ISE 307: Engineering Economic Analysis

Lecture 3: Time Value of Money — Part 1 (Chapter 2)

Term 253 (Summer)

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Lecture 3

Time Value of Money — Part 1: Interest and Cash Flow Diagrams

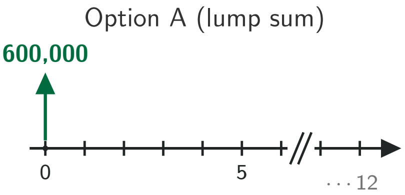
Lecture 3 Objectives

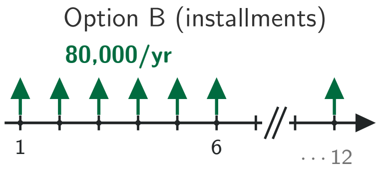
1. Explain why money has a **time value**
2. Identify the **elements** of a transaction involving interest
3. Plot a **cash flow diagram** and recognize its components
4. Calculate **simple** and **compound** interest, and explain the difference

Lump Sum or Installments?

A battery maker will buy refined lithium from your plant. They offer two ways to pay for the multi-year contract:

- **Option A:** pay **600,000 SAR** now, or
- **Option B:** pay **960,000 SAR** as 12 annual installments of 80,000 SAR





How to compare 600k SAR today vs. 960k SAR over 12 years?

Why Does Money Have a Time Value?

A sum of money's value depends on **(1) the amount** and **(2) when it arrives:**

1. **Earning power:** money can earn more money over time (invest it in a project, earn a return)
2. **Purchasing power:** what money can buy changes over time (inflation)

Quick check — which would you take?

- 200 SAR today or 150 SAR today? easy
- 100 SAR today or 100 SAR after 2 years? easy — earning power!
- 200 SAR today or 225 SAR after 2 years? **need a method!**

Time value of money is measured by the **interest rate**.

Interest: The Cost of Money

Interest is the cost of money:

- a **cost** to the borrower, and an **earning** to the lender

Why do lenders charge it? The rental price of money covers:

- Risk of loss · Administrative expenses · Pure gain/profit
- Risk of inflation · Opportunity cost

Earning vs. inflation (investing 100 SAR for 6% earning in a magnet company):

Case	**Account at $n=1$**	**Magnet costs**	**Outcome**
Inflation 8% $>$ earning 6%	106 SAR	108 SAR	losing money
Inflation 4% $<$ earning 6%	106 SAR	104 SAR	profitting

Elements of a Transaction Involving Interest

Symbol	Name	Meaning
P	Principal / present amount	initial amount of the transaction (at $n = 0$)
i	Interest rate	cost or price of money, % per period
N	Number of periods	duration of the transaction
A_n	Cash flow at period n	plan of receipts or disbursements over time
F	Future amount	cumulative effect of interest over N periods

Standing assumptions (unless stated otherwise):

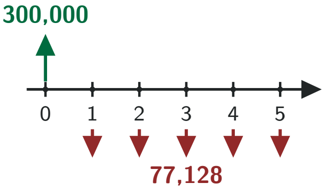
- interest rates are **market interest rates** (earning power *and* inflation)
- cash flows are in **actual riyals** (inflation already reflected in the amounts)

A 300,000 SAR Processing-Line Loan

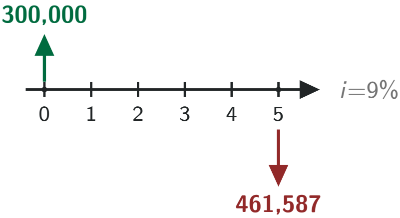
A minerals firm borrows **300,000 SAR** at **9%** annual interest to install a lithium processing line (plus a 3,000 SAR processing fee at n=0). The bank offers:

End of year	Plan 1 (installments)	Plan 2 (deferment)
0	3,000 SAR	3,000 SAR
1–4	77,127.66 SAR/yr	0
5	77,127.66 SAR	461,587.18 SAR

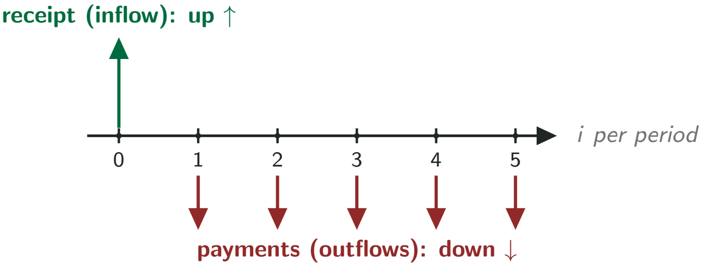
Plan 1



Plan 2



The Cash Flow Diagram



Reading the diagram:

- Horizontal axis = **time**, divided into interest periods ($n = 0, 1, \ldots, N$)
- **Up arrows** = cash inflows (receipts); **down arrows** = outflows (disbursements)
- Arrow lengths are *schematic*: longer ≈ larger amount
- Drawn from **one perspective** (borrower's or lender's — flip all arrows to switch)
- **End-of-period convention:** flows within a period are placed at its *end*

Two Ways to Calculate Interest

Simple interest

- interest **only on the principal**
- earned interest does not earn more interest

$$F = P(1 + iN)$$

Compound interest

- interest charged **on principal and accumulated interest**
- "interest on interest"

$$F = P(1 + i)^N$$

Engineering economic analysis uses **compound interest exclusively**, *which is the common practice in the real world.*

Simple Interest: Example

$P = 1{,}000$ *SAR*, $i = 8\%$, $N = 3$ *years*

Year	Begin	Interest	End
0			1,000.00
1	1,000.00	80.00	1,080.00
2	1,080.00	80.00	1,160.00
3	1,160.00	80.00	**1,240.00**

Interest is **80 SAR every year**: always 8% of the original 1,000.

Formula:

$$F = P(1 + iN)$$

$$F = 1{,}000\,[1 + (0.08)(3)]$$

$$F = \boxed{1{,}240 \text{ SAR}}$$

Compound Interest: Example

Same amount: $P = 1{,}000$ *SAR*, $i = 8\%$, $N = 3$ *years*

Year	Begin	Interest	End
0			1,000.00
1	1,000.00	80.00	1,080.00
2	1,080.00	86.40	1,166.40
3	1,166.40	93.31	**1,259.71**

Interest **grows each year**: 8% of an ever-larger balance.

Build-up:

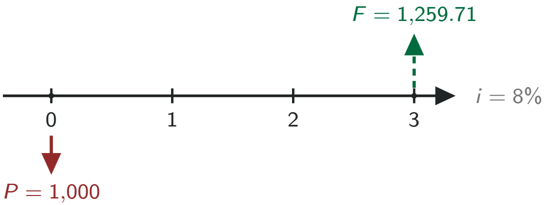
$$n{=}1 : F_1 = P(1+i)$$
$$n{=}2 : F_2 = F_1(1+i) = P(1+i)^2$$
$$n{=}N : F = P(1+i)^N$$

$$F = 1{,}000\,(1.08)^3 = \boxed{1{,}259.71}$$

Compound beats simple by 19.71 SAR over 3 years.

Compounding as a Cash Flow Diagram

$F = P(1+i)^N$



- Deposit (outflow) of 1,000 SAR at $n = 0$; withdrawal (inflow) of F at $n = 3$
- The diagram is drawn from **your** perspective as the depositor
- $(1+i)^N$ is the **growth factor** linking P to F

Next lecture: this growth factor becomes our first *interest formula*.

Summary

- Money has a **time value** because of **earning power** and **inflation**; it is measured by the **interest rate**
- Transactions are described by P, i, N, A_n, and F
- A **cash flow diagram** shows time on the axis, **inflows up**, **outflows down**, with the **end-of-period convention**
- **Simple interest:** $F = P(1 + iN)$ — interest on principal only
- **Compound interest:** $F = P(1 + i)^N$ — interest on interest

Before Class

To do before our class meeting:

1. **Read Section 2.1** of the textbook (Park, 3rd Ed.)
2. Complete the **Lecture 3 pre-class quiz** on Blackboard before class starts
3. Try this by hand: *a robotics fund holds 50,000 SAR at 6% simple interest — how many years to double? What if it's 5% compounded annually?*

In class: we'll practice drawing cash flow diagrams and compare simple vs. compound growth in a spreadsheet, and discuss how this apply in your projects.