ISE 307: Engineering Economic Analysis

Lecture 6: Equal-Payment Series — (A/P), (P/A), Deferred Annuities

Term 253 (Summer)

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Lecture 6

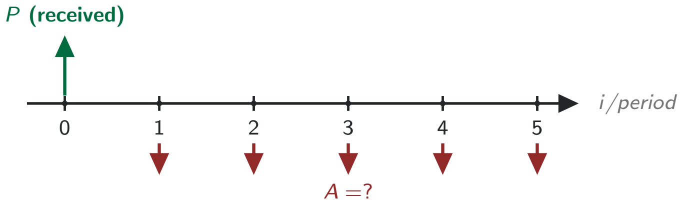
Equal-Payment Series, Part 2: Capital Recovery, Present Worth, and Deferred Annuities

Lecture 6 Objectives

1. Convert P into equal installments with **capital recovery** $(A/P, i, N)$
2. Value a series today with the **present worth factor** $(P/A, i, N)$
3. Price a **perpetuity** (payments forever): $P = A/i$
4. Handle a **deferred annuity**: a series that starts after a delay

Capital Recovery Factor $(A/P, i, N)$

Loans, leases, and machine ownership

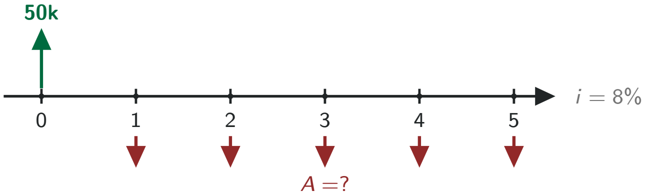


$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = P\,(A/P,\,i,\,N)$$

- You **receive** P today and **repay** it in N equal installments
- (A/P) also gives the annual cost of **owning** a machine bought for P

Example: Financing a Conveyor

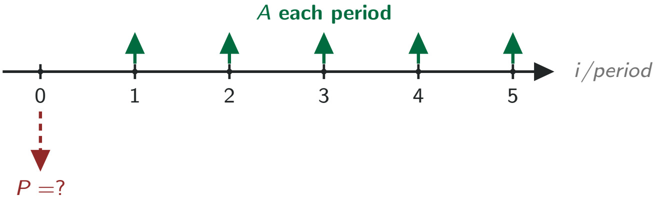
50,000 SAR financed at 8% over 5 years — find the payment



- Set up: $A = P\,(A/P, 8\%, 5) = 50{,}000\,(A/P, 8\%, 5)$
- Read the row $N = 5$ in the 8% table: $(A/P, 8\%, 5) = 0.25046$
- Multiply: $A = 50{,}000 \times 0.25046 = \boxed{12{,}523 \text{ SAR/yr}}$

Five payments of 12,523 total 62,615 SAR. The extra 12,615 is the interest cost of borrowing.

Present Worth Factor $(P/A, i, N)$



$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A\,(P/A,\ i,\ N)$$

Perpetuity $(N \to \infty)$: $\quad P = \dfrac{A}{i}$

- Read (P/A) as the **value today** of N equal future receipts
- (P/A) and (A/P) are **exact inverses** — one factor family, two directions

Interest Factor Table at $i = 8\%$

Now with the present-worth pair (P/A) and (A/P)

N	(P/A)	(A/P)
1	0.9259	1.08000
2	1.7833	0.56077
3	2.5771	0.38803
4	3.3121	0.30192
5	**3.9927**	**0.25046**
6	4.6229	0.21632
8	5.7466	0.17401
10	6.7101	0.14903
12	7.5361	0.13270

How to use it:

- Read (P/A) to **price** a stream today
- Read (A/P) to turn a loan into a **payment**

Note: (A/P) *is the reciprocal of* (P/A) *on the same row — check:* $1/3.9927 = 0.25046$.

Example: Pricing Two Income Streams

$i = 8\%$ throughout

- A maintenance contract pays a fund **9,000 SAR/yr for 10 years**:

$$P = 9{,}000\,(P/A,\ 8\%,\ 10) = 9{,}000\,(6.7101) = \boxed{60{,}391 \text{ SAR}}$$

- An endowment pays a lab **12,000 SAR/yr forever** (a perpetuity):

$$P = \frac{A}{i} = \frac{12{,}000}{0.08} = \boxed{150{,}000 \text{ SAR}}$$

A finite 10-year stream of 9,000 is worth 60,391 today; the *infinite* 12,000 stream is worth only 150,000. Distant payments worth little.

Lump Sum vs. Installments

An off-take buyer offers 60,000 now or 8,000/yr for 12 years

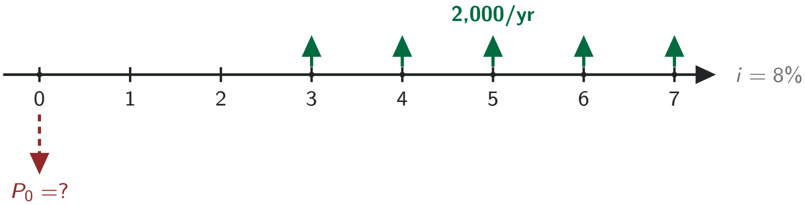
Value the installment stream today and compare with the 60,000 SAR lump sum:

i	$(P/A, i, 12)$	$P = 8{,}000 \times (P/A)$	verdict
6%	8.3838	67,071 SAR	installments win
8%	**7.5361**	**60,289 SAR**	$\approx$ **indifferent**
10%	6.8137	54,510 SAR	lump sum wins

Decision rule: if the firm's money earns **more than** $\approx$**8%**, take the 60,000 now; if **less**, take the installments. The "bigger total" (96,000) is meaningless without timing.

Deferred Annuity

A series that starts late.



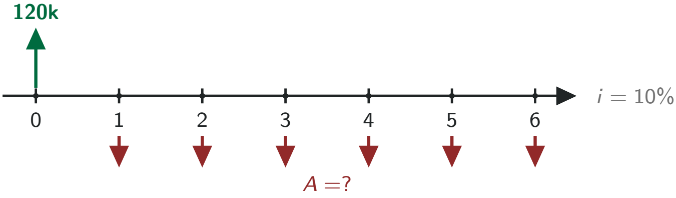
An equipment lease pays **2,000 SAR/yr at the ends of years 3–7**. Value it today in two steps:

1. (P/A) parks the value **one period before the first payment** (at $n = 2$):
   $P_2 = 2{,}000\,(P/A,\,8\%,\,5) = 2{,}000\,(3.9927) = 7{,}985$
2. (P/F) discounts that single amount home:
   $P_0 = 7{,}985\,(P/F,\,8\%,\,2) = 7{,}985\,(0.8573) = \boxed{6{,}846\ \mathrm{SAR}}$

Practice Problem 1

Loan payment (solve for A)

A robotics startup borrows 120,000 SAR at $i = 10\%$ and repays it in 6 equal year-end installments. Find the annual payment.



Apply the capital recovery factor:

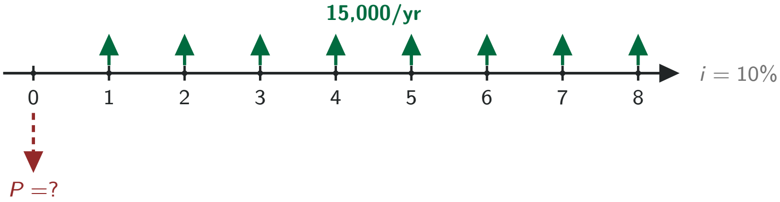
$$A = 120{,}000\,(A/P,\,10\%,\,6) = 120{,}000\,(0.22961)$$

$$= \boxed{\text{SAR } 27{,}553 \text{ per year}}$$

Practice Problem 2

Price a stream, then compare

A supplier will pay a refinery 15,000 SAR/yr for 8 years at $i = 10\%$. A competitor offers 85,000 SAR today instead. Which is worth more?



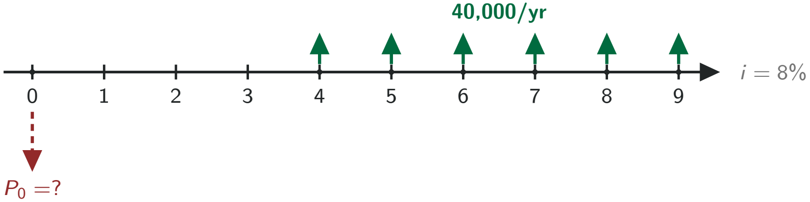
Value the stream today with (P/A):

$$P = 15{,}000\,(P/A,\ 10\%,\ 8) = 15{,}000\,(5.3349)$$
$$= 80{,}024\ \mathrm{SAR}\ <\ 85{,}000 \Rightarrow \boxed{\text{take the 85,000 today}}$$

Practice Problem 3

Deferred annuity (value a delayed stream)

A grant pays a robotics lab 40,000 SAR/yr at the ends of years 4–9 at $i = 8\%$. What is the grant worth today?



Park at $n = 3$ with (P/A), then discount 3 years with (P/F):

$$P_3 = 40{,}000\,(P/A,\ 8\%,\ 6) = 40{,}000\,(4.6229) = 184{,}916$$

$$P_0 = 184{,}916\,(P/F,\ 8\%,\ 3) = 184{,}916\,(0.7938) = \boxed{\text{SAR } 146{,}808}$$

Summary

- $(A/P, i, N)$ turns a loan P into equal installments A — **capital recovery**
- $(P/A, i, N)$ prices an N-payment stream today
- (P/A) and (A/P) are **reciprocals**
- **Perpetuity** (payments forever): $P = A/i$
- **Deferred annuity**: compute (P/A) before first A; finish with (P/F)
- Lump sum vs. installments: compare on a **common date**, not by total

Before Class

To do before our class meeting:

1. **Finish Section 2.3** (capital recovery through deferred annuities)
2. Complete the **Lecture 6 pre-class quiz** on Blackboard before class
3. Try this: *a 5-year 20,000 SAR loan at 7% — find the annual payment*

In class: we'll do a practice problem based on your team projects on financing equipment and pricing off-take streams. See you in class.