofit at 12%

Flows: $-55{,}000$; then $16{,}000 \times 4$; final year $16{,}000 + 8{,}000$ salvage:

n	0	1	2	3	4	5
A_n	$-55{,}000$	16,000	16,000	16,000	16,000	24,000
PB_n	$-55{,}000$	$-45{,}600$	$-35{,}072$	$-23{,}281$	$-10{,}075$	12,716

e.g. $PB_1 = PB_0(1 + 0.12) + A_1 = -55{,}000\,(1.12) + 16{,}000 = -45{,}600$

- **Risk exposure** (how deep the negatives run)
- **DPP** (first nonnegative PB: during year 5)
- **Terminal balance** $= 12{,}716 \approx$ NFW ✓

Projects That Never End

- A haul-road bridge, a dam, an endowment: the service has **no end date**
- The NPW sum needs a finite N — and the factor tables stop at $N = 100$
- But distant flows **shrink** when discounted at the MARR: at 8%, 100 SAR in year 100 is worth less than 5 SAR today
- So the PW of perpetual service **converges** to a finite number — we just need its limiting value

The **capitalized equivalent (CE)** is that limit, no N (similar to perpetuities from Chapter 2).

Capitalized Equivalent

Bridges, dams, endowments, waqf funds: the service horizon is **infinite**.
As $N \to \infty$:

$$(P/A,\, i,\, N) = \frac{(1+i)^N - 1}{i\,(1+i)^N} \;\longrightarrow\; \frac{1}{i}$$

Perpetual uniform A: $\quad CE(i) = \dfrac{A}{i} \quad$ (+ any initial cost)

A deposit of A/i earns interest of $i \times (A/i) = A$ each year (you withdraw exactly the interest, forever, and the principal never shrinks).

CE with Periodic Costs

A mining company endows a safety-training center **forever**: build for **200,000 SAR**; upkeep **12,000 SAR/yr**; refurbishment **40,000 SAR every 20 yrs**; $i = 8\%$.

Step 1 — convert the every-20-yr cost to an equal annual cost (one 20-yr cycle):

$$A_{\text{ref}} = \frac{40{,}000}{(P/A,\, 8\%,\, 20)} = \frac{40{,}000}{9.8181} = 874 \text{ SAR/yr}$$

Step 2 — capitalize all annual costs and add the initial cost:

$$CE = 200{,}000 + \frac{12{,}000 + 874}{0.08} = 200{,}000 + 160{,}926 = \boxed{\text{SAR } 360{,}926}$$

Practice Problem 1

A conveyor upgrade costs **40,000 SAR** and returns **15,000 SAR/yr** for **4 years**; MARR $= 10\%$. Build the project balance table for $n = 0, \ldots, 4$ and read off the discounted payback and the terminal balance.

Practice Problem 1: Solution

Recursion: $PB_n = PB_{n-1}\,(1.10) + A_n$, starting at $PB_0 = -40{,}000$:

n	0	1	2	3	4
A_n	$-40{,}000$	15,000	15,000	15,000	15,000
PB_n	$-40{,}000$	$-29{,}000$	$-16{,}900$	$-3{,}590$	$\boxed{+11{,}051}$

Some examples:

$PB_1 = PB_0(1 + 0.10) + A_1 = -40{,}000\,(1.10) + 15{,}000 = -29{,}000$ SAR

$PB_2 = PB_1(1 + 0.10) + A_2 = -29{,}000\,(1.10) + 15{,}000 = -16{,}900$ SAR

and so on.

DPP $= 3.26$ years $\Rightarrow$ 4 years, $PB_4 = +11{,}051$ SAR

Quick Check

A project's balance table ends with terminal balance $PB_N = +9{,}500$ SAR at the MARR. Which statement is correct?

(A) The project's NPW is 9,500

(B) The project's NFW is 9,500, so its NPW is $9{,}500\,(P/F,\ i,\ N)$

(C) The project never recovers its investment

(D) The discounted payback is N years

Answer: (B). The terminal project balance *is* the NFW; discount it back through $(P/F,\ i,\ N)$ to get the NPW. Both are positive, so the project is acceptable.

Summary

- **NFW** $=$ PW $\times (F/P,\ i,\ N)$, valuation when the project ends
- **Project balance**: $PB_n = PB_{n-1}(1+i) + A_n$ (grow then add), showing risk exposure, discounted payback, and terminal surplus ($=$ NFW)
- **Capitalized equivalent**: $CE = A/i$ ($+$ initial cost) for perpetual service
- Periodic every-k-yr costs enter via $(P/A,\ i,\ k)$ *before* dividing by i

Before Class

To do before our class meeting:

1. **Read Sections 5.3.3–5.3.5** (pp. 206–213: project balance, NFW, capitalized equivalent)
2. Complete the **Lecture 13 pre-class quiz** on Blackboard before class
3. Try this: what single deposit funds **25,000 SAR/yr forever** at 9%?