ISE 307: Engineering Economic Analysis

Lecture 10: Inflation, Real Money Value, and Equivalence (Ch 4)

Term 253 (Summer)

Instructor: Mansur M. Arief

Industrial and Systems Engineering, KFUPM

Office: 22-219 — Email: mansur.arief@kfupm.edu.sa

Lecture 10

Measuring Inflation, Real vs. Actual Money Value,
and Equivalence under Inflation

Lecture 10 Objectives

1. Explain what the **CPI** measures and its buyer's perspective
2. Compute **year-by-year** and **average** inflation rates from a price index
3. Convert between **actual** value and **constant** (real) value of money
4. Relate the **market rate** i, **real rate** i', and **inflation** $\bar{f}$
5. Apply the **pairing rule**: actual value uses i, constant value uses i'
6. Value a **constant-value perpetuity**

Measuring Inflation

A **price index** (e.g. CPI) tracks the cost of a goods over time, from the *buyer's* side.

$$\text{Year-}n\text{ rate: } f_n = \frac{\mathrm{CPI}_n - \mathrm{CPI}_{n-1}}{\mathrm{CPI}_{n-1}}$$

$$\begin{aligned}\text{Average: } \bar{f} &= \left(\frac{\mathrm{CPI}_N}{\mathrm{CPI}_0}\right)^{1/N} - 1 \\ &= \left(\prod_{i=1}^{N}(1+f_i)\right)^{1/N} - 1\end{aligned}$$

The **index** is a *level*; the **inflation rate** is its *year-to-year change*. You need two index values to get one rate.

Year-by-Year Inflation: A Worked Index

An index reads **100**, then **106**, then **116.6**, then **114.3** over three years. Each year's rate is the change over the previous level:

$$f_1 = \frac{106-100}{100} = \boxed{+6\%}$$
$$f_2 = \frac{116.6-106}{106} = \boxed{+10\%}$$
$$f_3 = \frac{114.3-116.6}{116.6} = \boxed{-2\%}$$

The third year *fell*, so its rate is negative. These are the three rates we average next.

(Geometric) Average Inflation

The average rate $\bar{f}$ is the single rate that, applied for N years, gives the same total growth. The yearly factors *multiply*:

$$\left(1+\bar{f}\,\right)^N = (1+f_1)(1+f_2)\cdots(1+f_N) = \frac{\text{CPI}_N}{\text{CPI}_0}$$

Using the rates $+6\%$, $+10\%$, -2%:

$$(1+\bar{f})^3 = (1.06)\,(1.10)\,(0.98) = 1.14268$$

$$\underbrace{\bar{f} = \left[(1.14268)^{1/3} - 1\right]}_{\text{previous equation}} = \boxed{4.55\%/\mathrm{yr}}$$

Not the arithmetic mean $(6+10-2)/3 = 4.67\%$. Rates compound, so factors multiply.

Actual vs. Constant Value

- **Actual** value A_n: the cash that actually changes hands in year n
- **Constant** value A'_n: the same flow in *today's* buying power

Deflate: $A'_n = A_n\,(1+\bar{f}\,)^{-n}$ Inflate: $A_n = A'_n\,(1+\bar{f}\,)^{n}$

An offtake contract pays **60,000 SAR in year 3** (actual). At $\bar{f} = 4\%$:

$$A'_3 = 60{,}000\,(1.04)^{-3} = \boxed{\text{SAR 53,340 in today's buying power}}$$

The buyer hands over 60,000, but it buys what 53,340 buys today.

Reverse direction (inflate): $A_3 = 53{,}340\,(1.04)^3 = \boxed{\text{SAR 60,000}}$ — exact opposite.

Inflation as a Tax on Idle Cash

Buying power is the (qty of goods)/(unit of money). Idle cash loses it at rate $\bar{f}$.

A 60,000 SAR reserve left untouched for 5 years at $\bar{f} = 4\%$:

$$\text{real value} = 60{,}000\,(1.04)^{-5} = \boxed{\text{SAR } 49{,}316 \text{ of today's power}}$$

Doing *nothing* loses about $60{,}000 - 49{,}316 = 10{,}684$ in buying power over five years.

Money has time value for **two** reasons:

- earning power (interest)
- buying power (inflation)

We combine them into one rate i in Ch 2 and the rest of this chapter.

Even Invested Cash Grows Only at the Real Rate

Now invest that **60,000 SAR** at a market rate of **6%** for 5 years, with $\bar{f} = 4\%$:

$$\text{actual balance} = 60{,}000\,(1.06)^5 = \boxed{\text{SAR } 80{,}294}$$

$$\text{buying power} = 80{,}294\,(1.04)^{-5} = \boxed{\text{SAR } 65{,}995}$$

The balance *looks* like it grew 6%, but buying power grew only at the **real rate**.

Idle cash is just the extreme case:
market rate $= 0$, so the real rate is negative and buying power falls.

Real Rate, Market Rate, and Inflation Rate

$$1 + i = (1 + i')(1 + \bar{f}\,)$$
$$i = i' + \bar{f} + i'\bar{f}$$
$$i' = \frac{i - \bar{f}}{1 + \bar{f}}$$

- i (**market** rate): what banks quote — carries *both* earning power and inflation
- i' (**real** / inflation-free rate): growth in true buying power
- e.g. $i = 9\%$, $\bar{f} = 4\%$ $\;\Rightarrow\;$ $i' = \dfrac{0.05}{1.04} = \boxed{4.81\%}$ $\quad$ ($\textit{not}$ $9 - 4 = 5\%$)
- again $i = 12\%$, $\bar{f} = 7\%$ $\;\Rightarrow\;$ $i' = \dfrac{0.05}{1.07} = \boxed{4.67\%}$ $\quad$ ($\textit{not}$ $12 - 7 = 5\%$)
- $i > i'$ whenever $\bar{f} > 0$; only deflation makes $i' \geq i$

The Pairing Rule

Cash flows in ...	**Discount at ...**	**Why**
Actual value	market rate i	i already carries inflation
Constant value	real rate i'	inflation already stripped out

- Both give the **same** present worth.
- Mixing them (constant value at i, or actual value at i') **double-counts** or **ignores** inflation.

Choose one: state which value you're in, use the matching rate, and proceed.

Both Approaches Give the Same Answer

A payment of **104** in **actual** value at year 1; $i = 9\%$, $\bar{f} = 4\%$, so $i' = 4.81\%$.

Approach 1 — actual value at the market rate:

$$P = \frac{104}{1.09} = \boxed{95.41}$$

Approach 2 — convert to constant, then real rate: $104/1.04 = 100$

$$P = \frac{100}{1.0481} = \boxed{95.41}$$

Same present worth, exactly. The identity guarantees the two approaches agree.

Breaking the Pairing Costs You

The correct present worth is **95.41**. See what each mismatch does:

$$\text{constant at market: } \frac{100}{1.09} = \boxed{91.74} \quad \text{(too low — inflation removed twice)}$$

$$\text{actual at real: } \frac{104}{1.0481} = \boxed{99.23} \quad \text{(too high — inflation never removed)}$$

The two wrong routes land on *opposite* sides of the right answer.

Procedure: (1) label each flow; (2) pick one route; (3) convert mismatched flows; (4) discount with the matching rate.

Comparing Two Offtake Contracts

Two 3-year supply contracts; $i = 8\%$, $\bar{f} = 5\%$, so $i' = 0.03/1.05 = 2.857\%$.

- **A:** 100k/yr (**constant**) $\Rightarrow$ $P_A = 100{,}000\,(P/A,\, 2.857\%,\, 3) = \boxed{283{,}640}$
- **B:** 105k/yr (**actual**) $\Rightarrow$ $P_B = 105k\,(P/A,\, 8\%,\, 3) = 105k(2.57) = \boxed{270{,}600}$

$P_A - P_B \approx 13{,}000$: the "smaller" constant-value contract wins.

Decision: "A; its inflation-protected payments worth ≈13k more."

Why Contract A Wins

Contract A is fixed in **constant** value, so its **actual** cheques rise with inflation (5%):

Actual cheque	**Year 1**	**Year 2**	**Year 3**
Contract A (100,000 constant)	105,000	110,250	115,763
Contract B (105,000 actual)	105,000	105,000	105,000

A and B start equal, but A's cheques climb while B's stay flat. That rising stream is what makes A worth about 13,000 more today.

Constant-Value Perpetuity

$$\text{Fixed perpetuity: } P = \frac{A}{i} \text{ (market rate).}$$

$$\text{Constant-value perpetuity: } P = \frac{A'}{i'} \text{ (real rate).}$$

A fund pays **3,000,000 SAR/yr** of *today's* buying power, forever, for mineral-research training; $i = 8\%$, $\bar{f} = 3\%$:

$$i' = \tfrac{0.05}{1.03} = 4.854\% \quad \Rightarrow \quad P = \frac{3{,}000{,}000}{0.04854} = \boxed{\text{SAR 61.8 million}}$$

Dividing by the *market* rate $(3{,}000{,}000/0.08 = 37.5\text{M})$ funds a stream whose real value decays $3\%/\text{yr}$ — the classic mistake.

Practice Problem 1

A published price index for a refinery's inputs reads **150.0** now and **178.65** three years from now. Find the average annual inflation rate over the three years.

Use the geometric form (the index ratio is the product of the yearly factors):

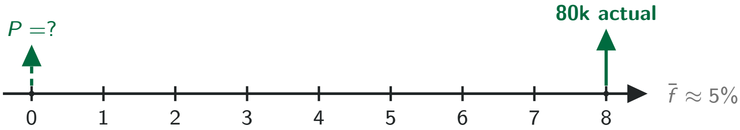
$$(1+\bar{f})^3 = \frac{\mathrm{CPI}_3}{\mathrm{CPI}_0} = \frac{178.65}{150.0} = 1.1910$$

$$\bar{f} = (1.1910)^{1/3} - 1 = \boxed{6.00\%/\mathrm{yr}}$$

Practice Problem 2

Supply prices rose **+4%**, **+5%**, then **+6%** in 3 years. A supply contract pays **80,000 SAR** in **actual money at the end of year 8**. Find its value in today's buying power.

$$\bar{f} = [(1.04)(1.05)(1.06)]^{1/3} - 1 = (1.1575)^{1/3} - 1 = 4.9968\% \approx \boxed{5\%}$$



Deflate the actual amount with the rounded rate (5% is a table rate):

$$P = 80{,}000\,(P/F,\,5\%,\,8) = 80{,}000\,(0.6768) = \boxed{\text{SAR } 54{,}144}$$

Practice Problem 3

Two 5-year supply plans, with market rate $i = 6\%$ and inflation $\bar{f} = 1\%$.

$$i' = \frac{i - \bar{f}}{1 + \bar{f}} = \frac{0.06 - 0.01}{1.01} = 4.95\% \approx \boxed{5\%} \text{ (round for the table)}$$

- **Plan X:** 30,000/yr in **constant** value $\Rightarrow$
  $P_X = 30{,}000\,(P/A,\,5\%,\,5) = 30{,}000(4.3295) = \boxed{\text{SAR } 129{,}885}$
- **Plan Y:** 32,000/yr in **actual** value $\Rightarrow$
  $P_Y = 32{,}000\,(P/A,\,6\%,\,5) = 32{,}000(4.2124) = \boxed{\text{SAR } 134{,}797}$

Constant value uses the real rate; actual value uses the market rate.
Plan X is cheaper today by about 4,900.

Quick Check

A payment of **90,000 SAR** will be made in **actual money** at year 5. With average inflation of 4% per year, which expression gives its value in **today's buying power**?

(A) $90{,}000\,(F/P,\,4\%,\,5)$

(B) $90{,}000\,(P/F,\,4\%,\,4)$

(C) $90{,}000$

(D) $90{,}000\,(P/F,\,4\%,\,5)$

Answer: (D). Going from actual money to today's buying power means deflating: multiply by $(1+\bar{f})^{-5}$, which is $(P/F,\,4\%,\,5)$.

Summary

- Avg inflation: $(1+\bar{f})^N = (1+f_1)\cdots(1+f_N) = \mathrm{CPI}_N/\mathrm{CPI}_0$ (geometric mean)
- **Actual** value/dollar = cash that changes hands
- **Constant** value/dollar = today's buying power; convert with $(1+\bar{f})^{\pm n}$
- Idle cash loses buying power at rate $\bar{f}$
- Identity: $1+i = (1+i')(1+\bar{f})$, so $i' = (i-\bar{f})/(1+\bar{f})$
- **Pairing:** discount actual value at market i, constant value at real i'
- Constant-value perpetuity: $P = A'/i'$

Before Class

To do before our class meeting:

1. **Read Sections 4.1–4.4** (inflation measures; equivalence under inflation)
2. Complete the **Lecture 10 pre-class quiz** on Blackboard before class
3. Try this: $i = 10\%$, $\bar{f} = 6\%$ — compute i' exactly, and check it is not 4%

In class: We will practice a few problems for Major 1 Exam preparation. Please be in-class for this session.