ISE 307: Engineering Economic Analysis

Lecture 5: Equal-Payment Series — (F/A) and (A/F) (Chapter 2)

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Instructor: Mansur M. Arief

Industrial and Systems Engineering, KFUPM

Office: 22-219 — Email: mansur.arief@kfupm.edu.sa

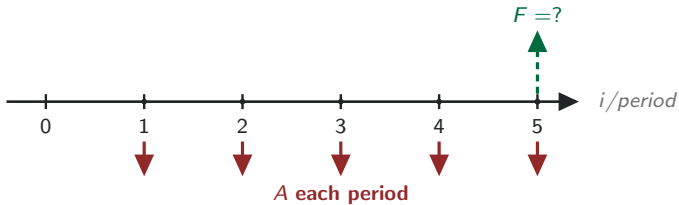
Lecture 5

Equal-Payment Series, Part 1: Future Worth and Sinking Funds

Lecture 5 Objectives

1. Recognize a **uniform (equal-payment) series** and its diagram
2. Use **equal-payment compound factor** $(F/A, i, N)$ to find F
3. Use **sinking fund factor** $(A/F, i, N)$ to calculate A for a given F
4. Read (F/A) and (A/F) from the **interest factor tables**

The Uniform (Equal-Payment) Series



- A is an **equal** amount at the end of **every** period, $n = 1$ to N
- **First payment lands at** $n = 1$, not at $n = 0$ (end-of-period convention)
- F is measured at the **same instant as the last payment** ($n = N$)

Almost every series error comes from misplacing the **first payment** or the **measurement point of** F.

Equal-Payment Compound Amount Factor $(F/A, i, N)$

Each deposit compounds for its own time

The deposit at $n = 1$ compounds $N - 1$ periods; the one at $n = N$ compounds zero:

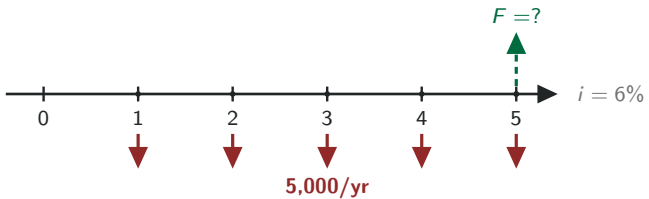
$$F = A(1+i)^{N-1} + A(1+i)^{N-2} + \cdots + A(1+i) + A$$

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A\,(F/A,\ i,\ N)$$

- $(F/A, i, N)$ is shorthand for the bracket — read as "find F given A"
- Tabulated in the interest factor tables (Appendix B), or computed in Excel

Example: A Robotics Reserve Fund

5,000 SAR deposited at each year-end for 5 years, $i = 6\%$. Find F!



- Write the formula in factor notation: $F = A\,(F/A,\ 6\%,\ 5) = 5{,}000\,(F/A,\ 6\%,\ 5)$
- Look up (or compute) the factor: $(F/A,\ 6\%,\ 5) = \dfrac{(1.06)^5 - 1}{0.06} = 5.6371$
- Multiply: $F = 5{,}000 \times (F/A,\ 6\%,\ 5) = 5{,}000 \times 5.6371 = \boxed{\text{SAR } 28{,}185.46}$

Only **25,000 SAR** was deposited. The extra **3,185 SAR** is the compound interest.

Interest Factor Table at $i = 8\%$

N	(F/P)	(F/A)	(A/F)
1	1.0800	1.0000	1.00000
2	1.1664	2.0800	0.48077
3	1.2597	3.2464	0.30803
4	1.3605	4.5061	0.22192
5	**1.4693**	**5.8666**	**0.17046**
6	1.5869	7.3359	0.13632
8	1.8509	10.6366	0.09401
10	2.1589	14.4866	0.06903

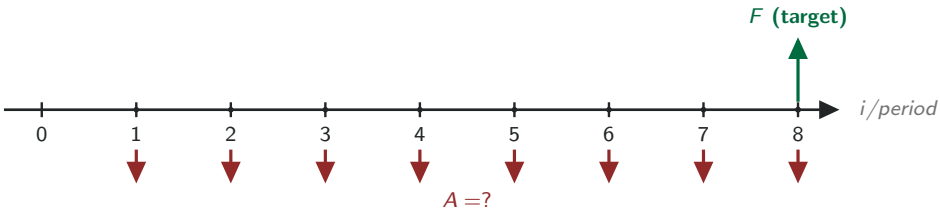
How to use it:

- Pick the table for $i = 8\%$
- Find the row for your N
- Read (F/A) to grow a series, (A/F) to size a deposit

Note: (A/F) *is the reciprocal of* (F/A) *on the same row — check:* $1/5.8666 = 0.17046$.

Sinking Fund Factor $(A/F, i, N)$

Work backwards from the target

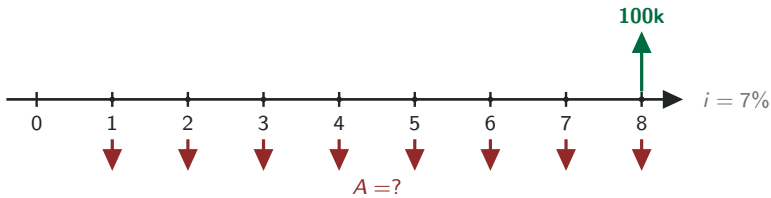


$$A = F\left[\frac{i}{(1+i)^N - 1}\right] = F\,(A/F,\ i,\ N)$$

- A **sinking fund** is a planned series of deposits that accumulates to a known future need
- Just the inverse of (F/A) — same table row, reciprocal column

Example: Funding an Overhaul

A haul-truck fleet needs 100,000 SAR in 8 years, $i = 7\%$. Find A!



- Set up: $A = F\,(A/F,\,7\%,\,8) = 100{,}000\,(A/F,\,7\%,\,8)$
- Read the row $N = 8$ in the 7% table: $(A/F,\,7\%,\,8) = 0.09747$
- Multiply: $A = 100{,}000 \times 0.09747 = \boxed{\text{SAR } 9{,}747/\text{yr}}$

Eight deposits of 9,747 (total 77,976) grow into 100,000, with interest covers the remaining 22,024 SAR.

Why Starting Early Wins

Two R&D funds each deposit 6,000 SAR/yr for **10 years** at 8%, then stop and let the balance ride to year 40:

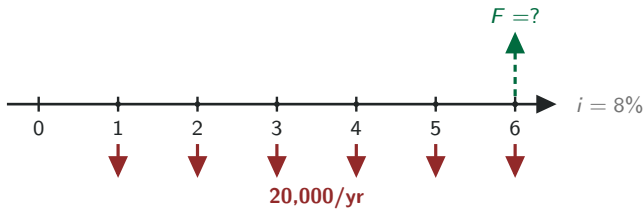
	Deposits during	Balance at year 40 (SAR)
Early fund	years 1–10	$6{,}000\,(F/A, 8\%, 10)(F/P, 8\%, 30) = \mathbf{874{,}640}$
Late fund	years 31–40	$6{,}000\,(F/A, 8\%, 10) = \mathbf{86{,}919}$

Same 60k deposited, but a $\mathbf{10}\times$ difference. Timing is the most powerful variable in $(F/A)(F/P)$.

Practice Problem 1

Find F of a series

A lithium-refinery training budget deposits 20,000 SAR at each year-end for 6 years at $i = 8\%$. What is the balance right after the 6th deposit?



Apply the equal-payment compound amount factor:

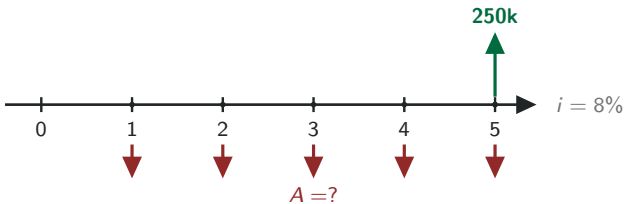
$$F = 20{,}000\,(F/A,\ 8\%,\ 6) = 20{,}000\,(7.3359)$$

$$= \boxed{\text{SAR } 146{,}718}$$

Practice Problem 2

Size a sinking-fund deposit (solve for A)

A flotation-cell replacement will cost 250,000 SAR in 5 years. The reserve account earns $i = 8\%$. What equal end-of-year deposit fully funds it?



Apply the sinking fund factor:

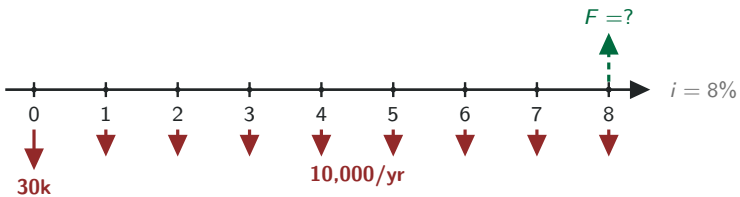
$$A = 250{,}000\,(A/F,\ 8\%,\ 5) = 250{,}000\,(0.17046)$$

$$= \boxed{\text{SAR } 42{,}615 \text{ per year}}$$

Practice Problem 3

A series plus a single existing balance

An automation fund already holds 30,000 SAR today and adds 10,000 SAR at each year-end for 8 years at $i = 8\%$. Find the balance at year 8.



Carry the lump sum with (F/P), the series with (F/A), then add:

$$F = 30{,}000\,(F/P,\ 8\%,\ 8) + \ 10{,}000\,(F/A,\ 8\%,\ 8)$$

$$= 30{,}000\,(1.8509) + 10{,}000\,(10.6366) = \boxed{\text{SAR } 161{,}893}$$

Summary

- A **uniform series**: equal A at the end of each period, **first at** $n = 1$; F sits **at** $n = N$
- $(F/A, i, N) = \dfrac{(1+i)^N - 1}{i}$: **future worth** of a series, $F = A\,(F/A, i, N)$
- $(A/F, i, N) = \dfrac{i}{(1+i)^N - 1}$: the **sinking fund** deposit, $A = F\,(A/F, i, N)$
- (A/F) and (F/A) are **reciprocals** on the same table row
- Mixed flows: value the **lump sum** and the **series** separately, then **add**

Before Class

To do before our class meeting:

1. **Read Section 2.3** (equal-payment series, through the sinking fund factor)
2. Complete the **Lecture 5 pre-class quiz** on Blackboard before class
3. Try this: *what equal yearly deposit at 8% for 10 years builds 150,000 SAR?*

In class: we'll do some practice problems built on your team projects. See you in class!