ISE 307: Engineering Economic Analysis

Lecture 4: Economic Equivalence & Single Cash Flows (Chapter 2)

Term 253 (Summer)

Instructor: Mansur M. Arief, PhD

Industrial and Systems Engineering, KFUPM

Office: 22-219 — Email: mansur.arief@kfupm.edu.sa

Lecture 4

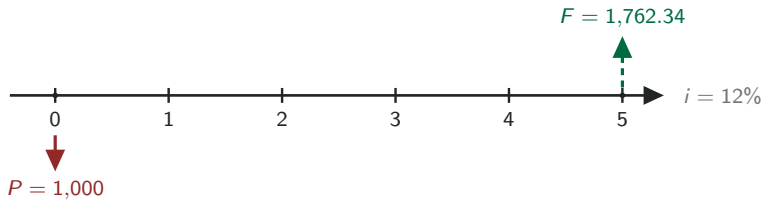
Economic Equivalence and Interest Formulas for Single Cash Flows

Lecture 4 Objectives

1. Explain what **economic equivalence** means
2. Use the **compound amount factor** $(F/P, i, N)$ to find F given P
3. Use the **present worth factor** $(P/F, i, N)$ to find P given F
4. Solve the **interest rate** i or **number of periods** N (incl. Rule of 72)

Economic Equivalence

Definition: two cash flows (or cash flow series) are **economically equivalent** if they have the **same value** at a common point in time, for a given interest rate i.



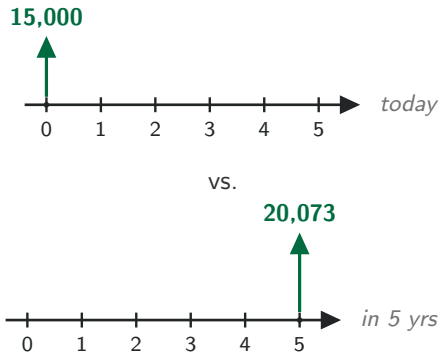
- At 12%, 1,000 SAR *now* is equivalent to 1,762.34 SAR *in 5 years*:

$$1{,}000(1.12)^5 = 1{,}762.34$$

- If two proposals are equivalent, we are **economically indifferent**
- That's why the bank doesn't care which loan-repayment plan we choose!

Finding the Equivalence Rate

15,000 SAR today or 20,073 SAR in five years?



At what i are these equivalent?

Set the value of 15,000 grown for 5 years equal to 20,073:

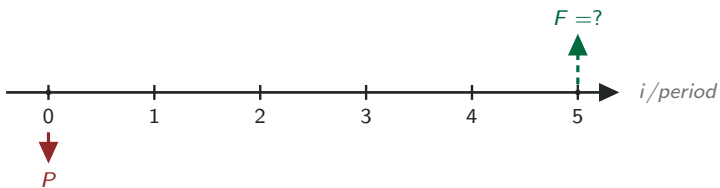
$$15{,}000(1+i)^5 = 20{,}073$$
$$(1+i)^5 = 1.338$$
$$1+i = 1.338^{1/5} = 1.06$$
$$i = \boxed{6\%}$$

Can we **for sure earn more** than 6%?

- If yes, take 15,000 now.
- If no, wait for 20,073.

Compound Amount Factor $(F/P, i, N)$

Find F, given P, i, N



$$F = P(1+i)^N = P\,(F/P,\ i,\ N)$$

- $(F/P, i, N)$ is shorthand for $(1+i)^N$ — read as "find F given P"
- Values are tabulated in the **interest factor tables** (Appendix B), or use Excel

Interest Factor Table at $i = 7\%$

N	$(F/P, 7\%, N)$	$(P/F, 7\%, N)$
1	1.0700	0.9346
2	1.1449	0.8734
3	1.2250	0.8163
4	1.3108	0.7629
5	1.4026	0.7130
6	1.5007	0.6663
7	1.6058	0.6227
8	**1.7182**	**0.5820**
9	1.8385	0.5439
10	1.9672	0.5083

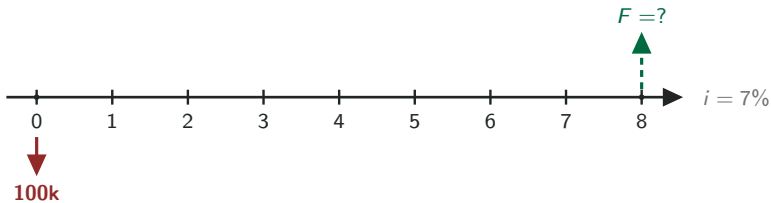
How to use it:

- Pick the table for rate $i = 7\%$
- Find row for $N = 8$ periods
- Read the column you need: (F/P) or (P/F)
- Record the factor for your use

Note: (P/F) *entries are reciprocals of the* (F/P) *entries on the same row — check:* $1/1.7182 = 0.5820$.

Example: Using the Table

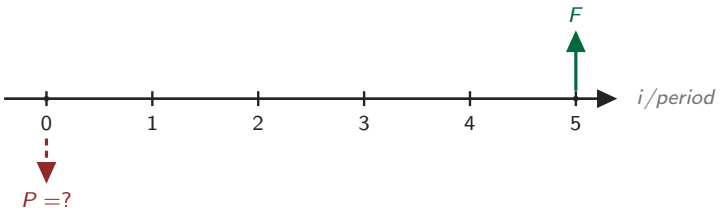
A robotics R&D fund of 100,000 SAR at $i = 7\%$ for $N = 8$ years — find F



- Write the formula in factor notation:
  $F = P\,(F/P,\, i,\, N) = 100{,}000\,(F/P,\, 7\%,\, 8)$
- Look up the row $N = 8$ in the 7% table: $(F/P,\, 7\%,\, 8) = 1.7182$
- Multiply: $F = 100{,}000 \times 1.7182 = \boxed{171{,}820\ \mathrm{SAR}}$

Present Worth Factor $(P/F, i, N)$

Find P, given F, i, N — "discounting"



$$P = F(1+i)^{-N} = F\,(P/F,\ i,\ N)$$

- Just the inverse of $(F/P, i, N)$; finding P from F is **discounting**
- **Example:** a plant worth 2,000,000 SAR in 5 years at $i = 9\%$ is worth $P = 2{,}000{,}000\,(1.09)^{-5} = \boxed{1{,}299{,}863\ \mathrm{SAR}}$ today

Using the Factor Table $(P/F,\ 9\%,\ 5)$

N	$(F/P, 9\%, N)$	$(P/F, 9\%, N)$
1	1.0900	0.9174
2	1.1881	0.8417
3	1.2950	0.7722
4	1.4116	0.7084
5	**1.5386**	**0.6499**
6	1.6771	0.5963
7	1.8280	0.5470
8	1.9926	0.5019
9	2.1719	0.4604
10	2.3674	0.4224

Read row $N = 5$, column (P/F):

$$(P/F,\ 9\%,\ 5) = 0.6499$$

Apply to the plant example:

$$P = 2{,}000{,}000 \times 0.6499$$
$$= \boxed{\text{SAR } 1{,}299{,}800}$$

Solving for N or i

$F = P(1+i)^N$ has **four** quantities — given any three, solve for the fourth:

Solve for N (take logs):

$$N = \frac{\ln(F/P)}{\ln(1+i)}$$

doubling at 6%: $N = \frac{\ln 2}{\ln 1.06} \approx 11.9$ yrs

Solve for i (take roots):

$$i = \left(\frac{F}{P}\right)^{1/N} - 1$$

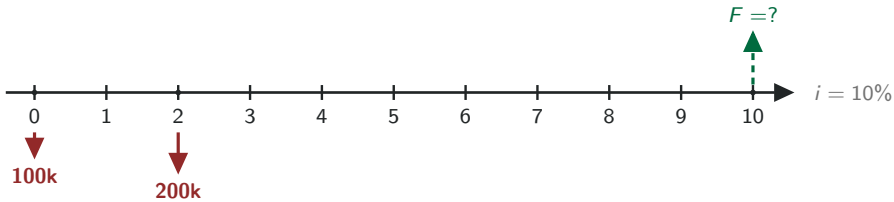
e.g. 1,000 $\to$ 1,200 in 2 yrs:
$i = 1.2^{1/2} - 1 = 9.54\%$

Rule of 72: years to double $\approx \dfrac{72}{\text{interest rate in \%}}$

e.g. at 12%: $72/12 = 6$ years

Practice Problem 1

A minerals sovereign fund deposits 100,000 SAR now and 200,000 SAR two years from now at $i = 10\%$. How much at the end of year 10?

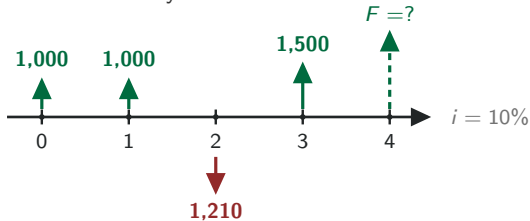


Each flow compounds for its own number of periods:

$$
\begin{aligned}
F &= 100{,}000\,(F/P, 10\%, 10) + 200{,}000\,(F/P, 10\%, 8) \\
&= 100{,}000\,(1.10)^{10} + 200{,}000\,(1.10)^{8} \\
&= 259{,}374 + 428{,}718 = \boxed{\text{SAR } 688{,}092}
\end{aligned}
$$

Practice Problem 2

A project reserve account earns $i = 10\%$. With the deposits and withdrawals below, what is the balance at the end of year 4?



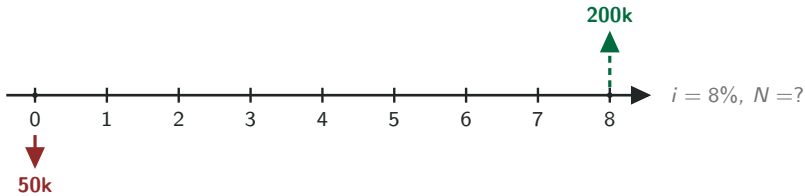
Treat each flow as its own (F/P) problem and sum (deposits $+$, withdrawals $-$):

$$
\begin{aligned}
F &= 1{,}000(1.1)^4 + 1{,}000(1.1)^3 - 1{,}210(1.1)^2 + 1{,}500(1.1)^1 \\
&= 1{,}000 \times 1.4641 + 1{,}000 \times 1.3310 - 1{,}210 \times 1.21 + 1{,}500 \times 1.1 \\
&= 1{,}464.10 + 1{,}331.00 - 1{,}464.10 + 1{,}650.00 \\
&= \boxed{\text{SAR } 2{,}981}
\end{aligned}
$$

Practice Problem 3

Solve for N — how long until the fund quadruples (multiplies by four)?

A petrochemical training endowment deposits 50,000 SAR at $i = 8\%$ compounded annually. How many years until the balance reaches 200,000 SAR?



Start from $F = P(1+i)^N$, take logs:

$$N = \frac{\ln(F/P)}{\ln(1+i)} = \frac{\ln(200{,}000/50{,}000)}{\ln(1+0.08)} = \frac{\ln 4}{\ln 1.08}$$

$$= \frac{1.3863}{0.07696} = \boxed{18.01 \text{ years}}$$

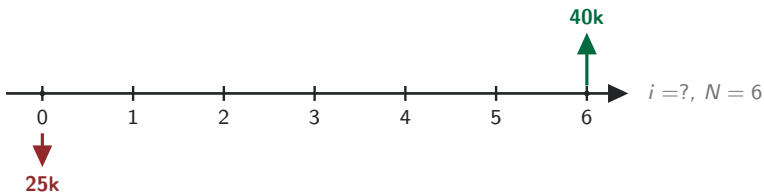
Rule of 72 sanity check:

- Doubling at 8% takes $\approx 72/8 = 9$ years
- Quadrupling = doubling twice ≈ 18 years ✓

Practice Problem 4

Solve for i (what effective rate did the fund earn?)

An engineering scholarship fund invests 25,000 SAR and grows to 40,000 SAR after 6 years. What effective annual interest rate i did the fund earn?



Start from $F = P(1+i)^N$, take the Nth root:

$$i = \left(\frac{F}{P}\right)^{\frac{1}{N}} - 1 = \left(\frac{40{,}000}{25{,}000}\right)^{\frac{1}{6}} - 1$$

$$= \frac{1.6^{\frac{1}{6}}}{1} - 1 = 1.0815 - 1 = \boxed{8.15\%}$$

Summary

- **Economic equivalence:** cash flows with the same value at a common time, at rate i — equivalence $\Rightarrow$ indifference
- $(F/P, i, N) = (1+i)^N$: **compounding** forward, $F = P(F/P, i, N)$
- $(P/F, i, N) = (1+i)^{-N}$: **discounting** backward, $P = F(P/F, i, N)$
- Given any 3 of $\{P, F, i, N\}$, solve for the 4th; **Rule of 72** for doubling time
- Uneven flows: convert **each flow separately** to the target date, then **add**

Before Class

To do before our class meeting:

1. **Read Sections 2.2–2.3** of the textbook (Park, 3rd Ed.)
2. Complete the **Lecture 4 pre-class quiz** on Blackboard before class
3. Try this: *what P today is indifferent to 150,000 SAR in 7 years at 6%?*

In class: we'll do timed practice problems based on your projects and pre-class assignments to help you prepare for the quiz next week and Major 1 exam. Please be ready and bring your calculator and your notes.