ISE 307: Engineering Economic Analysis

Lecture 11: Major Exam I Review (Chapters 1–4)

Term 253 (Summer)

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Lecture 11

Objectives Recap, the Formula Sheet,
and Fully Worked Practice

Major Exam I: Format and Rules

- **75 minutes**, in class, covering **Chapters 1–4**
- Some long problems (with sub-parts) plus a multiple-choice section
- Long problems give **partial credit for shown work**
- Provided: **formula sheet** and **factor tables**
- Non-programmable scientific calculator only

Lecture Summaries (Chapters 1–2, Lecture 2-7)

Lecture	You should be able to ...
L1–L2 (Ch 1)	define engineering economy; list decision types; state the four principles
L3 (Ch 2)	explain time value; draw a cash flow diagram; tell simple from compound interest
L4 (Ch 2)	move one amount in time with (F/P), (P/F); solve for i or N
L5 (Ch 2)	value a flat series with (F/A), (A/F), (P/A), (A/P)
L6 (Ch 2)	handle deferred series and combinations; sinking fund and capital recovery
L7 (Ch 2)	value linear gradients (P/G), (A/G) and geometric (%-growth) series

Lecture Summaries (Chapters 3–4, Lecture 8-11)

Lecture	**You should be able to ...**
L8 (Ch 3)	turn a nominal rate into an effective rate; handle mismatched periods
L9 (Ch 3)	find a loan payment; find a remaining balance; split interest from principal; spot the add-
L10 (Ch 4)	measure inflation; convert actual to constant value; pair the right rate; value a perpetuity
L11 (today)	review materials

The Formula Sheet

Single Payment: One Amount Moving in Time

Use when **one** lump sum moves to a different date.

You want ...	Formula	Trigger words
future value	$F = P\,(F/P, i, N)$	"grows to," "worth in N years"
present value	$P = F\,(P/F, i, N)$	"worth today," "deposit now"
the rate i	$i = (F/P)^{1/N} - 1$	"rate of return," two amounts + time
the periods N	$N = \frac{\ln(F/P)}{\ln(1+i)}$	"how long to double / triple"

Rule 72: doubling time $\approx 72/i(\%)$ — a quick check, not exact.

Equal-Payment Series: A Flat, Repeated Amount

Use when the **same** amount A repeats every period.

You have / want	**Formula**	**Trigger**
series $\rightarrow$ future	$F = A\,(F/A, i, N)$	"save each year, balance at end"
future $\rightarrow$ series	$A = F\,(A/F, i, N)$	"set aside to reach a target" (sinking fund)
series $\rightarrow$ present	$P = A\,(P/A, i, N)$	"worth today of a yearly payment"
present $\rightarrow$ series	$A = P\,(A/P, i, N)$	"loan payment," "recover a cost"

(P/A) places the value **one period before** the first payment. (F/A) places it **at** the last payment.

Gradients: Payments That Change Each Period

Use when the payment **changes** from period to period.

Pattern	**Formula**	**Key caution**
fixed step $+G$ each year	$P = G\,(P/G, i, N)$	gradient's *first* flow is 0
(linear)	$A = G\,(A/G, i, N)$	decompose: base A_1 + gradient
fixed %-growth g	$P = A_1 \left[\frac{1-(1+g)^N(1+i)^{-N}}{i-g}\right]$	special case $i = g$
(geometric)	$P = A_1 \frac{N}{1+i}$ if $i = g$	use the second form

A series like 100, 130, 160 is a **base of 100** plus a **gradient of 30** (whose first flow is 0).

Rates, Inflation, and Perpetuity

Situation	Formula	Trigger
nominal $\to$ effective	$i_a = (1 + \frac{r}{M})^M - 1$	"compounded monthly / quarterly"
continuous compounding	$i_a = e^r - 1$	"compounded continuously"
periods $\neq$ compounding	$i = (1 + \frac{r}{M})^C - 1$	"pay monthly, compound daily"
average inflation	$\bar{f} = [\prod(1 + f_k)]^{1/N} - 1$	several yearly rates
actual $\to$ constant	$A'_n = A_n(1 + \bar{f})^{-n}$	"today's-value money"
real rate	$i' = \frac{i - \bar{f}}{1 + \bar{f}}$	pairing with constant value
perpetuity	$P = A/i$ (or A'/i')	"forever," "in perpetuity"

Inflation pairing: **actual** value with i; **constant** value with i'.

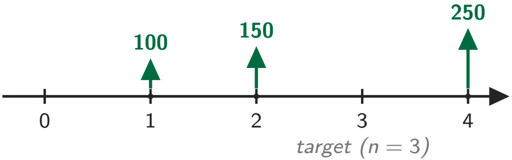
Practice Problems
(Major 1 Problem Set A)

Problem 1(a)

A project produces **100** at the end of year 1, **150** at the end of year 2, and **250** at the end of year 4. Draw a labeled cash flow diagram, and mark **year 3** as the reference point where you will later find one equivalent amount.

Try it: place each receipt on a time line and mark the target date.

Problem 1(a): Solution



Receipts are up-arrows at years 1, 2, and 4. The reference point, year 3, sits *between* the year-2 and year-4 flows. Two flows move forward to it, one moves back.

Problem 1(b)

Find the single amount at **year 3** equivalent to the whole series at $i = 10\%$.
(Flows before year 3 move forward; the flow after year 3 moves back.)

Try it: carry each flow to year 3 with (F/P) *or* (P/F)*, then add.*

Problem 1(b): Solution

Each flow travels to year 3 by its own distance:

$$
\begin{aligned}
V_3 &= 100\,(F/P, 10\%, 2) + 150\,(F/P, 10\%, 1) + 250\,(P/F, 10\%, 1) \\
&= 100(1.2100) + 150(1.1000) + 250(0.9091) \\
&= 121.00 + 165.00 + 227.27 = \boxed{\text{SAR } 513.27}
\end{aligned}
$$

The year-1 flow moves *forward* 2 years, year-2 forward 1 year, year-4 *back* 1 year.

Problem 1(c)

The plant buys equipment for **50,000** and sells it **6 years** later for **180,000**. Find the **rate of return** on this single investment.

Problem 1(c): Solution

One payment in, one payment out, so solve $F = P(1+i)^N$ for i:

$$i = \left(\frac{F}{P}\right)^{1/N} - 1 = \left(\frac{180{,}000}{50{,}000}\right)^{1/6} - 1$$
$$= (3.6)^{1/6} - 1 = \boxed{23.80\%}$$

No factor table needed — this rate is found by formula (a root), not a lookup.

Problem 1(d)

A **1 SAR** coin is placed in a trust earning **5% compounded annually** for **150 years**. Find its future value.

Problem 1(d): Solution

Single payment, but $N = 150$ is beyond the tables, so use the formula directly:

$$F = P(1+i)^N = 1\,(1.05)^{150}$$
$$= \boxed{\text{SAR } 1{,}507.98}$$

A small rate over a long time compounds into a large multiple — the power of compounding.

Problem 2(a)

Three accounts are quoted: **7.2% compounded quarterly**, **7.1% compounded monthly**, and **7.05% compounded continuously**. Find the **effective annual rate** of each and say which earns the most.

Try it: convert each nominal rate to an effective annual rate, then compare.

Problem 2(a): Solution

Convert each nominal rate to an effective annual rate:

$$
\begin{aligned}
\text{quarterly:} \quad & i_a = (1 + \tfrac{0.072}{4})^4 - 1 = \boxed{7.397\%} \\
\text{monthly:} \quad & i_a = (1 + \tfrac{0.071}{12})^{12} - 1 = \boxed{7.336\%} \\
\text{continuous:} \quad & i_a = e^{0.0705} - 1 = \boxed{7.305\%}
\end{aligned}
$$

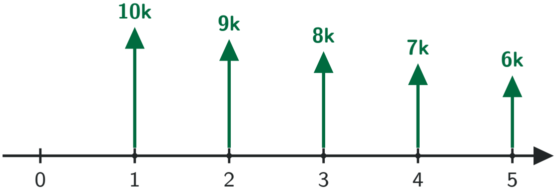
The **quarterly** account wins. A higher nominal rate can still win even with less frequent compounding.

Problem 2(b)

The workshop makes *decreasing* year-end deposits: **10,000** at year 1, then **1,000 less** each year (10,000, 9,000, 8,000, 7,000, 6,000). Draw a labeled cash flow diagram of the five deposits.

Try it: five down-steps; think "flat base minus a gradient."

Problem 2(b): Solution



Five deposits stepping down by 1,000 each year. Read it as a **flat 10,000** base *minus* a **1,000 gradient** (whose first flow is 0).

Problem 2(c)

The account earns **5% compounded annually**. Find the balance **immediately after the fifth deposit**.

Try it: future value of (flat base) minus future value of (gradient).

Problem 2(c): Solution

Flat base of 10,000 *minus* a 1,000 gradient, both carried to year 5:

$$\begin{aligned} F &= 10{,}000\,(F/A, 5\%, 5) - 1{,}000\,(A/G, 5\%, 5)(F/A, 5\%, 5) \\ &= 10{,}000(5.5256) - 1{,}000(1.9025)(5.5256) \\ &= 55{,}256 - 10{,}512 = \boxed{\text{SAR } 44{,}744} \end{aligned}$$

The gradient gives a level equivalent $A_G = G(A/G)$, then (F/A) carries it to year 5.

Problem 2(d)

An investment account earns **6% compounded semiannually**. Find how long it takes a deposit to **triple**.

Problem 2(d): Solution

Rate per period: $i = 6\%/2 = 3\%$. Solve $(1.03)^N = 3$ for N:

$$N = \frac{\ln 3}{\ln 1.03} = \frac{1.0986}{0.02956}$$
$$= 37.17 \text{ periods} = \boxed{18.6 \text{ years}}$$

N comes out in *periods* (half-years); divide by 2 to report years.

Problem 3(a): Financing an Upgrade

A factory borrows to finance an equipment upgrade.

180,000 SAR at **12% compounded monthly**, repaid in **24 equal monthly** payments. Four parts: the **payment**, a **remaining balance**, an **interest/principal split**, and **deflating** the final payment. Find the monthly payment.

Problem 3(a): Solution

Periods match: $i = 12\%/12 = 1\%$ per month, $N = 24$. Use (A/P):

$$\begin{aligned} A &= 180{,}000\,(A/P, 1\%, 24) \\ &= 180{,}000\,(0.04707) = \boxed{\text{SAR } 8{,}473.23} \end{aligned}$$

Problem 3(b)

Find the loan balance still owed **immediately after the 12th payment**.

Problem 3(b): Solution

After 12 payments, **12 remain**. Balance = present worth of those 12:

$$
\begin{aligned}
B_{12} &= 8{,}473.23\,(P/A, 1\%, 12) \\
&= 8{,}473.23\,(11.2551) = \boxed{\text{SAR } 95{,}366.80}
\end{aligned}
$$

Problem 3(c)

Split the **13th payment** into its **interest** and **principal** parts.

Problem 3(c): Solution

The balance just before payment 13 is $B_{12} = 95{,}366.80$:

$$\text{interest}_{13} = 0.01 \times 95{,}366.80 = \boxed{953.67}$$

$$\text{principal}_{13} = 8{,}473.23 - 953.67 = \boxed{7{,}519.56}$$

Each payment $=$ interest on the current balance $+$ principal that reduces it.

Problem 3(d)

The **24th (final) payment** is made at the end of **year 2**. If inflation averages **5% per year**, find that payment's value in **today's-value money**.

Problem 3(d): Solution

The payment is an **actual** amount at year 2; deflate to today at $\bar{f} = 5\%$:

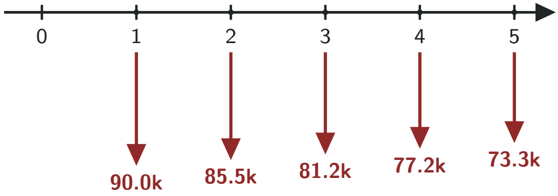
$$\begin{aligned} A_2' &= 8{,}473.23\,(P/F, 5\%, 2) \\ &= 8{,}473.23\,(0.9070) = \boxed{\text{SAR } 7{,}685.46} \end{aligned}$$

The cheque is still 8,473.23, but its *buying power* is about 7,685 in today's prices.

Problem 4(a)

A maintenance cost is **90,000** at the end of year 1 and *decreases* by **5% per year** through year 5. Draw a labeled cash flow diagram of the five costs.

Problem 4(a): Solution



Costs are down-arrows, each $0.95\times$ the previous: 90,000, then 85,500, 81,225, 77,164, 73,306. This is a **geometric** series with $g = -5\%$.

Problem 4(b)

At $i = 12\%$, find the **present worth** of that maintenance-cost series.

Problem 4(b): Solution

Geometric series, $A_1 = 90{,}000$, $g = -5\%$, $i = 12\%$, $N = 5$ (use the formula, $i \neq g$):

$$P = A_1 \frac{1 - (1+g)^N (1+i)^{-N}}{i - g} = 90{,}000 \frac{1 - (0.95)^5 (1.12)^{-5}}{0.12 - (-0.05)}$$
$$= 90{,}000 \frac{1 - 0.43882}{0.17} = \boxed{\text{SAR } 296{,}966}$$

With g negative, $i - g = 0.17$ in the denominator — subtracting a negative *adds*.

Problem 4(c)

Over three years the cobalt price changed by **+6%**, then **+9%**, then −4%. Find the **average annual inflation** rate.

Problem 4(c): Solution

Geometric mean of the yearly factors:

$$\bar{f} = [(1.06)(1.09)(0.96)]^{1/3} - 1$$
$$= (1.10923)^{1/3} - 1 = \boxed{3.51\%/\mathrm{yr}}$$

Problem 4(d)

At market rate $i = 10\%$ and inflation $\bar{f} = 6\%$, compare two 4-year plans: **Plan X** pays **40,000/yr in today's-value money**; **Plan Y** pays **43,000/yr in actual money**. Find each present worth and say which is cheaper today.

Problem 4(d): Solution

Real rate: $i' = \dfrac{0.10 - 0.06}{1.06} = 3.774\%$ (by formula). Pair each plan with its rate:

$$P_X = 40{,}000\,(P/A, 3.774\%, 4) = 40{,}000(3.6493) = \mathbf{145{,}974}$$
$$P_Y = 43{,}000\,(P/A, 10\%, 4) = 43{,}000(3.1699) = \mathbf{136{,}304}$$

In the actual exam, we will give you the P/A factors.

Problem 4(e)

A fund must pay **75,000/yr of today's purchasing power, forever**, at $i = 9\%$ and $\bar{f} = 2\%$. Find the **deposit required now**.

Problem 4(e): Solution

Constant-value perpetuity, so divide by the **real** rate:

$$i' = \frac{0.09 - 0.02}{1.02} = 6.863\%$$
$$P = \frac{A'}{i'} = \frac{75{,}000}{0.06863} = \boxed{\text{SAR } 1{,}092{,}857}$$

Dividing by the market rate ($75{,}000/0.09 = 833{,}333$) would underfund it — the classic mistake.

Multiple Choice Questions
(Major 1 Problem Set A)

MCQ 1

A series of receipts is **800** at year 1 and *increases* by **150** each year through year 5. At $i = 10\%$, which expression gives its present worth?

(A) $950\,(P/A, 10\%, 5)$

(B) $800\,(P/A, 10\%, 5) + 150\,(P/A, 10\%, 5)$

(C) $800\,(P/A, 10\%, 5) + 150\,(P/G, 10\%, 5)$

(D) $800\,(P/A, 10\%, 5) + 150\,(P/G, 10\%, 4)$

MCQ 1: Solution

Decompose: a **flat base of 800** plus a **gradient of 150** (first flow 0):

$$P = 800\,(P/A, 10\%, 5) + 150\,(P/G, 10\%, 5)$$

Answer: (C). The gradient uses (P/G) over the *same* $N = 5$. (B) wrongly uses (P/A) for the gradient; (D) uses the wrong N.

(Note: this evaluates to about 4,062 — but the exam only asks for the *expression*.)

MCQ 2

Four equal deposits of **5,000** are made in years 1–4 (8% annually). The fund is then emptied by **4 equal withdrawals** C in years 5–8. Which expression gives C?

(A) $C = 5{,}000$

(B) $C = 5{,}000\,(P/A, 8\%, 4)/(F/A, 8\%, 4)$

(C) $C = 5{,}000\,(A/F, 8\%, 4)(P/A, 8\%, 4)$

(D) $C = 5{,}000\,(F/A, 8\%, 4)/(P/A, 8\%, 4)$

MCQ 2: Solution

Build the balance at year 4, then spread it over the next 4 years:

$$\text{balance at yr } 4 = 5{,}000\,(F/A, 8\%, 4)$$

$$\text{this equals} \quad C\,(P/A, 8\%, 4) \;\Rightarrow\; C = \frac{5{,}000\,(F/A, 8\%, 4)}{(P/A, 8\%, 4)}$$

Answer: (D). Deposits accumulate to a balance at year 4; that balance is the present worth of the withdrawals that start at year 5.

MCQ 3

A payment of **75,000** will be made in **actual money** at the end of year 5. At average inflation **4%/yr**, which expression gives its value in **today's-value money**?

(A) $75{,}000\,(F/P, 4\%, 5)$
(B) $75{,}000\,(P/F, 4\%, 5)$
(C) $75{,}000\,(P/F, 4\%, 4)$
(D) 75,000

MCQ 3: Solution

Actual $\rightarrow$ today's value means **deflate**: multiply by $(1+\bar{f})^{-5}$, i.e. $(P/F, 4\%, 5)$:

$$A'_5 = 75{,}000\,(P/F, 4\%, 5)$$

Answer: (B). (A) inflates (wrong way); (C) uses the wrong N; (D) ignores inflation.

MCQ 4

The Consumer Price Index (CPI) measures price changes mainly from the perspective of the:

(A) seller of goods
(B) buyer (consumer) of goods
(C) government tax authority
(D) central bank only

MCQ 4: Solution

Answer: (B), the buyer. The CPI tracks the cost of a fixed basket of goods from the *consumer's* side, not the seller's.

MCQ 5

"Purchasing power" is best described as:

(A) the total amount of money a person holds
(B) the interest rate a bank pays on deposits
(C) the quantity of goods one unit of money can buy
(D) the rate at which prices rise each year

MCQ 5: Solution

Answer: (C). Purchasing power is the *quantity of goods* one unit of money can buy — not the amount held, not a rate.

(A) confuses power with quantity of money; (B) and (D) are rates, not buying power.

Before the Exam

1. Re-solve every part on these slides **without looking** at the solution
2. For each, write 4 steps: **diagram, factor notation, substitution, final answer**
3. Check the **pairing rule** and the **rate-per-period** conversions
4. Practice **under time**: 75 minutes is tight if you calculate factors instead of using the tables
5. Try also Major 1 Problem Set B in Blackboard

Bring your questions to class! We'll work these problems together and answer any questions.