ISE 307: Engineering Economic Analysis

Lecture 9: Loans — Payment, Balance, and the Add-On (Ch 3)

Term 253 (Summer)

Instructor: Mansur M. Arief

Industrial and Systems Engineering, KFUPM

Office: 22-219 — Email: mansur.arief@kfupm.edu.sa

Lecture 9

Amortized Loans, Remaining Balances, and the Add-On

Lecture 9 Objectives

1. Recap: **effective rate per payment period**
2. Compute the **payment** on an amortized loan with $(A/P, i, N)$
3. Find the **remaining balance** after any number of payments
4. Split a payment into **interest** and **principal**
5. Calculate the true cost of an **add-on loan**

Recap: Effective Rate per Payment Period

When payments and compounding don't match, calculate:

Effective rate **per payment period**:

$$i = \left(1 + \frac{r}{M}\right)^{C} - 1 \qquad C = \frac{M}{K}$$

K = payments/yr, M = compoundings/yr, C = compoundings/payment.

Example: pay **quarterly** (K=4), compounds **monthly** ($M = 12$), at $r = 8\%$:

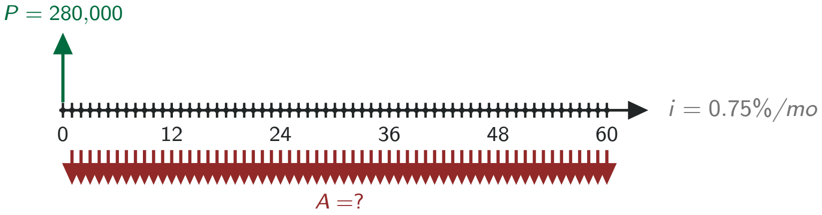
$$C = \frac{M}{K} = \frac{12}{4} = 3 \qquad i = \left(1 + \tfrac{0.08}{12}\right)^{3} - 1 = \boxed{2.0134\% \text{ per quarter}}$$

Once you have i, every chapter-2 factor works, with N = **number of payments**.

Amortized Loan: Finding the Payment

Amortized means equal payments that pay off both interest and principal over time

A plant finances a **280,000 SAR** lithium flotation cell line at **9% compounded monthly**, repaid over **60 monthly** payments.



Periods match (pay monthly, compound monthly): $i = 9\%/12 = 0.75\%$, $N = 60$.

$$A = 280{,}000\,(A/P,\,0.75\%,\,60) = 280{,}000\,(0.020758) = \boxed{\text{SAR } 5{,}812.34/\text{mo}}$$

Splitting a Payment: Interest vs. Principal

Each payment A covers interest on what you *still owe*; the rest pays down principal.

$$\text{Interest:} \quad I_t = i \cdot B_{t-1}$$
$$\text{Principal:} \quad PP_t = A - I_t$$

First payment ($B_0 = 280{,}000, \quad i = 0.75\%$):

$$I_1 = 0.0075\,(280{,}000) = 2{,}100 \quad \Rightarrow \quad PP_1 = 5{,}812 - 2{,}100 = \boxed{3{,}712}$$

Early payments are **interest-heavy**; **later payments become principal-heavy**.

Remaining Balance: Value What's Left

Balance after t = present worth of the **remaining** payments:

$$B_t = A\,(P/A,\, i,\, N - t)$$

Balance right after the 24th payment (36 payments remain):

$$\begin{aligned} B_{24} &= 5{,}812\,(P/A,\, 0.75\%,\, 36) \\ &= 5{,}812\,(31.4468) \\ &= \boxed{\text{SAR } 182{,}780} \end{aligned}$$

The Add-On Loan

A vendor offers a **50,000 SAR** haul-truck retrofit at "just **6% add-on**, 36 months."
Add-on charges interest on the **full** principal for the **whole** term:

$$\text{Owed} = 50{,}000\,(1 + 0.06 \times 3) = 59{,}000 \quad \Rightarrow \quad A = 59{,}000/36 = 1{,}638.89/\mathrm{mo}$$

True rate? Find i making 50,000 today equivalent to that payment stream:

$$(P/A,\, i,\, 36) = \frac{50{,}000}{1{,}638.89} = 30.508 \;\Rightarrow\; i \approx 0.92\%/\mathrm{mo} \;\Rightarrow\; \boxed{i_a \approx 11.66\%}$$

The "6%" add-on actually costs ≈**11.7% effective** — nearly double. You keep paying interest on principal you've already repaid.

Summary

- When the periods differ, use the effective rate per payment: $i = (1 + \frac{r}{M})^C - 1$
- **Balance:** $B_t = A\,(P/A,\, i,\, N - t)$ = present worth of remaining
- **Payment:** $A = P\,(A/P,\, i,\, N)$, with i per payment period
- **Split:** interest $I_t = i\, B_{t-1}$; principal $PP_t = A - I_t$
- Early payments are interest-heavy; later payments are principal-heavy
- **Add-on loans** charge interest on the full principal throughout; effective cost $\approx 2\times$ the quoted rate

Before Class

To do before our class meeting:

1. **Read Sections 3.4–3.5** (loan amortization; add-on loans)
2. Complete the **Lecture 9 pre-class quiz** on Blackboard before class
3. Try this: a **60,000 SAR** loan at **1% per month**, **24 payments** — find A and the balance after 12 payments

In class: We will practice comparing loans and different payment schedules for your group projects.